T=0 effective interaction in ^{14}N and ^{10}B .

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Abstract

We have calculated the 1^+ and 3^+, T=0 states in ^{14}N and ^{10}B . In a neutron-proton RPA model these two nuclei are described by the same set of equations. We first show that a bare Minnesota interaction leads to too weak binding in both nuclei. Furthermore it does not produce a 3^+ ground state in ^{10}B as it should. Including medium effects as an exchange of phonons between the neutron-proton pair cures the desagreement in ^{14}N but still gives a 1^+ ground state in ^{10}B with the 3^+ as an excited state. The same study with a Gogny effective interaction reproduces nicely the properties of both nuclei: same agreement in ^{14}N as previously when medium effeccts were introduced but now the 3^+ in ^{10}B becomes the ground state. This success suggests that through its density dependent term the Gogny interaction takes account of the presence of a three-body force which, in a shell model calculation, has been shown to be essential to give a 3^+ ground state in ^{10}B .

I. INTRODUCTION.

The T=0 neutron-proton pairing correlations in N=Z nuclei have motivated a number of theoretical and experimental works. In odd-odd N=Z nuclei, T=0 and T=1 states are both present so that they offer a good place to make a comparative study of the two types of correlations. We have focused our attention on the two odd-odd nuclei of ^{10}B (N=Z=5) and ^{14}N (N=Z=7) for several reasons. First ^{14}N and ^{10}B have a 1^+, T=0 and a 3^+ T=0 ground states respectively while the 0^+, T=1 states are excited states and therefore suggesting stronger correlations in the T=0 channel. Furthermore because of charge invariance of the nucleon-nucleon interaction, the neutron-proton and neutron-neutron effective interactions should be the same in the T=1 channel. Therefore an effective interaction and a nuclear model able to explain the 0^+, T=1 states in a core plus two neutrons system (like ^{14}C) should also describe well the same T=1 states in the core plus a neutron-proton pair (like ^{14}N). We have then an opportunity to make a further test of our previous work in collaboration with J.C. Pacheco on N=8 nuclei [1]. With the Gogny effective interaction [2, 3]or its zero range density dependent substitute [4], and in a two-neutron RPA model we were able to reproduce the 0^+ states of ^{14}C -^{10}C, ^{12}Be-^{8}Be and ^{11}Li-^{7}Li. The nucleus ^{14}N (^{10}B) described as a core of ^{12}C plus (minus) a neutron-proton pair is the analog of ^{14}C (^{10}C) described as a core of ^{12}C plus (minus) a neutron-neutron pair and our first aim is to check if the same interactions and the same RPA model where the two-neutron pair is replaced by a neutron-proton pair, will lead also to a good description of the 0^+ states in ^{14}N and ^{10}B .

In the two-nucleon RPA model, successful in the description of the T=1 states, we calculate the T=0, 1^+ and 3^+ states in ^{14}N and ^{10}B . First we perform the calculation with a simple zero range force fitted to reproduce the deuteron binding energy. The results obtained with this simple potential show a too weak binding. Because a zero range force is a crude simplification of a realistic one, we have made the same calculation with the Minnesota bare interaction [5] which has one short range component, two long range components and all exchange terms. The results are very close to the previous ones and show again a lack of binding. It is not surprising that bare interactions could not describe well bound many-body systems for which we know that medium effects are important. Medium effects on the bare nucleon- nucleon interaction may arise, at least partly, from the exchange of phonons between the two-nucleon pair. Indeed strong two-body correlations in nuclei manifest themselves as very collective vibrationnal states at low excitation energies. These vibrations, or phonons, may be exchanged between the two nucleons and induce a modification of the nuclear interaction. The presence of these phonons, besides their effect on the nuclear two-body force, has an other manifestation by modifying the average interaction of a single nucleon with the core due to the coupling between the nucleon and the phonons of the core. Such couplings modify the Hartree-Fock average potential and are so strong in nuclei like ^{11}Be and ^{10}Li that they are, at least partly, responsible for the inversion of the $1/2^+$ and $1/2^-$ states [6, 7, 8]. Both effects, on the two-nucleon force and on the nucleon-core interaction will be included in our two-particle RPA model for the description of ^{14}N and ^{10}B .

The spectra obtained in ^{14}N and ^{10}B will be compared to those derived with the phenomenological T=0 Gogny effective interaction [2, 3]. From this comparison and comparison with shell model calculations [9] using two-body and three-body interactions, we will try to understand better what is implicitly contained in this empirical force.

In section II we present briefly the two-particle RPA model applied to a neutron-proton pair and precise our choice of single neutron and proton basis and our choice of two-body interactions. In section III we first present the results for the 0^+, T=1 states. Then we show the results obtained for the T=0 states with the two bare nucleon-nucleon interactions and discuss the contribution of phonon exchanges between the neutron-proton pair. Then we make the same study with the effective Gogny interaction. Comparing our results with those of shell model calculations we can give a qualitative interpretation of the medium effects contained in this very successful effective interaction. Section IV is devoted to our general conclusions.

II. THE NEUTRON-PROTON RPA MODEL

We describe ^{14}N and ^{10}B as a core of ^{12}C in its ground state plus or minus a neutron-proton pair respectively and define two-body amplitudes as:

$$X_a^+ = \langle {}^{14}N|A_a^+|{}^{12}C\rangle \quad (1)$$

$$Y_\alpha^+ = \langle {}^{14}N|A_\alpha^+|{}^{12}C\rangle \quad (2)$$

$$X_\alpha^- = \langle {}^{10}B|A_\alpha|{}^{12}C\rangle \quad (3)$$

$$Y_a^- = \langle {}^{10}B | A_a | {}^{12}C \rangle \tag{4}$$

where a,b... and α,β,... represent configurations where the neutron and the proton are respectively in states unoccupied and occupied in the Hartree Fock ground state of the ^{12}C core. $A^+_{a(\alpha)}$ ($A_{a(\alpha)}$) are operators which create (annihilate) a neutron-proton pair coupled to given spin and isospin (for simplicity we omit them in our equations). $|^{12}C>$ represents the correlated ground state of the ^{12}C-core. Note that the Y-amplitudes,named the small RPA amplitudes, would be zero for an uncorrelated core.

The two-nucleon RPA model has been described and used in a number of papers and we remind briefly that:

-the same system of equations determines the amplitudes and energies of ^{14}N and ^{10}B which then are not independent. For a given spin and isospin these equations write as:

$$(\Omega - \epsilon_a)x_a - \sum_b < a|V|b > x_b - \sum_\beta < a|V|\beta > x_\beta = 0 \tag{5}$$

$$(\Omega - \epsilon_\alpha)x_\alpha + \sum_b < \alpha|V|b > x_b + \sum_\beta < \alpha|V|\beta > x_\beta = 0 \tag{6}$$

where the eigenvalues,x_a and x_α, are related to the amplitudes of eqs.(1-4) as explained below. The ϵ_a and ϵ_α are the unperturbed energies of the neutron-proton pair in states a and α respectively.The matrix elements of the neutron-proton interaction V have to be antisymmetrised.

-if the model subspace contains N configurations a,b,.. and M configurations α, β ..., the RPA equations have N+M eigenstates with eigenvalues Ω and eigenvectors x_a and x_α. N of them correspond to ^{14}N with:

$$\begin{aligned} &E_n(^{14}N) - E_0(^{12}C) = \Omega_n \\ &X_a^{+(n)} = x_a^{(n)} \\ &Y_\alpha^{+(n)} = x_\alpha^{(n)} \end{aligned} \tag{7}$$

and M to ^{10}B with:

$$\begin{aligned} &E_m(^{10}B) - E_0(^{12}C) = -\Omega_m \\ &X_\alpha^{-(m)} = x_\alpha^{(m)} \\ &Y_a^{-(m)} = x_a^{(m)} \end{aligned} \tag{8}$$

$E_0(^{12}C\ )$ is the ground state energy of the ^{12}C-core. The separation into two sets of solutions is unambigous and based on energy considerations and relative values of x_a and x_α amplitudes.

- the amplitudes x_a and x_α are normalised according to:

$$\sum_a |x_a|^2 - \sum_\alpha |x_\alpha|^2 \ = \ 1 \quad for \ \ ^{14}N \tag{9}$$

$$= \ -1 \quad for \ \ ^{10}B \tag{10}$$

-the only inputs of the calculation are the individual neutron and proton energies and the effective two-body interaction.

A. Choice of the single particle basis

We make a semi-phenomenogical approach to the RPA model. We replace the Hartree Fock average potential by a Saxon-Woods potential plus a spin-orbit force plus a phenomenological surface potential fitted to reproduce the experimental single neutron energies in the field of ^{12}C and write the one neutron hamiltonian as:

$$h_n = t_n + V_0 \left(f(r) - 0.44 r_0^2 (\mathbf{l.s}) \frac{1}{r} \frac{df(r)}{dr} \right) + \delta V_n \tag{11}$$

where:

$$f(r) = [1 + exp(\frac{r - R_0}{a})]^{-1} \tag{12}$$

with V_0=-50.5 MeV, a=0.75 fm, R_0=$r_0(12)^{1/3}$ with r_0=1.27 fm. The last term δV_n is added to simulate medium effects due to the coupling of the neutron with the phonons of the core.. The shape of δV_n is suggested by a semi-microscopic calculation of neutron- phonon couplings [7, 10]. This contribution to the average one body potential depends on the neutron state and is written as:

$$\delta V_n = \alpha_n \left(\frac{df(r)}{dr} \right)^2 \tag{13}$$

The coefficients α_n are fitted on the experimental neutron energies for $1p_{3/2}$, $1p_{1/2}$, 2s and $1d_{5/2}$ states and put to zero for higher states. The proton energies are calculated by adding the Coulomb potential to the neutron hamiltonian of eq.(11). In Table I are given the experimental energies and the corresponding calculated energies for the lowest, known,

	$1p_{3/2}$		$1p_{1/2}$		2s		$1d_{5/2}$	
	exp.	cal.	exp.	cal.	exp.	cal.	exp.	cal.
neutrons	-18.72	-18.71	-4.95	-4.95	-1.85	-1.86	-1.1	-1.1
protons	-15.96	-15.54	-1.94	-1.98	0.42	0.25	1.61	1.35

TABLE I: Experimental and calculated individual energies (in MeV) for neutrons and protons.

neutrons and protons states. The results presented below have been obtained with the $1p_{3/2}$ proton energy replaced by the experimental value. However it has no effect on the ^{14}N spectrum and in ^{10}B gives both 1^+ and 3^+ energies are lowered by about 0.5 MeV what does not change qualitatively our discussion of results. The effect of the Coulomb potential on the wave functions is neglected and neutron and proton wave functions are assumed to be the same.

B. Choice of two-body interactions.

We have first used a zero range density dependent interaction. The general form of such a force writes as:

$$V(\mathbf{r_1},\mathbf{r_2}) = -V_0\left\{1-\eta\left(\frac{\rho((\mathbf{r_1}+\mathbf{r_2})/\mathbf{2})}{\rho_0}\right)^{\alpha}\right\}\delta(\mathbf{r_1}-\mathbf{r_2}) \tag{14}$$

With the parameters of Garrido et al. [11], V_0=500 MeV.fm^3, α=0.47 ,η=-0.1 we get much too weak binding. These parameters were fitted so that to reproduce the gap in nuclear matter calculated using the Paris potential, then they do not take account of the medium effects which are expected to be important. Therefore we have proceeded in a different way. We first use a density independent zero range neutron-proton interaction where the strength V_0 is fitted to give the binding energy of deuteron. The calculation leads to a relation between the T=0 and T=1 strengths given by [12]:

$$V_0(T=0) = V_0(T=1)\left\{1-\left(\frac{-\epsilon_b}{\epsilon_c}\right)^{1/2} Arctg\left(\frac{\epsilon_c}{-\epsilon_b}\right)^{1/2}\right\}^{-1} \tag{15}$$

where ϵ_b is the deuteron binding energy and ϵ_c the cut-off on the nucleon energies which in a nucleus should be counted relative to the bottom of the average one-nucleon potential. In our calculations we take for a nucleon in the field of our ^{12}C-core a cut-off of 10 MeV which is equivalent to ϵ_c=60 Mev and corresponds to:

$$V_0(T=0) = 1.36V_0(T=1) \quad (16)$$

$$= 682MeV.fm^3 \quad (17)$$

A similar ratio between the strengths of the T=0 and T=1 pairing interactions has been found by Satula and Wyss [13] in their study of even-even nuclei. However a zero range force can be considered only as a simple substitute to a more realistic one, and before introducing medium effects in our calculations we have tested this substitution and made the calculation with the Minnesota free nucleon-nucleon force [5]. This force has one short range repulsive components and two long range attractive ones with all exchange terms and has been fitted on nucleon-nucleon scattering lengths and deuteron binding energy. The results with the two forces show very similar results with the same underestimation of two-body correlations suggesting the necessity to take account of medium effects on the neutron-proton interaction. Indeed when two nucleons are added to a core their mutual interaction will be modified by the presence of the other nucleons. Strong two-body correlations which manifest themselves as very collective low energy vibrationnal states can induce a modification of the interaction through an exchange of these collective phonons between the pair of nucleons. Such phonon exchange contribution to the two-neutron pairing force have been studied by Barranco et al. [14] and found to be responsible for about half of the gap in the isotopes ^{A}Ca, ^{A}Ti and ^{A}Sn. In our case the ^{12}C core has a low 2^+, T=0 state at 4.4 MeV with a very strong collective transition amplitude β_2=0.6 and a less collective 3^-, T=0 state at 9.6 MeV with β_3=0.4 [15]. We can expect that these two states will give most of the effect and we include both of them in our calculation.

The diagrams corresponding to the exchange of phonons are represented in Fig.1 for the three types of matrix elements entering in the RPA equations. The diagrams a) -b) concern the matrix elements $< a|V|b >$ while the diagrams c) -d) and e) -f) concern respectively the matrix elements $< \alpha|V|a >$ and $< \alpha|V|\beta >$ of eqs.(5-6) . The calculation of their contribution to the RPA matrix elements is given in the Appendix when each vertex of the diagrams is replaced by a phenomenological anzats. These matrix elements which have to be added to the bare matrix elements depend on the eigenvalues of the RPA equations which therefore are now nonlinear equations and will be solved by iteration.

Once one has seen the effect of phonon exchanges on the ^{14}N and ^{10}B spectra, the last

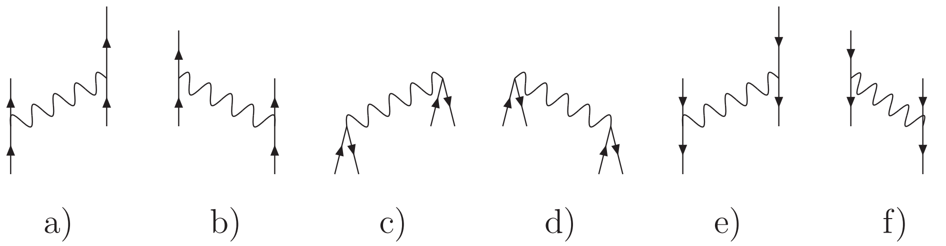

FIG. 1: Diagrams corresponding to a one phonon exchange between neutron-proton pairs appearing in the RPA equations.

step is to look if a phenomenological effective interaction such as the Gogny interaction may be interpreted as a bare interaction corrected by the diagrams of Fig.1. In the T=0 channel the Gogny interactions, D1 [2] and D1S [3], have a density dependent component of zero range. As any zero range interaction, it diverges and its fitted strength depends on the cut-off of the single nucleon energies. Since our cut-off for neutrons and protons is quite low (10 MeV) we have used a slightly increased strength, as will be discussed below.

III. RESULTS

Our configuration subspace is restricted to neutron states up to 10 MeV and to the corresponding proton states.

A. 0^+, T=1 states

Previous calculations [1] have shown that the 0^+, T=1 states in the Li, Be and C isotopes are very well described in the two-neutron RPA model with the Gogny forces or their zero range density dependent substitute [4]. ^{14}N and ^{10}B are the analogs of ^{14}C-^{10}C where the two-neutron pair is replaced by a neutron-proton pair added or substracted from a ^{12}C-core. Because of charge invariance of strong interactions the same T=1 effective interactions should be able to describe the two kinds of nuclear systems. The calculation has been performed for ^{14}N and ^{10}B and yields to the results reported in Table II for the zero range density dependent force and the Gogny interactions D1 and D1S. We see that the three forces give close results as already found in ref[1] for ^{14}C and a good agreement with the

	a)	b)	c)	exp.)
^{14}N	-10.12	-10.18	-10.03	-10.14
	-3.28	-3.10	-3.06	-3.8
^{10}B	29.7	29.0	29.24	29.15

TABLE II: Energies(in MeV) of the 0^+,T=1 states with respect to the ground state of ^{12}C in ^{14}N and ^{10}B obtained with: a) the zero range density dependent force, b) and c) the D1 and D1S Gogny interaction respectively. In the last column are given the experimental energies.

measured energies [16, 17, 18] even though the excited 0^+ state is slightly too high as it was in ^{14}C. We may conclude that we have a good test of the validity of our two-body RPA model as well as a further confirmation of the efficiency of the Gogny effective interactions to describe light nuclei for which it was not designed.

B. 1^+ and 3^+, T=0 states

All results of this section are presented in Tables III-V and Figures 2 and 3 for ^{14}N and ^{10}B .

With the zero range interaction of eq.(14) and the parameters of ref.[11], V_0=500 MeV.fm^3, η =-0.1 and α=0.2, our results desagree with experimental spectra. In particular the lowest 1^+ state in ^{14}N is above the lowest 0^+ T=1 which is then the ground state in desagreement with experiments [16].

Instead we have first made the calculations using the bare density independent zero range neutron-proton interaction with the strength V_0=682 MeV.fm^3 of eq.(17) and with the Minesota force [5]. The energies of the lowest states referred to the theoretical ground state energy of ^{12}C as defined in eqs.(7-8) are given in the table III. We see that the energies are very similar for the two bare interactions but that there is a significant desagreement with experimental energies for both nuclei. The desagreement is still more pronounced for ^{10}B where the ground state is found as a 1^+ state while experimentally it is a 3^+ state. As expected we see clearly that a free neutron-proton interaction is not sufficient to give the binding of two nucleons inside a nucleus. Then we have introduced medium effects due to the exchange of phonons as explained in section II.B and in the Appendix and have

J^{ϖ}	a)	b)	c)	d)	e)	exp.
^{14}N 1^+	-11.7	-11.9	-12.4	-12.8	-12.5	
1^+	-5.8	-4.61	-5.75	-6.9	-6.9	-6.3
3^+	-3.26	-4.27	-5.55	-7.25	-7.2	-6.05
^{10}B 1^+	27.6	28.7	28.1	29.1	28.4	28.1
3^+	30.3	29.	28.6	28.3	27.7	27.4

TABLE III: Energies of the lowest 1^+ and 3^+,T=0 states in ^{14}N and ^{10}B with respect to the ground state energy of ^{12}C obtained with different interactions: a) the bare zero-range interaction -b) the bare Minnesota interaction -c)the Minnesota force plus the exchange of phonons -d) and e) the Gogny force with respectively the experimental and calculated $1p_{3/2}$ proton energy . In the last column are given the experimental energies.

calculated the induced matrix elements according to eqs.(21-29). The results are again very similar for the two interactions and we show in the tables and figures those obtained with the Minnesota interaction. The excitation spectra are shown in Fig.2 for ^{14}N and Fig.3 for ^{10}B for the bare and bare plus induced interaction. At the bottom of the figures are given the separation energies of a neutron-proton pair in ^{14}N and ^{12}C which are directly related to the lowest RPA energies in ^{14}N and ^{10}B respectively.

We see an improvement for the ^{14}N excited spectrum as well as for the neutron-proton separation energies. The excitation spectrum of ^{14}N (see Figure 2) shows now a quite good agreement with the experimental spectrum. Since we take in the definition of our neutron-proton subspace nucleon states up to 10 MeV we have in the higher part of the ^{14}N spectrum a large number of states but we have given only those called pairing vibrational states which have amplitudes on several neutron-proton configurations. The other levels have a smaller probability to be seen experimentally [19]. For these high energy levels we have not calculated the contribution of phonon exchange. Our code is inefficient when we have several very close eigenstates but we expect the induced matrix elements to be weak because of energy denominators. The contribution of phonon exchange is very important for the low energy states and improves significantly the energy spectrum where now the ground state energy, or equivalently the neutron-proton separation energy, is very good. At about 6 MeV we can reproduce the experimental 1^+- 3^+ doublet with the right order of levels

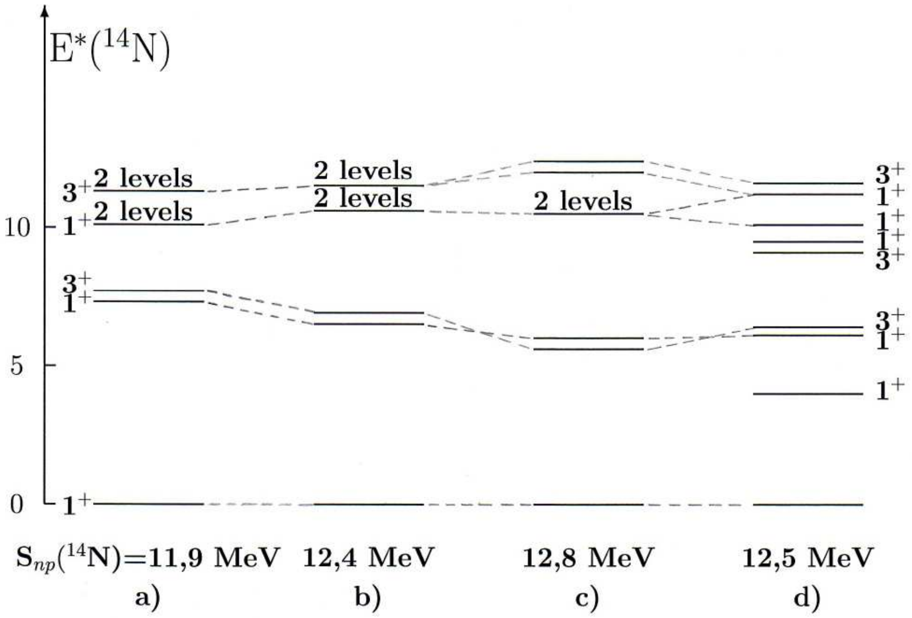

FIG. 2: Excited spectrum of ^{14}N calculated with: a) the bare Minnesota interaction-b) the same with the exchange of phonons-c) the Gogny interaction (D1S with t_3=1600 MeV.fm^3. The experimental spectrum is given in the last column. Below are given the ccalculated and experimental neutron-proton separation energies.

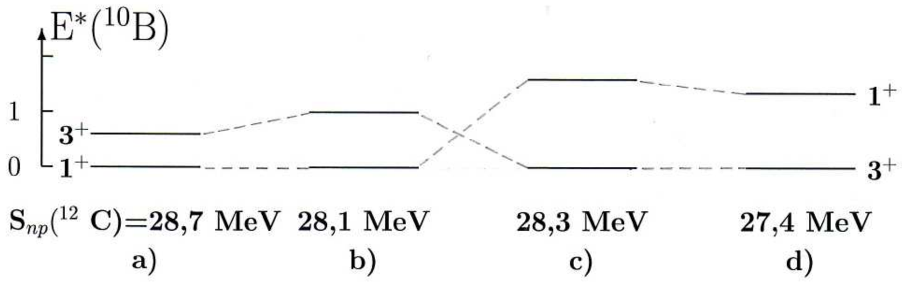

FIG. 3: Same legend as Fig.2 for ^{10}B . The numbers at the bottom of the figure are the neutron-proton separation energies, calculated and experimental, in ^{12}C.

and energies very close to the experimental ones. The main RPA amplitudes are given in Table IV for the lowest states . We see that the RPA Y-amplitudes with two nucleons in the $1p_{3/2}$ shell are not negligible what indicates the presence in the wave function of the ^{12}C ground state of configurations with at least two holes in this shell, in agreement with the shell model calculation of Cohen and Kurath for example [20] . In the higher part of the spectrum, above 10 MeV, we get a group of two 1^+ and two 3^+ states which may be

J^{ϖ}	E*(MeV)	$(1p_{1/2})^2$	$(1d_{5/2})^2$	$(2s)^2$	$(2s,1d_{5/2})$	$(1p3/2)^2$
1^+	0	0.88	0.24	0.1		-0.24
1^+	6.6	-0.22	0.50	0.74		-0.11
3^+	6.8		0.50		0.78	-0.13

TABLE IV: RPA amplitudes in ^{14}N

	$(1p_{1/2})^2_{1^+,0}$	$(1p_{3/2})^2_{1^+,0}$	$(1p_{3/2})^2_{3^+,0}$
Min	-3.42	-3.74	-5.19
Min'	-3.42	-4.03	-5.49
PWBT	-3.45	-4.16	-6.04

TABLE V: Diagonal matrix elements (in MeV) for the two nucleons in the $1p_{1/2}$ or $1p_{3/2}$ shells coupled to $J^{\varpi}=1^+$ or 3^+ calculated with the bare Minnesota interaction (Min), when the phonon exchange contribution is added (Min') and those fitted in ref.[24] by Warbuton and Brown(PWBT).

identified with the same experimental group. However we miss the experimental 1^+ level at 3.95 MeV which is very likely formed mainly of a ^{12}C core excited to its 2^+ state at 4.4 MeV coupled to the n-p pair in its 1^+ ground state. This is also suggested by the analysis of ^{12}C (^{6}Li,α)^{14}N [21] and ^{16}O(γ,np)^{14}N reactions [22, 23]. RPA is not able to describe such state since it relies on the assumption of an inert core in its ground state. In the same way we miss the 1^+ and 3^+ states at about 9 MeV which again are very likely due to the coupling of the same 2^+ in ^{12}C with the neutron-proton pair in the 1^+ or the 3^+ excited states at about 6 MeV. This assumption is supported by the presence in the experimental spectrum [16]of two T=0 levels, a 2^+ at 8.98 MeV and a 5^+ at 8.96 MeV which very likely belong to the same multiplet.

In ^{10}B the results are improved by phonon exchanges but not enough to get the 3^+ state as the ground state. The energy of the 1^+ when referred to the core ground state energy, is close to the experimental value but the 3^+ is too high and appears again as an excited state. This 3^+ state is a nearly pure $(p_{3/2})^{-2}$ state weakly affected by RPA correlations because the lowest possible configuration with unoccupied neutron and proton unperturbed states (anomalous RPA configuration) is the $(2s,1d_{5/2})$ configuration, much higher in energy. The correlated energy is therefore mostly due to the diagonal matrix element $\langle (p_{3/2})^2|V|(p_{3/2})^2\rangle$ which appears to be too weak as is discussed now. Indeed we compare our matrix elements with those of Warburton and Brown [24] which are determined by least-squares fits to 216 levels in A=10-22 nuclei. They use a (1p,2s,1d) shell model space, compatible with our subspace, at least for the lowest states. In Table V are reported their fitted matrix elements and ours calculated with the Minnesota plus phonon exchange when the n-p pair is in the 1p-shell. We show the diagonal matrix elements for the n-p pair in the $1p_{3/2}$ and $1p_{1/2}$ shells coupled to spin 1^+ and 3^+ which are for a large part responsible for the binding energy of the lowest 1^+ states in ^{14}N and ^{10}B and the 3^+ state in ^{10}B . The same comparison has been made with the matrix elements fitted by Cohen and Kurath [20] in a smaller model space but the conclusion is qualitatively the same. We see a good agreement when the two nucleons are coupled to J=1^+ but a too weak binding for J=3^+. It shows that something is missing in our force and a possible reason why it is too weak could be found in more recent shell model calculations. Indeed in a no-core shell model and with the Argonne V8' two-nucleon force , Aroua et al [26] get a good description of the 1^+ ground state of ^{14}N while Caurier et al.[25] and Navratil and Ormand [9] show that with the same force and the same model it is not possible to get the ground state spin of ^{10}B . The authors of ref.[9] show that it is correctly obtained only when they include the Tucson-Melbourne three-body interaction. The necessity to use a three-body force in the description of ^{10}B is also clear from quantum Monte Carlo shell model calculations of ^{10}B [27, 28] A genuine three-body force is out of the scoop of the RPA since we work with two nucleons only. However this three-body term could induce an effective two-body contribution which is not included when we use the Minnesota force plus phonon exchange. Intuitively we expect this term to be more important for two $1p_{3/2}$-nucleons inside the ^{12}C, therefore for the description of ^{10}B , where they will interact with a third nucleon off the six other nucleons in the same shell than for the two nucleons added in higher shells to describe ^{14}N which will have less interaction with a third nucleon inside ^{12}C. This might explain why our calculations failed in ^{10}B but are very satisfying in ^{14}N .

At last we have made the same RPA calculations with the D1 and D1S Gogny forces [2, 3]. The two forces give the same results and we discuss only those obtained with D1S. In the T=0 channel this force has a zero range density dependent component with a strength t_3=1390.6 MeV.fm^3. Any zero range interaction is divergent and the fitted strength depends on the cut-off on single-particle energies. Since our cut-off is somewhat low we have increased t_3 in order to get close to the measured energy of the 1^+ ground state in ^{14}N . With t_3=1600 MeV.fm^3, a value slightly stronger than the genuine value, we get a binding energy for the n-p pair of -12.7 MeV instead of -12.5 MeV, the experimental value. In the third columns of Figs.2 and 3 we show the excited spectra obtained for ^{14}N and ^{10}B respectively. We see that for ^{14}N the levels are in very close agreement with experiment and very close to those obtained with the Minnesota interaction plus the phonon exchange contribution with however an inversion of the 1^+- 3^+ states at about 6 MeV. This result suggests that the Gogny interaction, often thought as a G-matrix, includes implicitly the phonon exchange contribution. In ^{10}B the 3^+ state is obtained as the ground state as it should, contrarly to what we got previously with an effective two-body interaction. The 1^+ state is now the first excited state with an excitation energy in agreement with the experimental value and with shell model calculations of ref[9]. Therefore according to our previous discussion on shell model results we may conclude that, through its density dependent term, the Gogny interaction includes implicitly an effective two-body contribution coming from three-body forces.However the neutron-proton separation energy in ^{12}C given as the difference between the ground state energies in ^{10}B and ^{12}C is too high by 0.8 MeV but a still larger overestimation is observed in the shell model calculations (see their tables V and VII). Note that if we use the calculated energy for a $1p_{3/2}$ proton (see Table 1) instead of the experimental energy this discrepancy is strongly reduced.

IV. CONCLUSIONS

In the framework of a two-particle RPA model applied to the description of ^{10}B and ^{14}N formed of a correlated core of ^{12}C in its ground state minus or plus a neutron-proton pair, we have performed a detailed analysis of the T=0 effective nucleon-nucleon interaction. First we have shown that for the 0^+ T=1 states the results are as good as they were for ^{14}C-^{10}C (where the neutron-proton pair is replaced by a two-neutron pair) when the Gogny interactions, D1 and D1S, or their zero range equivalent are used. The same two-nucleon RPA model has been applied to 1^+ and 3^+, T=0 states and an attempt to analyse the contains of an effective interaction has been made. We have shown that an effective interaction constructed as a bare interaction, from Minnesota or of zero range, fitted to the deuteron binding energy, complemented by medium effects due to phonon exchanges between the neutron-proton pair gives a very good representation of the ^{14}N levels but fails to reproduce the 3^+ ground state of ^{10}B . By comparing with shell model calculations we are able to suggest that this is due to the presence of a three-body component in the interaction which are not included in our derivation. This additionnal component will not spoil our good results in ^{14}N but will improve those in ^{10}B . At last our study suggests that the T=0 Gogny interaction which yields good agreement with measurements in both nuclei, includes empirically both the effect of phonons exchange in the effective interaction and the effect of a two-body component coming from the presence of a three-body interaction and included in the density dependent term.

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APPENDIX A: MATRIX ELEMENTS OF THE INTERACTION INDUCED BY PHONON EXCHANGE.

The matrix elements of the induced interaction of Figure I are calculated between two neutron-proton pairs coupled to spin J,M and isospin T=0.

Each vertex in the diagrams is replaced by a phenomenological expression. Let's call L,M_L the angular momentum of the phonon, the transition density from a zero-phonon state to the one-phonon state is written as:

$$< 1ph|V|0ph >= \frac{1}{\hat{L}}\beta_L R_0 \frac{dU(r)}{dr} Y_L^{M_L *}(\omega) \tag{A1}$$

where U(r) is the average one-body potential assumed to be a Saxon-potential such as:

$$\frac{dU(r)}{dr} = -U_0 \frac{df(r)}{dr} = \frac{U_0}{a} g(r) \tag{A2}$$

where the function f(r) and the values of the parameters are given in section II. The collective amplitudes, β_L, can be fitted on the experimental values of the B(EL) or on proton inelastic scattering cross sections.

The two-nucleon wave function is constructed by coupling the two-nucleon states to a total spin (JM) as:

$$|j_1 j_2, JM >= \sum_{m_1 m_2} < j_1 j_2 m_1 m_2 | JM > |l_1 j_1 m_1 > |l_2 j_2 m_2 > \tag{A3}$$

where the nucleons 1 and 2 are either in occupied states, α_1 α_2, or unoccupied states, a_1 a_2 with energies ϵ_{α_1}, ϵ_{α_2} or ϵ_{a_1} ϵ_{a_2} respectively. The total wave function has to be antisymmetric so that for T=0 states the spin-space wave function has to be symmetric. Therefore the general expression for the antisymmetrised matrix element of the induced interaction, V_{ind}, is obtained as:

$$< 12|V_{ind}|34 >= \sum_L \{V_{dL}D_{dL} + V_{eL}D_{eL}\} \tag{A4}$$

with:

$$\begin{aligned}
V_{dL} &= \frac{1}{2}[1+(-1)^{l_2+l_4+L}](-1)^{j_1+j_3+J}\frac{\beta_L^2 R_0^2 U_0^2}{4\pi a^2}\frac{1}{\sqrt{(1+\delta_{12})(1+\delta_{34})}}\hat{j}_1\hat{j}_2\hat{j}_3\hat{j}_4 \\
&\begin{pmatrix} j_1 & j_3 & L \\ 0.5 & -0.5 & 0 \end{pmatrix}\begin{pmatrix} j_2 & j_4 & L \\ 0.5 & -0.5 & 0 \end{pmatrix}\begin{Bmatrix} j_1 & j_2 & J \\ j_4 & j_3 & L \end{Bmatrix} < 1|g|3 >< 2|g|4 > \qquad \text{(A5)} \\
V_{eL} &= \frac{1}{2}[1+(-1)^{l_2+l_3+L}](-1)^{j_1-j_3}\frac{\beta_L^2 R_0^2 U_0^2}{4\pi a^2}\frac{1}{\sqrt{(1+\delta_{12})(1+\delta_{34})}}\hat{j}_1\hat{j}_2\hat{j}_3\hat{j}_4 \\
&\begin{pmatrix} j_2 & j_3 & L \\ 0.5 & -0.5 & 0 \end{pmatrix}\begin{pmatrix} j_1 & j_4 & L \\ 0.5 & -0.5 & 0 \end{pmatrix}\begin{Bmatrix} j_2 & j_1 & J \\ j_4 & j_3 & L \end{Bmatrix} < 2|g|3 >< 1|g|4 > \qquad \text{(A6)} \\
< m|g|n > &= \int_0^\infty g(r)\phi^*_{l_m j_m}(r)\phi_{l_n j_n}(r) r^2 dr \qquad \text{(A7)}
\end{aligned}$$

where the ϕ's are the one-nucleon radial wave functions.

While the quantities V_{dL} and V_{eL} have the same expressions whatever are the initial and final two-nucleon states, occupied or unoccupied, the expressions of D_{dL} and D_{eL} which come from energy denominators have to be derived in three different cases corresponding to the diagrams of Fig.I:

If (12)=(a_1 a_2), (34)=(a_3 a_4) (diagrams a) and b))

$$D_{dL} = [\Omega - (\epsilon_{a_2} + \epsilon_{a_3} + \omega_L)]^{-1} + [\Omega - (\epsilon_{a_1} + \epsilon_{a_4} + \omega_L)]^{-1} \tag{A8}$$

$$D_{eL} = [\Omega - (\epsilon_{a_1} + \epsilon_{a_3} + \omega_L)]^{-1} + [\Omega - (\epsilon_{a_2} + \epsilon_{a_4} + \omega_L)]^{-1} \tag{A9}$$

If (12)=(a_1 a_2), (34)=(α_1 α_2) or the inverse (diagrams c) and d)):

$$\begin{aligned}
D_{dL} &= -[\epsilon_{a_2} - \epsilon_{\alpha_2} + \omega_L]^{-1} - [\epsilon_{a_1} - \epsilon_{\alpha_1} + \omega_L]^{-1} \\
D_{eL} &= -[\epsilon_{a_1} - \epsilon_{\alpha_2} + \omega_L]^{-1} - [\epsilon_{a_2} - \epsilon_{\alpha_1} + \omega_L]^{-1} \qquad \text{(A10)}
\end{aligned}$$

It is straightforward to show that:

$$< a_1 a_2 | V_{ind} | \alpha_1 \alpha_2 > = < \alpha_1 \alpha_2 | V_{ind} | a_1 a_2 > \tag{A11}$$

If (12)=(α_1 α_2), (34)=(α_3 α_4) (diagrams e) and f)):

$$\begin{aligned} D_{dL} &= -[\Omega - \epsilon_{\alpha_2} - \epsilon_{\alpha_3} + \omega_L]^{-1} - [\Omega - \epsilon_{\alpha_1} - \epsilon_{\alpha_4} + \omega_L]^{-1} \\ D_{eL} &= -[\Omega - \epsilon_{\alpha_1} - \epsilon_{\alpha_3} + \omega_L]^{-1} - [\Omega - \epsilon_{\alpha_2} - \epsilon_{\alpha_4} + \omega_L]^{-1} \end{aligned} \tag{A12}$$

In these equations Ω is the eigenvalue of the RPA eqs.(5-6) and ω_L is the energy of the phonon L. When we add these phonon exchange contributions to the bare matrix elements, the RPA equations become non-linear and will be solved by iteration.

Note that the approximation of eq.(18) implies that one can defined an equivalent two-body term which has to be added to the bare neutron-proton interaction which is the sum of separable terms of the following form:

$$\delta V = \frac{d\rho(r)}{dr} \frac{d\rho(r')}{dr'} Y_L^M(\omega) Y_L^M(\omega') \tag{A13}$$

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[1] J.C.Pacheco and N.Vinh Mau, Phys. Rev. **C65** 044004 (2002)

[2] J. Decharge and D. Gogny Phys. Rev. **C21** 1568 (1980)

[3] J.F. Berger, M. Girod and D. Gogny Computer Physics Communiccations **63** 365 (1991)

[4] E.Garrido et al., Phys. Rev. **C60** 064312(1999)

[5] D.R.Thomson, M.Lemere and Y.C. Tang Nucl. Phys. **A286** 53 (1977)

[6] H. Sagawa et al. Phys. Lett. **B309**, 1, (1993)

[7] N. Vinh Mau Nucl. Phys. **A592** 33(1995)

[8] G. Gori et al., Phys. Rev. **C69** 041302 (2004)

[9] P. Navratil and W. E. Ormand Phys. Rev. **C68** 034305 (2003)

[10] N. Vinh Mau and J.C. Pacheco, Nucl. Phys. **A607**, 163 (1996)

[11] E. Garrido et al., Phys. Rev. **C63**, 037304 (2001)

[12] G. F. Bertsch and H. Esbensen Ann. of Phys. (N.Y.), 209 (1991)

[13] W. Satula and R. Wyss Phys. Rev. Lett. **86** 4488 (2001)

[14] F. Barranco et al. Phys. Rev. Lett. **83**, 2147 (1999)

[15] S.A. Satchler, Nucl. Phys. **A128**, 401 (1969)

[16] F. Ajzenberg-Selove Nucl. Phys. **A523**, 1 (1991)

[17] A.H. Wapstra and G. Audi, Nucl. Phys. **A565** 1 (1993)

[18] F. Ajzenberg-Selove Nucl. Phys. **A490**, 1 (1988)

[19] A. Bouyssy and N. Vinh Mau, Phys. Lett.**35B**,269 (1971)

[20] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965)

[21] A.J. Mendez et al., Phys. Rev. **C51**, 651 (1995)

[22] L. Isaksson et al., Phys. Rev. Lett. **83**, 3146 (1999)

[23] K.R. Garrow et al., Phys. Rev. **C64**, 064602 (2001)

[24] E.K. Warburton and B.A. Brown, Phys. Rev. **C46**, 923 (1992)

[25] E. Caurier,P. Navratil, W.E. Ormand and J.P. Vary, Phys. Rev. **C66**, 024314 (2002)

[26] S. Aroua et al., Nucl. Phys. **A720**, 71 (2003)

[27] S.C. Pieper, K. Varga and R.B. Wiringa, Phys. Rev. **C66**, 044310 (2002)

[28] R.B. Wiringa and S.C. Pieper, Phys. Rev. Lett. 89,1825501 (2002)