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Résumé. — La population des bandes rotationnelles du niveau fondamental des noyaux déformés pairs ^{166,168}Yb, ^{170,172,174}Hf et ^{178,180}W a été observée en étudiant des réactions induites par des protons de 18 jusqu'à 57 MeV. On a analysé en détail la désexcitation des noyaux dans le cadre du modèle statistique et on peut affirmer que les résultats de l'expérience sont bien reproduits.

On a déterminé les valeurs moyennes des moments d'inertie des noyaux résiduels à 6 MeV d'excitation et on les trouve inférieures à 50 % des valeurs correspondantes pour une sphère rigide. Cette détermination est en bon accord avec les prévisions du modèle de Lang et Le Couteur.

Abstract. — The population of ground state rotational bands in the even-even deformed nuclides ^{166,168}Yb, ^{170,172,174}Hf, and ^{178,180}W has been observed in reactions induced by 18 to 57 MeV protons. Detailed calculations are performed on the basis of the statistical model of the compound-nucleus-reaction de-excitation process, and it is concluded that the experimental results can be satisfactorily explained.

The mean moments of inertia of the nuclei produced in the reactions are deduced, for an estimated residual excitation energy of 6 MeV, and found to amount to no more than 50 % of the corresponding rigid sphere figures. The obtained values are compared, and shown to be in agreement, with the previsions of the independent pairing model of Lang and Le Couteur.

1. Introduction. — It is by now well established that the study of the population of angular momentum states in compound-nucleus reactions can give valuable information on nuclear structure and reaction mechanisms. A number of papers dealing with various aspects of this subject have appeared, and to introduce the relevant literature we may quote the recent article by J. O. Newton *et al.* [1] where an extensive bibliography can be found.

In the present contribution we report new experimental data on the population of the ground state rotational band (gsb) in the doubly even nuclides ^{166,168}Yb, ^{170,172,174}Hf and ^{178,180}W, produced by proton bombardment of ^{169}Tm, ^{175}Lu and ^{181}Ta. Then we calculate, in the framework of the statistical model [2], the spin distribution of the gsb states reached at the end of the de-excitation process of the compound-nucleus reaction products. By comparing the calculated results with the experimental data, we deduce the spin cut-off parameters, and therefrom the moments of inertia of the product nuclei at an average excitation energy of some 6 MeV.

It is worth recalling that while in general the develop-ment of this work follows the line used in analysing isomeric ratios in reactions of similar type [3], the observation of the relative intensities of lines excited in gs rotational bands provides broader (and often more homogeneous) information than that obtainable from the study of isomers.

2. Experimental method and results. — The gamma rays accompanying proton induced reactions at various incident energies in ^{169}Tm, ^{175}Lu and ^{181}Ta have been measured by the in-beam technique.

The external proton beams of the AVF cyclotrons in Milan and Grenoble were used : the first for the energy interval from 18 to 44 MeV, the second from 38 to 57 MeV. Care was taken to minimize any difference in experimental conditions in the two locations, whose layouts are sketched in figure 1. The beam energy was varied in steps of 2 MeV in the Milan runs, and of 3 MeV in Grenoble ; the energy spread was less than 300 keV fwhm. The beams were focussed into spots of 5 mm diameter inside small scattering chambers.

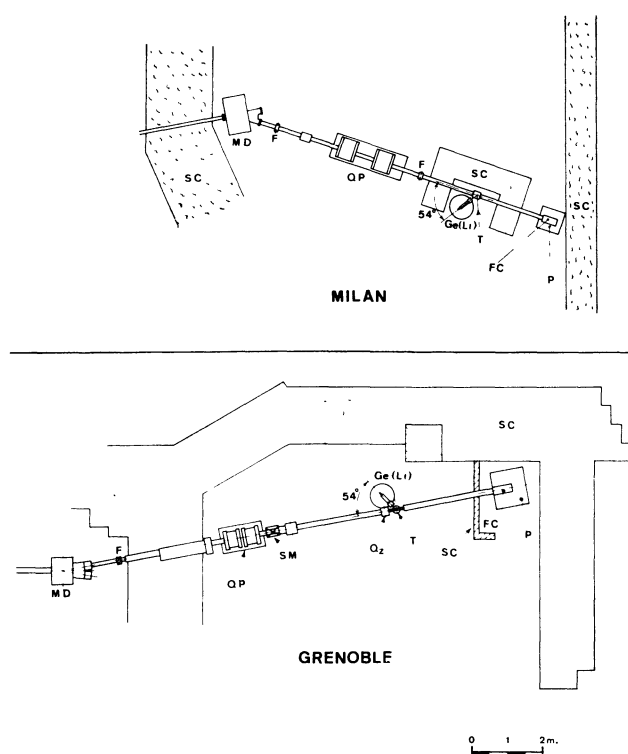To reduce the neutron background, the beam

FIG. 1. — Experimental layouts in Milan and in Grenoble. SC, concrete shielding ; MD, deflecting magnet ; F, slit ; QP, quadrupole ; SM, steering magnet ; Q_z, quartz probe ; T, target ; FC, Faraday cup ; P, paraffin absorber.

defining slits and the Faraday cup have been placed far from the target, at a distance greater than 5.5 and 2 m respectively ; both were shielded by concrete walls. Low beam currents of about 1 nA were used. Self-supporting foils of natural Tm (100 % ^{169}Tm), Lu (97.4 % ^{175}Lu) and Ta (99.9 % ^{181}Ta) with thicknesses of 15, 24 and 40 mg.cm^{-2} were used as targets. The gamma rays were detected in 35 and

10 cm^3 Ge(Li) counters, having a resolution of 2.3 keV at 600 keV. The counters were placed at an angle of about 54° with respect to the beam, in order to infer from measurements at only one angle the absolute populations of the gsb levels [4].

The data considered in the present paper regard the determination of the intensities of gamma rays arising from J^+-$(J - 2)^+$ transitions between states of the gsb of the product nuclei of the reactions ^{169}Tm, ^{175}Lu, ^{181}Ta(p, 2 n), (p, 4 n) and ^{175}Lu(p, 6 n). Examples of typical gamma spectra are shown in figures 2 and 3. The gamma rays of interest were identified by their energies, as known from the literature ; the correctness of the attributions was verified, when needed, by inspecting the pertinent excitation functions. This type of test proved quite useful also to discriminate against radiation from competing reactions. The transition intensities were obtained from the observed gamma intensities corrected for electron conversion by means of the coefficients tabulated by Sliv and Band [5], and for self-absorption in the targets by direct measurements with calibrated sources.

The results are summarized in table I and figures 4 to 10, where for the seven reactions studied are reported the energies of the gsb transitions (Table I) and, in function of the incident proton energies, the intensities divided by the yields of the $J = 4^+$-$J = 2^+$ transitions (Fig. 4 to 10). Notice, however, that in the case of the ^{181}Ta(p, 2 nγ)^{180}W reaction (Fig. 9) the 4^+-2^+ transition intensity could not be reliably measured, due to the superposition of the 233 keV gamma-peak of ^{179}W produced in the competing reaction ^{181}Ta(p, 3 nγ). Therefore the ^{180}W transition intensities are expressed as ratios to the 2^+-0^+ transition intensity. Also reported is the intensity of the strong gamma transition from the $J = 8^-$ 5.2 ms isomeric state at 1.525 MeV to the $J = 8^+$ gsb state in ^{180}W.

The precision of the intensity ratios is estimated

TABLE I

Measured energies (keV) *of ground state rotational band transitions. Errors are less than* ± 0.4 keV *when a digit after the decimal point is given, and less than* ± 1.0 keV *in all other cases*

Reaction	$(2^+$-$0^+)$	$(4^+$-$2^+)$	$(6^+$-$4^+)$	$(8^+$-$6^+)$	$(10^+$-$8^+)$	$(12^+$-$10^+)$
^{169}Tm(p, 2 nγ)^{168}Yb	87.9 (^{a})	199.1	299.0	384	—	—
^{169}Tm(p, 4 nγ)^{166}Yb	102	228	338	430	508 (^{b})	569
^{175}Lu(p, 2 nγ)^{174}Hf	91.0 (^{a})	206.5	311.0	401.0	476	
^{175}Lu(p, 4 nγ)^{172}Hf	95	213.5	318	408.5	483	543
^{175}Lu(p, 6 nγ)^{170}Hf	100.0 (^{c})	221	320	400	462	
^{181}Ta(p, 2 nγ)^{180}W	103.4	233.3	350	449.0		
^{181}Ta(p, 4 nγ)^{178}W	105.9	236.3	352	446.5		

(^{a}), (^{b}), (^{c}) Reliable measurements of gamma intensities could not be made for these transitions due to :

(^{a}) superposition of peaks from competing (p, 3 nγ) reactions ;

(^{b}) closeness to the 511 keV β^+ annihilation peak ;
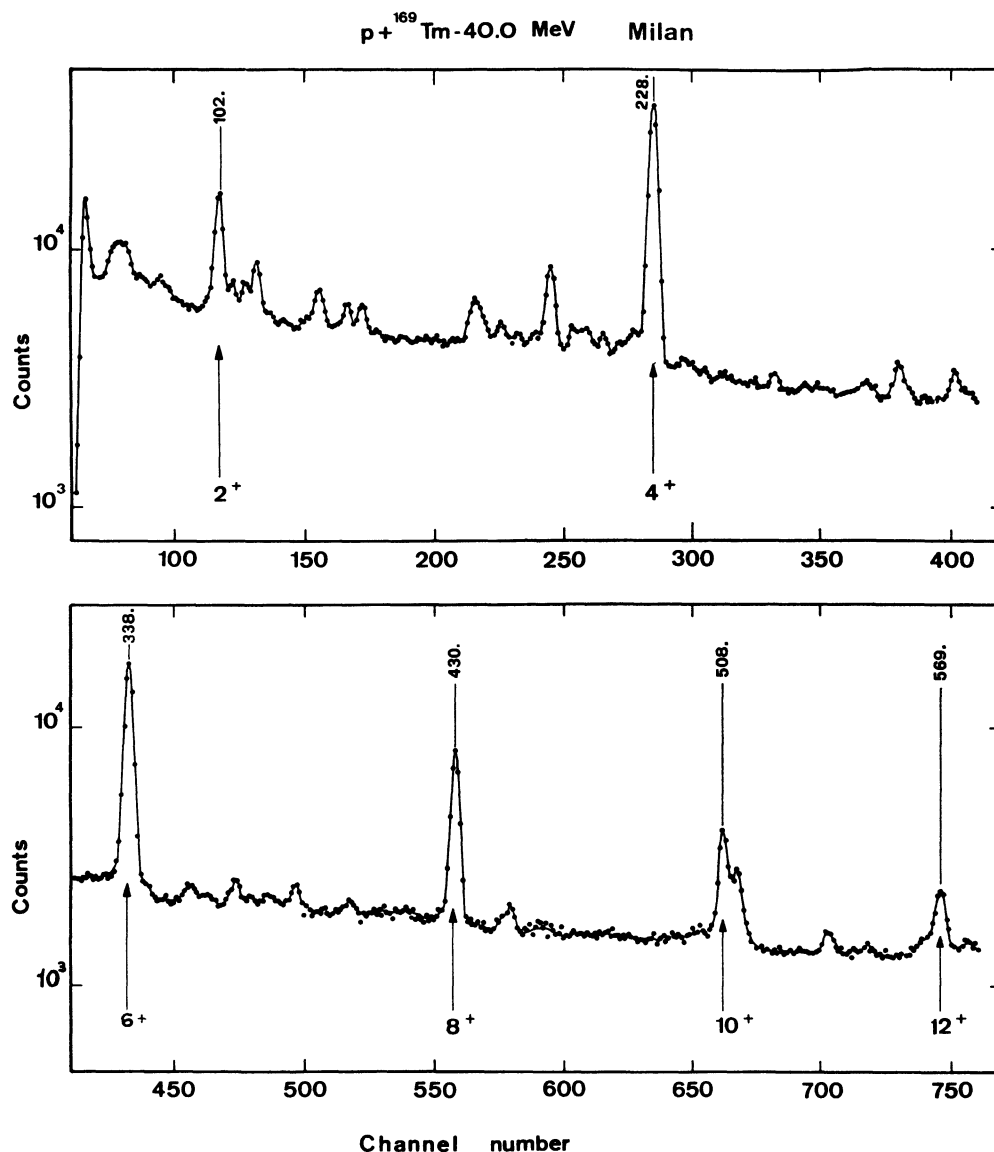
(^{c}) occasional failure in determination of detector efficiency.

FIG. 2. — Gamma ray spectrum from bombardment of [169]Tm with 40.0 MeV protons.
The peaks whose energies are indicated are assigned to transitions from gsb levels in [166]Yb.

to be generally of the order of 10 % ; in very unfavourable cases, e. g. for some measurements near reaction thresholds, the uncertainties can reach up to about 20 %.

3. **Calculation procedures.** — It is assumed, as usual [2], that the reactions considered here can be described with the statistical model. For the (p, 6 ny) reaction and for the (p, 4 ny) reactions produced by incident particles of energy lower than, say, 42 MeV, this assumption is probably quite safe. For the (p, 2 ny) reactions at $E_p \gtrsim 25$ MeV, and for the (p, 4 ny) at $E_p \gtrsim 42$ MeV, it can be reasonably expected, however, that the first neutron is emitted in a pre-equilibrium process [6], rather than statistically

evaporated. We believe that this effect can influence but slightly the main results of a statistical model treatment for the group of reactions here examined. A short discussion of this point will be presented in section 4.1.

The development of the calculation is outlined below. The de-excitation process of the coumpound-nucleus (CN) reactions is supposed to take place in three steps. The nuclear thermal energy is removed first by neutron evaporation. There follows, when the excitation energy of the product nucleus falls below the neutron binding energy, an essentially continuous emission of gamma rays. The de-excitation process is then completed with the emission of low energy characteristic gamma rays.
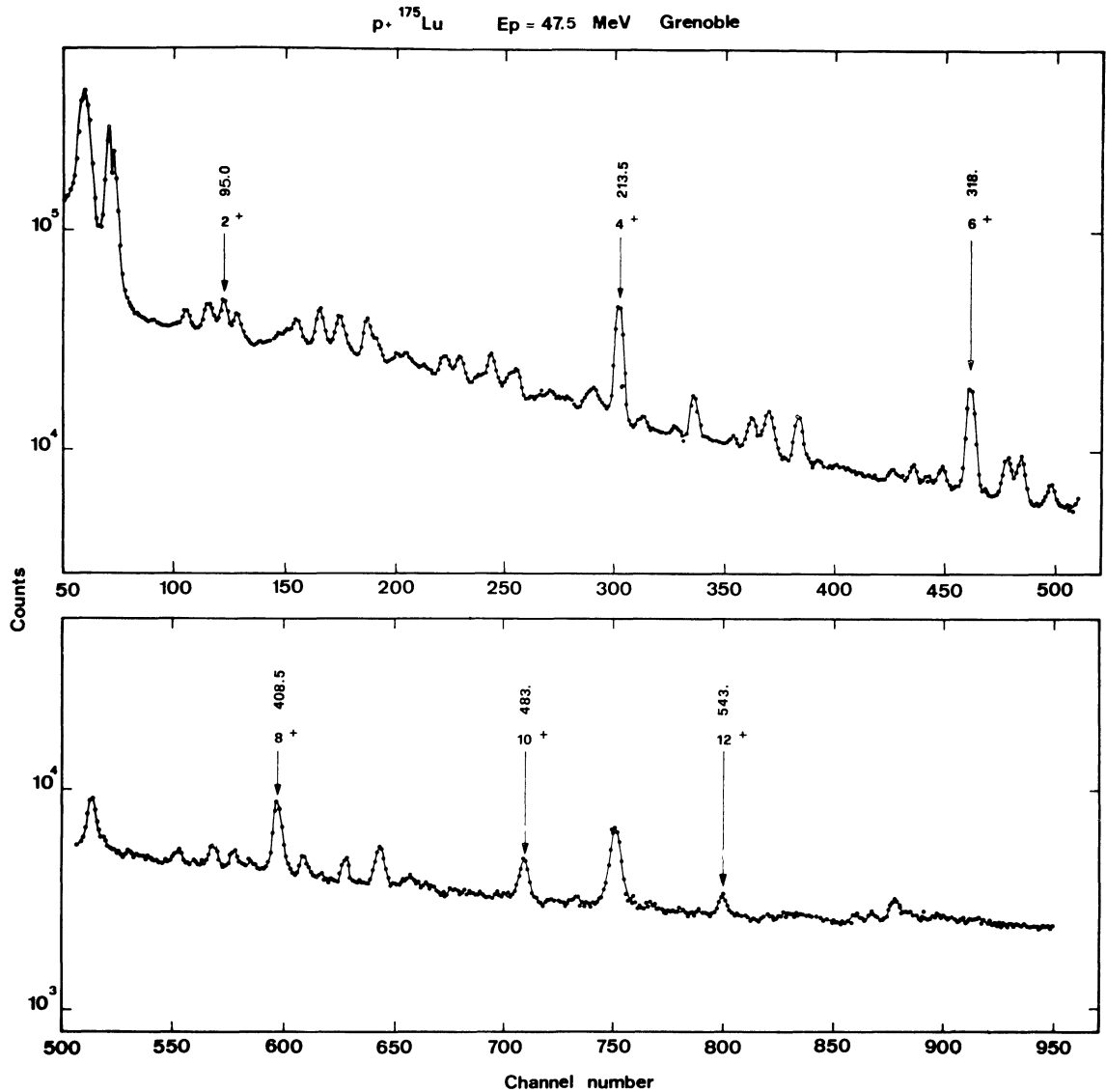
p+ ^{175}Lu   Ep = 47.5 MeV   Grenoble

FIG. 3. — Gamma ray spectrum from bombardment of ^{175}Lu with 47.5 MeV protons. The peaks whose energies are indicated are assigned to transitions from gsb levels in ^{172}Hf.
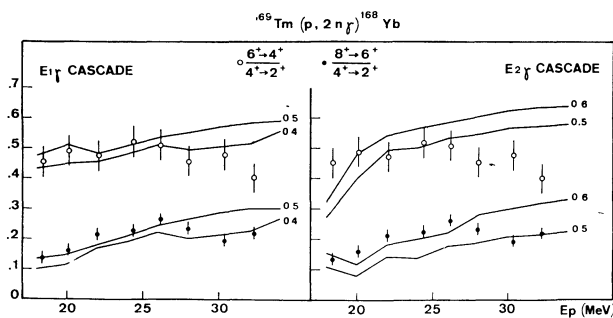

FIG. 4. — Observed and calculated intensity ratios of gsb transitions in the product nucleus of the ^{169}Tm(p, 2 nγ)^{168}Yb reaction. The plots shown have been computed with the k parameter values indicated on the right end of each curve.

3.1 CN SPIN DISTRIBUTION.

— The normalized spin distribution of the CN excited levels is given by [2]

$$P_{CN}(J) = \frac{\sigma_{CN}^{J}}{\sum_J \sigma_{CN}^{J}} \qquad (1)$$

with

$$\sigma_{CN}^{J} = \pi \lambdabar^2 \frac{2 J + 1}{(2 I_T + 1)(2 i + 1)} \sum_{|I_T - i|}^{I_T + i} \sum_{|J - s|}^{J + s} T_l^P \qquad (2)$$

where $\lambdabar$ and i are respectively the wavelength and spin of the incident proton, T_l^P its transmission coefficients, J the spin of the CN state under consideration, and I_T the spin of the target nucleus.
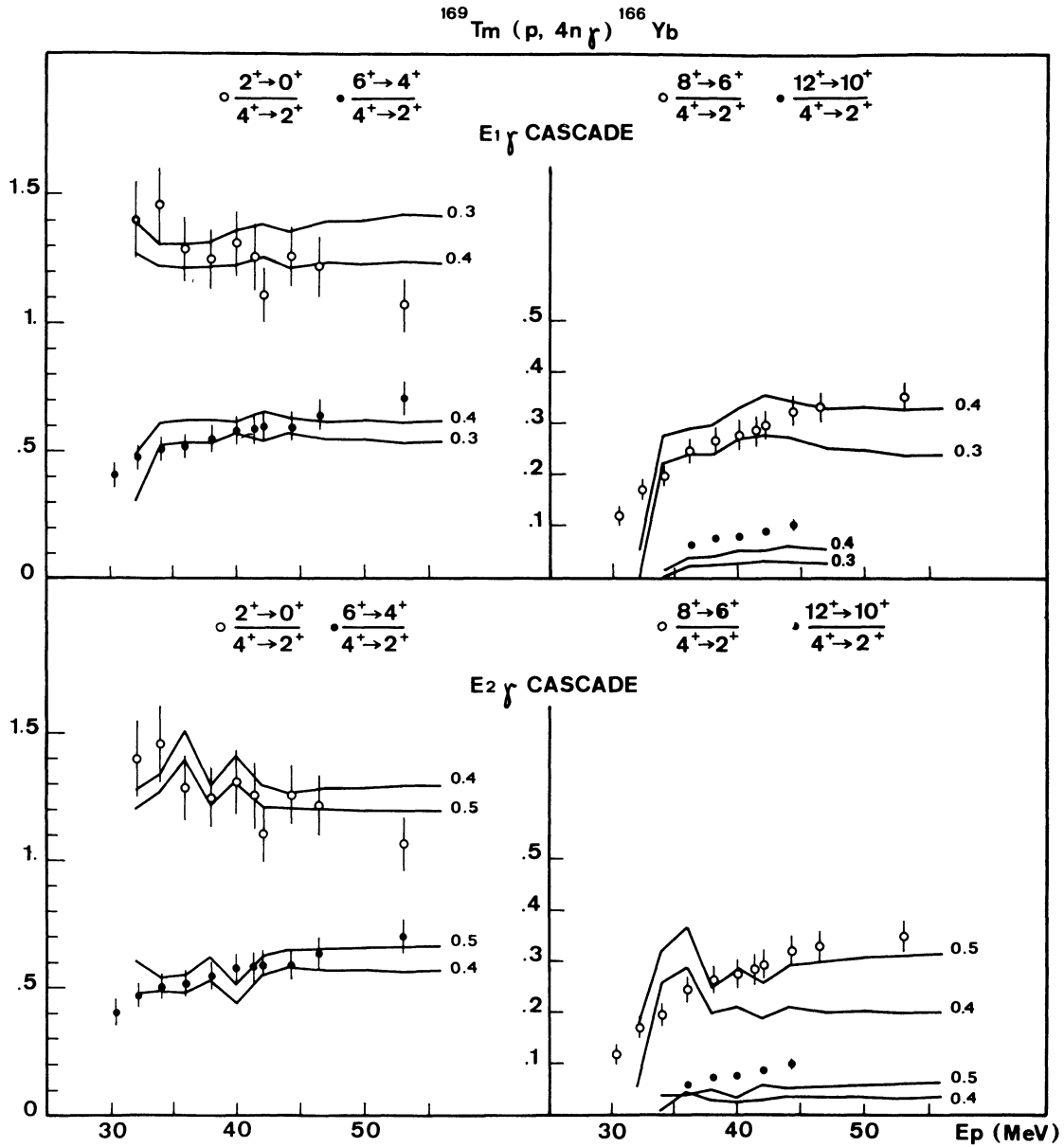
FIG. 5. — The same as figure 4 for the ^{169}Tm(p, 4 nγ)^{166}Yb reaction.
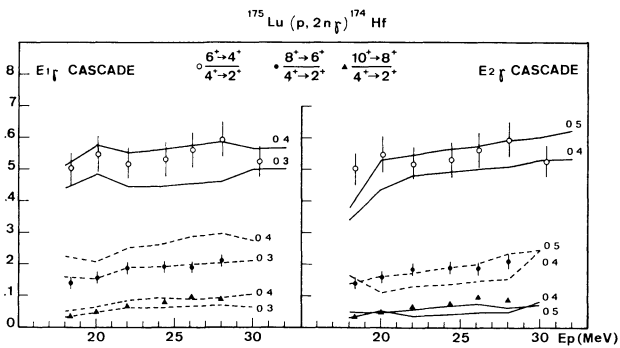
FIG. 6. — The same as figure 4 for the ^{175}Lu(p, 2 nγ)^{174}Hf reaction.

3.2 SPIN DISTRIBUTION AFTER PARTICLE EMISSION. — The CN neutron emission width whereby the residual nucleus is brought to a state of spin J_R and energy E_R is given by

$$\Gamma(J, \varepsilon, J_R) \, d\varepsilon =$$

$$= \frac{1}{2 \pi \rho_{CN}(E, J)} \, \rho_1(E_R, J_R) \sum_{|J_R-i'|}^{J_R+i'} \sum_{|J-s|}^{J+s} T_l^n(\varepsilon) \, d\varepsilon, \quad (3)$$

where E is the CN energy, $\varepsilon = E - B_n - E_R$ the kinetic energy of the emitted neutron with B_n denoting its binding energy, i' the spin of the neutron and J_R that of the residual nucleus ; and T_l^n are the neutron transmission coefficients.
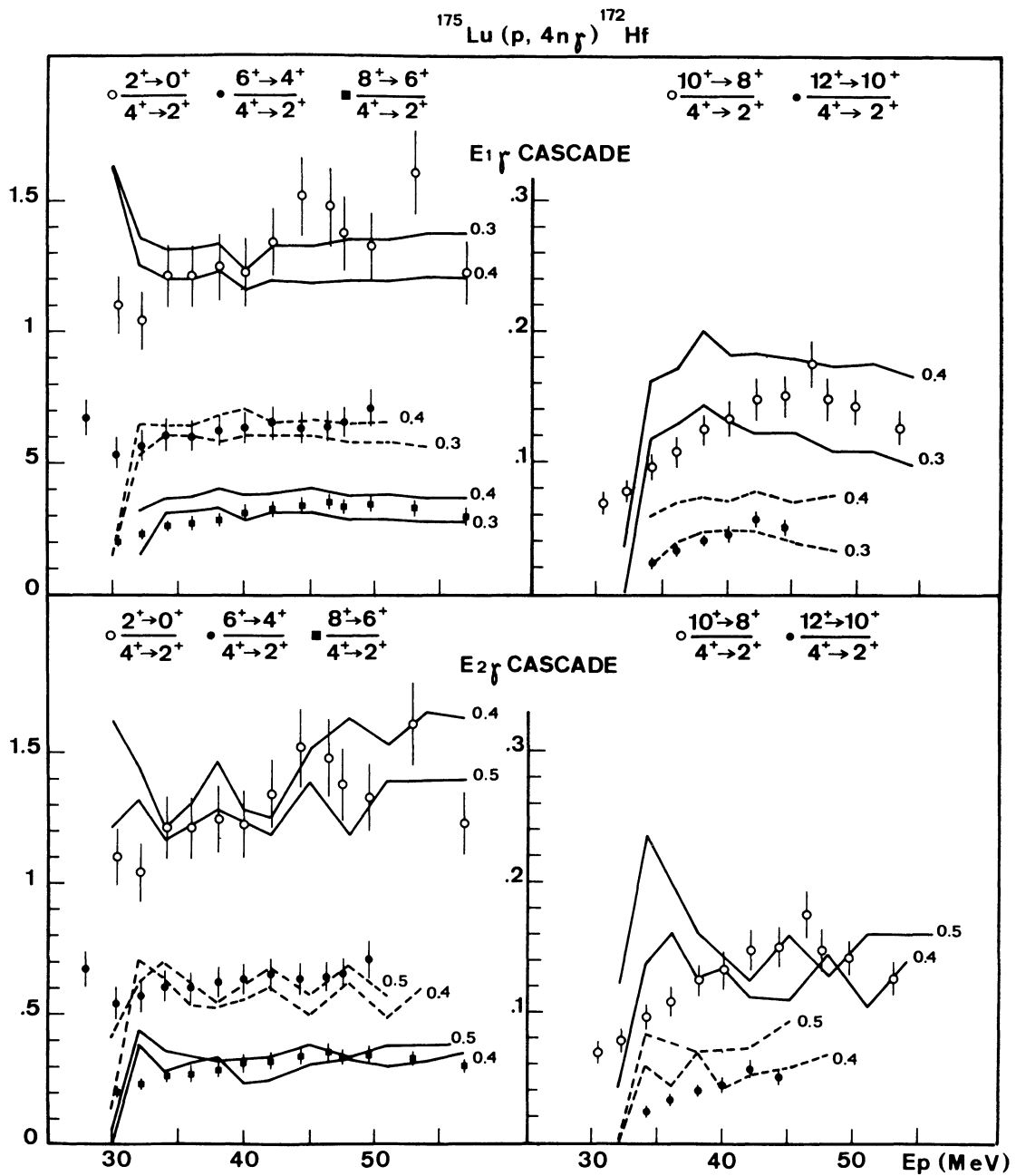
FIG. 7. — The same as figure 4 for the ^{175}Lu(p, 4 nγ)^{172}Hf reaction.

For every energy E_R the condition is imposed that J_R cannot exceed a maximum value J_R^* given by the rule [7]

$$E_R' = (E_R - \Delta) \approx \frac{\hbar^2 \, J_R^*(J_R^* + 1)}{2 \, \mathfrak{J}}$$

$$\approx \frac{\hbar^2 (J_R^* + \frac{1}{2})^2}{2 \, \mathfrak{J}} \qquad (4)$$

where $\mathfrak{J}$ is the moment of inertia, and Δ the pairing energy of the residual nucleus.

In order that in the reaction considered x neutrons can be emitted, the following condition should be satisfied

$$\varepsilon \leqslant E - \sum_1^x B_n(k) - \Delta_x = \varepsilon_{max}^1 , \qquad (5)$$

where $B_n(k)$ is the binding energy of the kth emitted neutron, and Δ_x the pairing energy of the residual nucleus after emission of x neutrons.

^{175}Lu $(p, 6n\gamma)$ ^{170}Hf

FIG. 8. — The same as figure 4 for the ^{175}Lu(p, 6 nγ)^{170}Hf reaction.

^{181}Ta $(p, 2n\gamma)$ ^{180}W

FIG. 9. — The same as figure 4 for the ^{181}Ta(p, 2 nγ)^{180}W reaction. Also reported is the intensity ratio of the 389.4 keV 8$^-$-8$^+$ transition in ^{180}W.
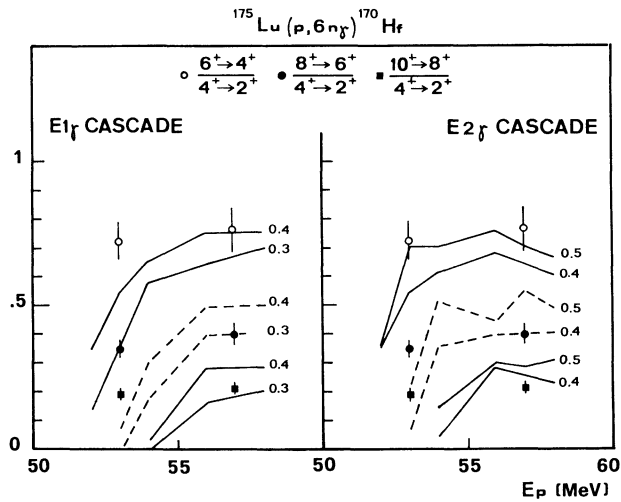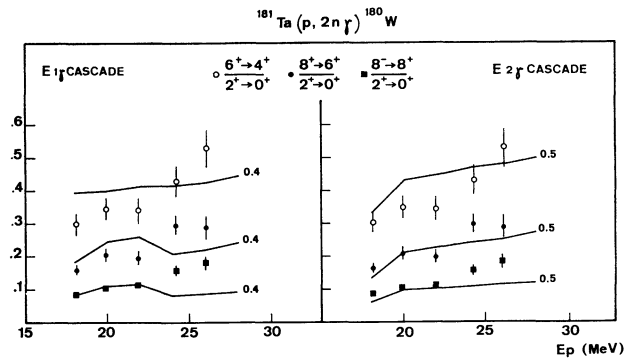
The mean energy of a neutron emitted from a CN state of spin J is given by

$$\bar{\varepsilon}(J) = \frac{\int_0^{\varepsilon^1_{max}} \varepsilon \left\{ \sum_{J_R} \Gamma(J, \varepsilon, J_R) \right\} d\varepsilon}{\int_0^{\varepsilon^1_{max}} \left(\sum_{J_R} \Gamma(J, \varepsilon, J_R) \right) d\varepsilon} . \quad (6)$$

To simplify the calculations, we assume that at each step of the neutron cascade the neutrons are emitted with their average energy. For the first emission, the average energy is

$$\bar{\varepsilon}_1 = \sum_J P_{CN}(J) \, \bar{\varepsilon}(J) . \quad (7)$$

After the first emission, the normalized spin distribution of the residual nuclei, which are supposed to have the excitation energy $E_R^* = E - B_n(1) - \bar{\varepsilon}_1$, is given by

$$P_1(J_1) = \sum_J P_{CN}(J) \, \Gamma_1^*(J, J_1) \quad (8)$$

where the expression for $\Gamma_1^*(J, J_1)$ is

$$\Gamma_1^*(J, J_1) = \frac{\int_0^{\varepsilon^1_{max}} \Gamma(J, \varepsilon, J_1) \, d\varepsilon}{\sum_{J_1} \int_0^{\varepsilon^1_{max}} \Gamma(J, \varepsilon, J_1) \, d\varepsilon} . \quad (9)$$

The computation is iterated for the second emission, with $P_1(J_1)$ substituted for $P_{CN}(J)$, $\rho_1(E_R^*, J)$ for $\rho_{CN}(E, J)$, and $\rho_2(E_R, J_R)$ for $\rho_1(E_R, J_R)$. ε^1_{max} is replaced by

$$\varepsilon^2_{max} = E_R^* - \sum_{2}^{x} B_n(k) - \Delta_x . \quad (10)$$

^{181}Ta $(p, 4n\gamma)$ ^{178}W

FIG. 10. — The same as figure 4 for the ^{181}Ta(p, 4 nγ)^{178}W reaction.

The new spin distribution is given by

$$P_2(J_2) = \sum_{J_1} P_1(J_1) \, \Gamma_2^*(J_1, J_2) \,. \qquad (11)$$

The iteration is continued up to the xth-emission. For the last emission, the lower limit in the integrals of the neutron widths is zero only when

$$\varepsilon_{min}^x = E - \sum_k^{x+1} B_n(k) - \sum_k^{x-1} \bar{\varepsilon}(k) - \Delta_{x+1} \leqslant 0 \,,$$

otherwise is equal to ε_{min}^x.

3.3 GAMMA CASCADE. — After the neutron evaporation, a gamma cascade is initiated. Emission of gamma rays of both $E\,1$ and $E\,2$ multipolarity is considered possible. Each gamma is assumed to be emitted with the energy [2]

$$\bar{\varepsilon}_\gamma = \frac{\displaystyle\int_0^{E^*} \varepsilon^{2l+2} \, \rho(E^* - \varepsilon) \, d\varepsilon}{\displaystyle\int_0^{E^*} \varepsilon^{2l+1} \, \rho(E^* - \varepsilon) \, d\varepsilon} \qquad (12)$$

where E^* is the effective excitation energy of the nucleus prior to gamma emission, $\rho(E^* - \varepsilon)$ the overall density of accessible levels, $l = 1$ for $E\,1$ and $= 2$ for $E\,2$ transitions. Denoting by $Q(J)$ the expression

$$Q(J) = (2\,J + 1)\exp\left[\frac{-\,J(J + 1)}{2\,\sigma^2}\right] \qquad (13)$$

and by $P_0(J)$ the spin distribution before gamma emission, the spin distribution after gamma emission is

$$P(J) = Q(J) \sum_{|J-l|}^{J+l} \left\{ \frac{P_0(J')}{\displaystyle\sum_{|J'-l|}^{J'+l} Q(J'')} \right\}. \qquad (14)$$

This formula is based on the assumption that no gamma transition is privileged, and that the final states are populated with weights proportional to their spin distributions (13). As in the case of the neutron cascade, for every residual excitation energy $E_R' = E^* - \bar{\varepsilon}_\gamma$ it is assumed that spins J up to a maximum value J^* given by (4) are possible. Let J^+ be the spin value corresponding to the intersection of the yrast line [7], defined in the E_R vs. J^* plane by the function (4) $E_R = E_R(J^*)$, with the gsb. There are three possible situations concerning the $P_0(J)$ population that leads to states with energy E_R. If there are states with $J > J^* + l > J^+$, these states will decay by low energy gamma emission to states near the yrast line ; a cascade of successive low energy gamma transitions (mainly of $E\,2$ multipolarity) to states always near the yrast line will in the end populate the J^+ state of the gsb. This process is known to be of importance for (HI, xnγ) reactions [1], [7] but is nearly negligible in (p, xnγ) reactions. If there are states with

$$J^+ > J > J^* + l \,,$$

these states will populate directly, and with equal probability, the accessible states of the gsb. If neither of the above situations occurs, the gamma cascade ends when E^* becomes smaller than some energy E^+ (called

the deciding energy) ; the end states of the cascade are then assumed to populate, again directly and with equal probabilities, the gsb.

The transitions feeding the gsb are considered of $E\,1$ or $E\,2$ multipolarity, except when the decaying level is a 0^- state, which is presumed to populate the 2^+ state of the gsb by means of $M\,2$ gamma rays. In the case of the $^{181}\text{Ta}(p, 2\,n\gamma)$ reaction a slight modification must be introduced to take into account the presence of the 8^- state which is populated in competition with the 10^+, 8^+, 6^+ states ; the 8^- state eventually decays to the 8^+ state via $E\,1$ gamma transitions.

3.4 LEVEL DENSITIES. — The level density appearing in eq. (3) is given by

$$\rho(E, J) = \frac{(2\,J + 1)}{2(2\,\pi)^{1/2}\,\sigma^3}\exp\left[\frac{-\,J(J + 1)}{2\,\sigma^2}\right]\rho(E)\,, \qquad (15)$$

with $\rho(E)$ expressed according to the Fermi gas model [8] by

$$\rho(E) = \frac{6^{1/4}}{24}\,g^{-1/4}\,\frac{\exp 2\,\sqrt{a(E - \Delta)}}{(E - \Delta + t)^{5/4}}\,, \qquad (16)$$

where g is the single particle level density near the Fermi energy, and t the thermodynamic temperature bound to the energy by the equation of state

$$E - \Delta = a t^2 - t \,,$$

with $a = \pi^2\,g/6$. $\rho(E^* - \varepsilon)$ in expression (12) is also given by eq. (16), with obvious substitutions. The spin cut-off parameter, σ^2, which determines the spin distribution, is the most influential parameter in the present calculation ; it is connected to $< m^2 >$, the average value of the square of the projection of the total angular momentum of excited nucleons on the z-axis, by the relation $\sigma^2 = v < m^2 >$. Here v is the number of excited nucleons which, in the Fermi gas model, is given by gt. At high energies

$$\sigma^2 = v < m^2 > = \frac{\Im_R \, t}{\hbar^2}\,, \qquad (17)$$

where $\Im_R = 2/5\,AR^2$ is the moment of inertia of the nucleus taken as a rigid sphere whose radius R is assumed, to the purpose of this paper, equal to $1.27\,A^{1/3}$ fm. At low energies, eq. (17) can still define a moment of inertia $\Im$ of the nucleus, provided it is regarded as a parameter whose value will depend on the energy as well as on the particular nucleus. Unfortunately, the function $\Im = \Im(E)$ is model dependent, and the literature does not offer as yet satisfactory evidence to justify a preference for anyone model. Therefore we limit ourselves to obtain for each reaction an average value of $\Im$, not dependent on energy, such that $\Im = k\Im_R$. It is these k-values that will be used to express the results of our analysis.

4. Comments on the calculation. — It seems appropriate to comment on some questions encountered in the development of the calculation procedure reported in the previous section.

4.1. NON-STATISTICAL EMISSION OF THE FIRST NEUTRON. — If the first neutron is emitted via a pre-equilibrium process instead of statistically evaporated, the $P_1(J_1)$ distribution of eq. (8) will be altered because (i) the average energy of the emitted neutron is higher than that given by eq. (7), and (ii) the angular momentum of the neutron may be aligned with the spin of the CN. The last mentioned effect may be important under certain conditions, as when the differential cross-section for pre-equilibrium neutron emission is strongly forward peaked, and I_T is much lower than the angular momentum of the incident proton. The alterations in the $P_1(J_1)$ distribution will influence all the following distributions, and the eventual outcome will be a reduction in the population of the high spin states of the gsb in favour of the population of the low spin states. This fact should show up in the experimental data by a reduction of the gamma intensity ratios (Fig. 4 to 10) for transitions from high spin levels, at energies beyond the maxima of the excitation functions. Moreover, the reduction should be greater for the reactions on ^{169}Tm (spin 1/2) than for those on ^{175}Lu and ^{181}Ta (spin 7/2). Indeed, the effect begins to be noticeable only in the ^{169}Tm(p, 2 nγ) reaction at $E_p > 25$ MeV and in the ^{175}Lu(p, 4 nγ) reaction at $E_p > 42$ MeV. We can thus expect that while in general neglecting the precompound emission can be justified in the present work, some slight disagreement between calculated and experimental results might be found at the highest energies.

4.2 SPIN FRACTIONATION EFFECT. — The possible weight of this effect, often quoted in the literature, has to be evaluated since it can be suspected of causing a growth in the population of the high spin states of the gsb. When studying a (p, xnγ) reaction, the effect should intervene at energies slightly over the threshold of the (p, (x + 1) nγ) reaction. It is due to the fact that near the threshold, even if the available energy suffices for the emission of $x + 1$ neutrons, the product nucleus may have no low energy excited states with high enough spin to be populated. If this is the case, some of the excited nuclei reached after the emission of x neutrons will, in place of emitting a further neutron, de-excite through gamma decay. The gammas, which arise from high spin states, populate states of high spin in the gsb of the nucleus produced in the (p, xn) reaction.

We have evaluated with apposite calculations the importance of the spin fractionation effect, and found it to be negligible in all the cases of interest here.

4.3 GAMMA-CASCADE MULTIPOLARITY. — Next to σ^2, the most influential parameters in the calculation are the number of gamma rays issued in the cascade, and their multipolarity. The number is determined once the mean excitation energy of the nucleus when starting the cascade of gamma rays, their mean energy and their multipolarity are fixed. Choosing the multi-

polarity, however, raises a difficult problem. Theoretically, for deformed nuclei a strong enhancement of $E2$ in comparison with $E1$ transitions is expected [9]. The experimental evidence for nuclei in this mass region regards mainly reactions produced with alpha particles of 30-40 MeV, and indicates a complex situation. For instance, the recent work by Williamson et al. [10] suggests a predominance of $E2$ gamma rays at comparatively low energies ($E_\gamma < 1.0$ MeV), that is, for transition between levels close to the yrast lines, and a strong competition between $E1$ and $E2$ gammas at higher energies. We fail to see any reason why the situation should be markedly different in the case of proton-induced reactions. Therefore, to test the extremes of assuming either $E2$ or $E1$ cascades, we have performed the calculations in both cases, and will present the results in section 5.2.

4.4 DECIDING ENERGY. — The assumption of the termination of the gamma cascade for $E^* < E^+$ is introduced to simplify the computations. If a constant value is chosen for E^+, the estimate of the intensity ratios of the gsb transitions may sometimes fluctuate. This could be the case when for two initial excitation energies close to each other it so happens that, after emission of a certain number of gammas, in one instance E^* turns out to be slightly lower, and in the other slightly higher than E^+. There one would infer, in spite of the closeness of the excitation energies available to the gamma cascade, different numbers of gammas.

With E^+ set equal to 1 MeV, we have tested the effect of these fluctuations on the determination of the spin cut-off parameter, and found it to be greater for $E2$ than for $E1$ cascades. Even in the first case, however, we came to the conclusion that the uncertainties involved are of no practical consequence in the present calculations.

5. Results of the analysis and discussion. — **5.1 CALCULATION PARAMETERS.** — The transmission coefficients of the incident protons, T_l^p, have been obtained with the optical potential [11]

$$U(r) = - V(e^x + 1)^{-1} - iW(e^{x'} + 1)^{-1} +$$

$$+ \left[\frac{\hbar}{m_\pi c}\right]^2 V_s \, \sigma \times 1\frac{1}{r} \frac{d}{dr} (e^x + 1)^{-1} + V_c(r)$$

where

$$x = \frac{r - R_0 A^{1/3}}{a}, \quad x' = \frac{r - R_0' A^{1/3}}{a'},$$

$$R_0 = 1.20 \text{ fm}, \quad R_0' = 1.428 \text{ fm},$$

$$a = 0.65 \text{ fm}, \quad a' = 0.704 \text{ fm},$$

$$V = (V_0 - 0.2 E) \text{ MeV}, \quad W = (W_0 - 0.05 E) \text{ MeV},$$

$$E = E_p \text{ MeV}, \quad V_0 = 58 \text{ MeV},$$

$$W_0 = 9 \text{ MeV}, \quad V_s = 6.6 \text{ MeV};$$

$V_c(r)$ is the Coulomb potential of a uniformly charged sphere of radius $1.20\ A^{1/3}$ fm. Other optical potentials [12] have been tried, with no appreciable effect on the calculated spin distribution $P_{CN}(J)$.

The transmission coefficients of the emitted neutrons have been computed with the optical potential of Wilmore and Hodgson [13]. The level density parameters used were :

$$a = (0.210\ N + 0.025)\ \text{MeV}^{-1} \cong A/8\ \text{MeV}^{-1} ;$$

$$A = A_z + A_n ,$$

with $A_z = A_n = 0$ for doubly odd nuclei, and
$A_z = (1.654\text{-}0.009\ 58\ Z)$ MeV and
$A_n = (1.374 - 0.005\ 16\ N)$ MeV

for doubly even nuclei [14]. The neutron binding energies were taken from the compilation of Wapstra and Gove [15]. For the deciding energy E^+ the value of 1 MeV was chosen.

5.2 COMPARISON WITH THE EXPERIMENTAL DATA. —

The deduced $k = \mathfrak{J}/\mathfrak{J}_R$ values, resulting from the comparison of the calculated plots with the experimental data, are summarized in table II, for both $E\,1$ and $E\,2$ gamma cascades multipolarities. In determining the k values, particular account was taken of the experimental results in the E_p energy ranges centered around the maxima of the excitation functions. Examples of typical plots are shown together with the experimental points, in figures 4 to 10. It is apparent that the assumption of $E\,2$ gamma cascades indicates systematically k values slightly higher than the assumption of $E\,1$ cascades. In both cases, however, an almost equally good overall fit to the experimental data is obtained. Therefore, an intermediate value $< k > = 0.4$ can be assumed for all the reactions considered.

TABLE II

*Values of $k = \mathfrak{J}_R/\mathfrak{J}$ deduced from comparison
of calculation and experiment*

Residual Nucleus	k ($E\,1\ \gamma$ cascade)	k ($E\,2\ \gamma$ cascade)
^{168}Yb	0.4-0.5	0.5-0.6
^{166}Yb	0.3-0.4	0.5
^{174}Hf	0.3	0.5
^{172}Hf	0.3	0.4
^{170}Hf	0.3-0.4	0.4-0.5
^{180}W	0.4	0.5
^{178}W	0.2	0.35

5.3 DISCUSSION. —

The above result, that the predictions of a statistical model calculation are consistent with the observed relative intensities of rotational radiations from residual heavy nuclei excited in (p, xn) reactions, is at variance with the conclusions reached by Williamson et al. [10] for reactions induced by alpha particles. These authors have suggested that the observed spin population might be perturbed by non-statistical cascades in higher rotational bands prior to entering the gsb. As a consequence, the spin cut-off parameters deduced by a simple statistical model treatment would be irrelevant. Also Mills and Rautenbach [2], finding difficulties in accounting for the energy variations of the gsb line intensities in (α, xn) reactions, mention the possible intervention of privileged non statistical transitions in high energy bands. If such effects are present, their importance should be lesser for proton than for alpha-induced reactions, since in the present work both the gsb line intensities and their energy variations can be satisfactorily understood using a purely statistical treatment with the same set of parameters for several nuclei. Besides, no theory has been developed as yet to deal in a quantitative way with the effects mentioned above, so that it is impossible to test whether our results would be compatible also with the suggestions of Williamson et al. Thus, while not excluding that the actual physical situation might be more complicated than the one summarized by the statistical model, for the time being our discussion will be limited to its implications.

According to our definition of k in section 3.4, $k \approx 0.4$ means a sizable reduction of the nuclear *average* moment of inertia in comparison of the rigid sphere value $\mathfrak{J}_R$. That this reduction should refer to the final nuclei produced in the reactions is not implied by the calculation procedure. However, at incident proton energies exceeding the reaction thresholds by some 10 MeV, where the precision of the calculations is expected to be the highest, it is found that the spin population $P_x(J_x)$ at the end of the neutron cascade is very much the same whether $k = 1$ or 0.4-0.5. It seems therefore reasonable to assume that the k values apply to the moment of inertia of the gamma decaying residual nuclei. If this is so, it can be further assumed that the values obtained correspond to a mean effective excitation energy

$$\overline{E}^* = (E^*_{max} + E^*_{min})/2 \cong (B_n + E^+)/2 \cong 4\ \text{MeV} .$$

Since $\overline{E}^* = \overline{E} - A$, where $\overline{E}$ is the true excitation energy, there results $\overline{E} \cong 6$ MeV. Reduced values for the moment of inertia of doubly even deformed nuclei at low excitation energy have been predicted several years ago by Lang and Le Couteur [16] with their independent pairing model (ipm). Up to this time however, only in few instances the previsions of the model have been compared with experimental findings (see for instance [17]) ; this was probably due to the scarcity of pertinent experimental information.

The ipm is based on the assumption of a strong

coupling between any two nucleons whose states differ only owing to the fact that the projections of their total angular momenta on the symmetry axis of the nucleus are opposite. If the pairing energy $\Delta \gtrsim t$, where t is the mean excitation energy of a single nucleon (approximately equal, in the Fermi gas model, to the thermodynamic temperature), the excitation of single nucleons will be hindered and the greatest part of excited states should be accounted for by pair excitations. The contribution of each pair to $< m^2 >$, however, ought to be a minor one, since the two components along the symmetry axis are supposed to be opposite. Using the notations of Lang and Le Couteur ($c\hbar^2 = \Im_R$, $c''\hbar^2 = \Im$), the ipm provides the relation

$$c'' = (c')^{1/3} \cdot (\tfrac{1}{2} c + \tfrac{1}{2} c')^{2/3} ,$$

where $c' t^* \approx ct^* \exp(- 0.437 \Delta/t^*)$, and t^* is defined by

$$E = at^{*2} \left\{ \frac{1}{4} + \frac{3}{4} \left(\frac{1+0.437 \Delta}{t^*} \right) \exp\left(\frac{-0.437 \Delta}{t^*} \right) \right\} - t^* .$$

The k used in this work is given, in the same notations, by

$$\Im t = kc\hbar^2 t = c'' \hbar^2 t^* ,$$

whence $k = c'' t^*/ct$, with t calculated from

$$E - \Delta = at^2 - t .$$

Taking the following average values for the nuclei of interest here, $A = 173$, $a \cong A/8$ MeV^{-1}, $E = 6$ MeV, $\Delta = 1.8$ MeV, we obtain $c' \cong 0.2\ c$, $t^* \cong 0.5$ MeV, $c'' \cong 0.42\ c$, $t \cong 0.46$ and finally $k \cong 0.44$, in very good agreement with the results of section 5.2. This conclusion should not be effected by the fact that the ipm contemplates at low energies also a modification of the expression (16) for the level density energy dependence, since at low energies we use eq. (16) only to evaluate, with effective parameter values appropriate to the energy range in question, the mean energy of the emitted gammas. The agreement found with the predictions of the ipm appears to be better than that obtainable with the superconductor model [18], [19]. This result might perhaps have been expected in the case of the present data, since up to now the latter model has produced its best achievements for odd-mass nuclei. In the notations of Vonach et al. [19], and choosing for A, a, E, Δ the same values used in the comparison with the ipm, the superconductor model estimate yields

$$\theta_0 \approx 1.17 , \qquad t_c \approx 0.669 ,$$

$$t/t_c \approx 0.882 , \quad A(U) \approx 0.663 ,$$

and, deriving t' again from $E - \Delta = at'^2 - t'$,

$$A(U)\ t/t' = k \approx 0.78 .$$

A brief comment may be added concerning the ^{181}Ta(p, 4 nγ)^{178}W reaction, the only case in which the analysis indicates a comparatively small k value (it can also be argued that the corresponding $\Im_R$ might be even smaller, by some 10 or 15 %, than indicated by k, which in this case is unlikely to apply only to the residual nucleus). In the ipm this reduction might be explained if for ^{178}W the pairing energy were somewhat higher than the mass averaged value of 1.8 MeV used above ; actually, by inspecting the original tabulations [20] it appears that for a nucleus in the $A \sim 178$ mass region an upwards fluctuation in Δ as high as 40 % is allowable. Alternatively, the slight ^{178}W anomaly might be a signal of some presence of the privileged transitions cited earlier in this discussion.

6. Conclusion.

— It is confirmed that the basic features of the de-excitation process of compound nuclei in the heavy mass region can be adequately predicted by the statistical model. To this conclusion the study reported here contributes by presenting a consistent set of results on proton induced reactions, over a wide range of bombarding energies.

No indication was found in these reactions of the presence of privileged cascades perturbing the supposedly statistical nuclear cooling process : it should be pointed out, however, that this experiment was not designed for the exacting task of measuring any such radiations. The analysis of the experimental data suggests that for even-even deformed nuclei, with A from 166 to 180, at a residual excitation energy of about 6 MeV, the moments of inertia should be reduced to some 40 % of the rigid body figures.

The values obtained are sufficiently precise to allow for a comparison with the previsions of the independent pairing model of Lang and Le Couteur, and a very satisfactory overall agreement is found.

To conclude, we would like to recall that the rather sparse results available to date for some doubly even deformed nuclei concur with the present findings. Hansen et al. and Jägare [2] report from the study of the ^{181}Ta(p, 2 nγ) reaction at $E_p \approx 12$ MeV a spin cut-off value which in our notation corresponds to $k = 0.4$. Again Jägare [2] suggests the same value, although affected by rather great uncertainty, for ^{170,172}Hf produced in heavy ion reactions. Mills and Rautenbach [2] by examining (α, 2 nγ) reactions in ^{178,180}Hf conclude that the spin cut-off parameters of ^{180}W and ^{178}W should be in about the same ratio as found here.

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