Deformation or spherical symmetry in ^{10}Be and the inversion of $1/2^{-}$-$1/2^{+}$ states in ^{11}Be .

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Abstract

For a core plus one neutron system like ^{11}Be we have calculated the energies of the $1/2^{-}$ and $1/2^{+}$ states assuming a deformation of the core deduced from the low energy 2^{+} state properties or taking into account the coupling of the neutron with this 2^{+} state interpreted as a spherical one-phonon state. We have shown that the two derivations yield identical results if the phonon energy is neglected in the second derivation and close results in the general case.

The problem of the $1/2^-$-$1/2^+$ states inversion in ^{11}Be has been for long a challenge for theoreticians and has been the subject of a large number of publications assuming deformation [1–4] or spherical symmetry [5–8]. However we restrict our discussion to papers by Nunes et al. [1]and Vinh Mau [5] which are directly concerned with the inversion problem and have proposed simple models to examine its origin. These two theoretical papers which relate the inversion of the $1/2^-$-$1/2^+$ states in ^{11}Be to the existence of a low energy 2^+ state in ^{10}Be are often considered as conflicting . Indeed the first one relies on the assumption that this 2^+ state at 3.36 MeV is a rotational state implying a deformation of the nucleus while the other assumes it to be a one-phonon vibrational state therefore works with a spherical nucleus. The parameter β_2 deduced from the measured B(E2) is in the first case interpreted as a deformation parameter and in the second case as a collective transition amplitude for the vibrational phonon. Intuitively we could already say that these two interpretations are not independent or contradictory because deformation comes from strong correlations between nucleons inside the nucleus which are taken into account implicitly when β_2 is interpreted as a collective amplitude. This equivalence shows up in large basis shell model calculations which are able to reproduce rotational as well as vibrational states [10]. Moreover a recent analysis of p(^{11}Be , ^{10}Be)d reaction [11] where the wave functions of the last neutron in ^{11}Be were taken from the two models leads to the same agreement with experiments.

The aim of this note is to show explicitly and analytically that there is a simple relation between the two derivations for $1/2^+$ and $1/2^-$ states. The basic assumptions of the two methods [1, 5] are schematised in Fig.1.

Let's start with the left-hand side of the figure which illustrates the model used by Nunes et al.. The neutron one-body potential is then written as a deformed Woods-Saxon potential by replacing the spherical radius R_0 by the deformed one, $R_0(\theta,\phi)$. It writes as:

$$V_{nc}^{def}(r,\theta,\phi) = V_0\left(1+exp[(r-R_0(\theta,\phi))/a]\right)^{-1} \quad (1)$$

$$R_0(\theta,\phi) = R_0\left(1+\beta_2 Y_2^0(\theta,\phi)\right) \quad (2)$$

what, by performing an expansion around R_0, leads to the potential:

$$V_{nc}^{def}(r,\theta,\phi) \simeq V_{WS}(r,R_0) - \beta_2 R_0 \frac{dV_{WS}(r,R_0)}{dr} Y_2^0(\theta,\phi) \quad (3)$$

The second term of eq.(3) is a correction to the spherical Woods-Saxon potential, $V_{WS}(r,R_0)$, and can be considered as a small perturbation. Therefore $\delta\epsilon_n$, the corresponding modification

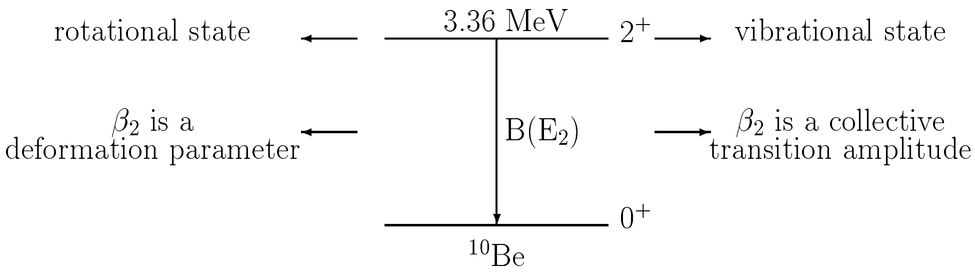

FIG. 1: Interpretation of the 2^+ state in ^{10}Be as proposed in ref.[1] (left side of the diagram) and in ref[5] (right side of the diagram).

of the single neutron energy for a neutron state represented by the index n, may be calculated to lowest order in perturbation theory. For $1/2^+$ and $1/2^-$ states the first order term is zero and one has to go to second order which leads to:

$$\delta\epsilon_n = \beta_2^2 R_0^2 \sum_{\lambda \neq n} \frac{< n|\frac{dV_{WS}}{dr} Y_2^0|\lambda >< \lambda|\frac{dV_{WS}}{dr} Y_2^0|n >}{\epsilon_n - \epsilon_\lambda} \tag{4}$$

where $|n >, \epsilon_n$ and $|\lambda >, \epsilon_\lambda$ are the eigenvectors and eigenvalues of the spherical potential, V_{WS}, for states n and λ respectively. The summation over λ runs over the complete set of neutron states except state n. Performing the calculation of matrix elements leads to:

$$\delta\epsilon_n = \frac{1}{10} \beta_2^2 R_0^2 \sum_{\lambda \neq n} (-1)^{j_n - j_\lambda} \frac{R_{n\lambda}^2}{\epsilon_n - \epsilon_\lambda} |< l_n, j_n||Y_2||l_\lambda, j_\lambda >|^2 \tag{5}$$

where $< l_n, j_n||Y_2||l_\lambda, j_\lambda >$ is the reduced matrix element of Y_2 and $R_{n\lambda}$ the radial integral:

$$R_{n\lambda} = \int_0^\infty r^2 \, dr \; \phi^*_{\alpha_n l_n j_n}(r) \; \phi_{\alpha_\lambda l_\lambda j_\lambda}(r) \frac{dV_{WS}(r)}{dr} \tag{6}$$

with obvious notations.

Going to the right hand side of Fig.1 which illustrates the model of ref.[5] and after calculation of the particle-phonon coupling diagrams for phonons of angular momentum L and energy E_L, one obtains the modified one-body potential [5] as:

$$V_{nc}^{sph}(\mathbf{r}, \mathbf{r}') \; = \; V_{WS}(r)\delta(\mathbf{r}, \mathbf{r}') + \sum_L \frac{\beta_L^2 R_0^2}{2L+1} \sum_{\lambda, M} F_{L,\lambda} \phi^*_\lambda(\mathbf{r}) \; \frac{dV_{WS}(r)}{dr} Y_L^{M*}(\theta\phi)$$

$$\phi_\lambda(\mathbf{r}')\frac{dV_{WS}}{dr}|_{r=r'}\; Y_L^M(\theta',\phi') \tag{7}$$

$$F_{L,\lambda} = \frac{1-n_\lambda}{\epsilon_n-\epsilon_\lambda-E_L}+\frac{n_\lambda}{\epsilon_n-\epsilon_\lambda+E_L} \tag{8}$$

where n_λ is the occupation number of state λ and $\phi_\lambda(\mathbf{r})$ the three dimensional wave function of the neutron calculated in the Woods Saxon field. The potential is non local, has spherical symmetry and looks very different of the deformed potential of eq.(3). However it is again a perturbation to V_{WS} and to first order gives for the contribution of a 2^+ phonon as:

$$\delta\epsilon_n = \frac{1}{5}\beta_2^2 R_0^2 \sum_{\lambda,M} F_{2,\lambda} < n|\frac{dV_{WS}}{dr}Y_2^M|\lambda >< \lambda|\frac{dV_{WS}}{dr}Y_2^{M*}|n > \tag{9}$$

$$= \frac{1}{10}\beta_2^2 R_0^2 \sum_{\lambda\neq n}(-1)^{j_n-j_\lambda}(\frac{1-n_\lambda}{\epsilon_n-\epsilon_\lambda-E_2}+\frac{n_\lambda}{\epsilon_n-\epsilon_\lambda+E_2})|R_{n\lambda}|^2\,|< l_n,j_n||Y_2||l_\lambda,j_\lambda >|^2 \tag{10}$$

where the term $\lambda = n$ is automatically eliminated for $j_n = 1/2$ because of angular momentum coupling. By comparing the two formulae, eqs.(5) and (10), one sees that they are identical if one takes E_2, the phonon energy, equal to zero. Because E_2 appears in the denominators only, it is easy to see that eq.(10) yields a correction which is smaller than given by eq.(5). However the phonon energy is small (few MeV) and in the limit of very strong collectivity is close to zero then it should not introduce a too large difference between the two derivations. This result tells us that one may not consider the two models as contradictory and that it is not justified to reject one or the other as it is sometimes done.

For other states with $j_n \neq 1/2$ we have not found such a simple relation between the two derivations because the perturbative first order contribution to $\delta\epsilon_n$ is non zero for the potential of eq.(3).

The theoretical expressions of the potentials have been used in different ways. In ref.[1] β_2 was taken from the measured value of the B(E2) and the strengths of Woods-Saxon and spin-orbit potentials fitted to the experimental neutron energies in ^{11}Be. In the papers following ref.[5] the Woods-Saxon and spin-orbit potentials were fixed accordingly to their known properties in normal nuclei [12] and the strength of the second term of eq.(7) was parametrised assuming a surface form factor, $(\frac{dV_{WS}(r)}{dr})^2$, as suggested by theory. When applied to reaction problems involving ^{11}Be, the two potentials give the same (good) results [11] not only for $1/2^+$ and $1/2^-$ but also for $5/2^+$ states.

We have shown that assuming a deformed mean field model or taking account of two-body correlations in a spherical model leads to close results for the inversion of $1/2^+$ and

$1/2^-$ in core plus one neutron systems. We think that this equivalence between the two models is also visible, at least qualitatively, in the work of Li and Heenen [4]. In a deformed Hartree-Fock calculation they get the energy minimum in ^{10}Be and ^{11}Be for a spherically symmetric configuration and a $1/2^-$ ground state in ^{11}Be . The same result comes out from a Hartree-Fock-Bogolioubov calculation [9]. The authors of ref.[4] introduce two-body correlations by angular momentum projection of Hartree-Fock wave functions. The effect of projection on ^{11}Be is not enough to bring the $1/2^+$ state below the $1/2^-$ state but reduces the energy difference between them. The minimum of the projected ^{10}Be energy is then obtained for a deformed configuration . We think that this work is another proof of the close equivalence between the models of ref.[1] and [5] and a justification of both: in a mean field approximation ^{10}Be and ^{11}Be are spherical (then justifying the starting assumption of ref.[5]) but when one adds correlations (what is analogous to the treatment of ref.[5]) one improves the results in ^{11}Be and one gets a deformed configuration (what is a justification of the treatment of ref.[1]).

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