

H_∞ deflection control of a unimorph piezoelectric cantilever under thermal disturbance

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Abstract—The effect of the temperature variation on a unimorph piezoelectric cantilever is studied. Its influences on the thermal expansion, the piezoelectric constant, the transient part and the creep are experimentally analyzed. Afterwards, a H_∞ controller is synthesized in order to reject the thermal disturbance and to reach performances required in micromanipulation. Finally, the closed-loop experiments end the paper.

I. INTRODUCTION

Nowadays, miniaturized systems, integrating more intelligence and functionalities are more and more required in our every day life. These systems are either micro-mechanisms (micro ball bearings, microgears, micromotors), micro-optical systems (switches, laser) or hybrid Micro-Opto-Electro-Mechanical-Systems (MOEMS) like micro-scanners, micro-mass spectrometer, or micro-coils [1][2]. Their small sizes impose the use of the micromanipulation and microassembly techniques.

One of the main requirements of micromanipulation systems is their ability to automatically perform efficient, reliable and precise tasks. For instance, fixing a micro lens at the tip of an optical fiber with $1 \mu\text{m}$ of relative positioning error or $0,4 \mu\text{rad}$ of orientation error causes a loss of 50 % of the light flux [3].

Such a positioning accuracy is commonly required and can only be obtained using active materials as actuators. Among them, piezoelectric materials are widespread and their use keeps growing due to their fast response time, resolution and sensor capabilities.

Unfortunately, performances of piezoelectric actuators like other active materials are strongly dependent on the environment conditions: vibrations, evolution of ambient temperature, etc. For instance, the microassembly system presented in the Fig. 1 is currently used at the LAB - Automatic Laboratory of Besançon. This system notably includes a unimorph piezoelectric actuated microgripper [4], a 2DoF stick-slip microsystem [5], a stereo microscope, DC motors and a tool changer [6]. The whole microassembly system is put under a glass box to reduce the influence of the outside environment. Nevertheless, the temperature inside the box (working temperature) generally varies in the range of 19°C to 27°C during

the day. Deported lights i.e. "cold lights", the system of tool changer and the working of positioning motors are the main causes of such variations. Fixing systems (active gluing, laser) and other peripheral systems (backlight for visual servoing) could also be other sources of temperature variations. Some of these can generate fast localized heating or cooling of 1 to 10° whereas others are responsible of slow and weak influence. These changes are the cause of a high inaccuracy notably due to thermal expansion and temperature dependancy of piezoelectric constants. It is possible to use a controlled environment [7] but this method does not reject the localized heating. Then, it is necessary to have a microassembly station whom the microrobots and microsystems are robust relative to the environmental disturbances. Moreover, such approach seems less expensive than a controlled environment approach.

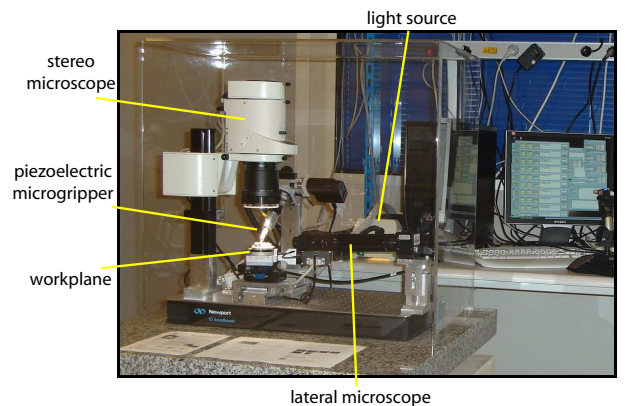


Fig. 1. Photography of the microassembly system used at the LAB. The piezoelectric microgripper is moved by a linear-angular axis. The workplane enables $X - Y - \theta$ motions, a stereomicroscope equipped with a deported light offers a view from the top and a PC is used to control all these devices together.

This paper deals with the analysis and H_∞ control of unimorph piezoelectric cantilevers when the working temperature varies from 20°C up to more than 30°C .

First, the thermal effects on unimorph piezoelectric cantilevers are analyzed. Then, a model is presented and a H_∞ controller is computed. The external force applied to the cantilever and the working temperature represent the disturbances and are taken into account during the controller synthesis. Finally, the experimental results of the closed-loop system are presented.

II. THERMAL EFFECTS ON A UNIMORPH PIEZOELECTRIC CANTILEVER

In our application, a unimorph piezoelectric cantilever has been used because of the low voltage required to produce high deflection and because of the simplicity of the electric wiring. A unimorph piezoelectric cantilever is made up of a piezoelectric layer and a passive layer glued themselves (Fig. 2). In addition with its converse piezoelectric effect, a bending of the cantilever can also be got when the working temperature is changed. On the one hand, a temperature change ΔT will result in expansion of both elements, but generally with different amounts (due to different thermal expansion coefficients), and then resulting a bending of the cantilever [8]. On the other hand, the piezoelectric constant d_{31} of the piezo layer is also influenced by the temperature [9] so that the equivalent piezoelectric constant d_p of the cantilever will also be influenced. In this section, we analyze experimentally the effect of the temperature on a unimorph piezoelectric cantilever.

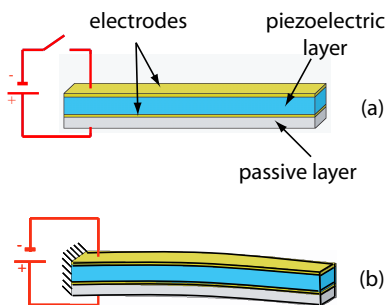


Fig. 2. Constitution of a unimorph piezoelectric cantilever.

For all the experiments, we use a cantilever made up of a PZT layer and a Copper layer. The sizes is $15\text{mm} \times 2\text{mm} \times 0.3\text{mm}$ (length, width and thickness). Fig. 3 shows the experimental setup. A laser sensor is used to measure the deflection while two thermocouples are used to measure the temperature at the left and right sides of the cantilever. A hotplate is used to heat the whole system. The sensors, the cantilever and the hotplate are placed inside a glass box in order to prevent external perturbations. A computer is used to control the whole setup. During the experiments, the temperature analysis range is $[20^\circ\text{C}, 30^\circ\text{C}]$. Experiments show that the laser sensor is insensitive to such range of temperature.

A. General effect of the temperature on the cantilever

The first experiment consists in analyzing the effect of the temperature on the cantilever bending when the applied voltage U is constant. Fig. 4 shows the evolution of the deflection when the temperature varies and when a step voltage of 15V has been applied at $t = 0\text{s}$. It can be deduced that the deflection due to the temperature can reach nearly four times more than the deflection obtained with the voltage.

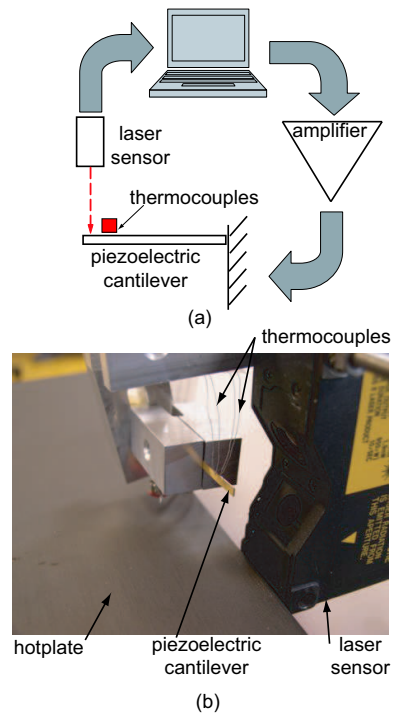


Fig. 3. The experimental setup.

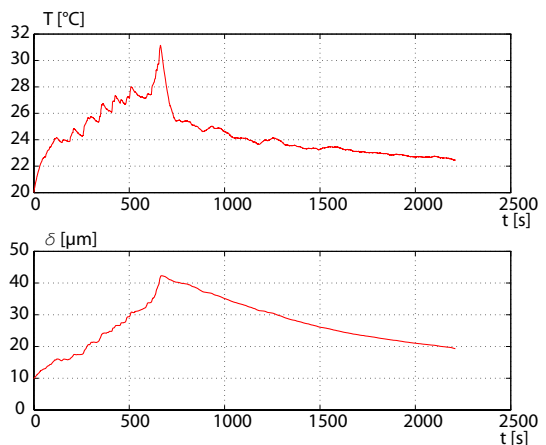


Fig. 4. Evolution of the deflection when the temperature varies (with $U = 15\text{V}$).

A temperature-deflection (T, δ) curve (Fig. 5) is then performed for three values of voltage: $U = 0\text{V}$ (short-circuited electrodes), $U = 15\text{V}$ and $U = 30\text{V}$. The figure shows that the static mode characteristics relating the temperature and the deflection is nearly hysteretic. In fact, the elements of the unimorph are very temperature sensitive so that the used temperature range makes the cantilever work in the plastic deformation.

B. Effect of the temperature on the (U, δ) -plane

The objective of this analysis is to evaluate the effect of the temperature on the piezoelectric constant d_p of the cantilever. In this aim, the static electromechanical domain $\delta = d_p \cdot U$ is plotted. Experiments with

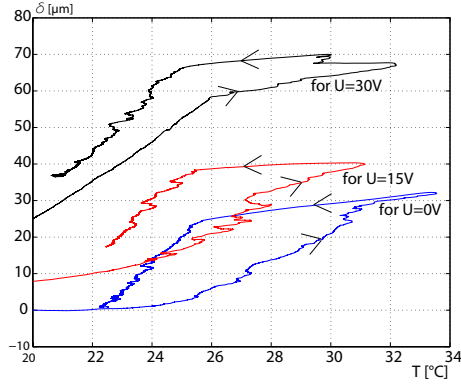


Fig. 5. Deflection versus temperature for different values of voltage.

three working temperatures (20°C , 30°C and 35°C) were performed. For each temperature, the experiment consists in applying a sine voltage to the cantilever and measuring the resulting deflection δ . The corresponding (δ, U) -curves are presented in the Fig. 6. In order to compare the three curves, the offsets of the deflection due to the voltage have been removed. The curves are slightly hysteretic but a linear form with a slope $d_p = \frac{\partial \delta}{\partial U}$ can approximate them. According to the results, the partial slopes for the three temperatures are almost equivalent. In conclusion, the effect of the temperature on the piezoelectric constant is negligible and the general effect given in the previous section mainly concerns the thermal expansion.

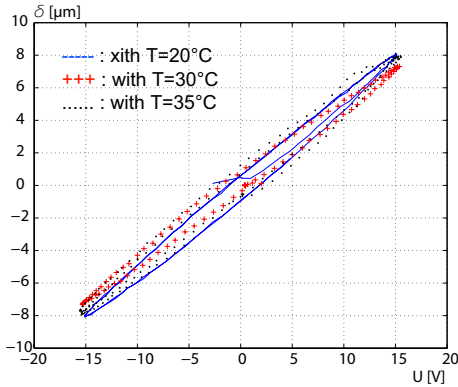


Fig. 6. Deflection versus voltage with different temperatures.

C. Effect of the temperature on the response

Here, we analyze the effect of the temperature on the response, i.e. the dynamic characteristic, of the piezoelectric cantilever. For that, a step voltage ($U = 15\text{V}$) is applied. Fig. 7 presents the step responses for two working temperatures. The two corresponding curves well line up. It can be deduced that the electromechanical response is not influenced by the temperature.

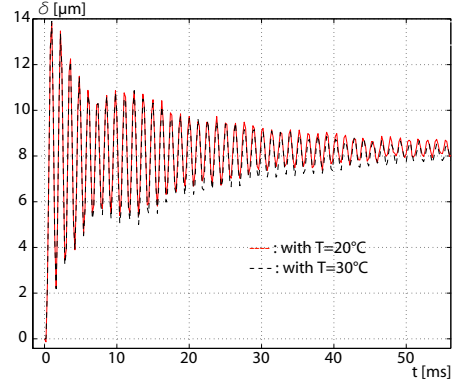


Fig. 7. The temperature does not affect the response of the unimorph piezoelectric cantilever.

D. Effect of the temperature on the creep phenomenon

Finally, the effect of the temperature on the creep phenomenon is analyzed. When a step voltage is applied to a piezoelectric cantilever, a drift appears after the response ends. This phenomenon is called 'creep' and can be modelled with various manners [10][11][12]. On the one hand, the amplitude of the creep depends on the amplitude of the step voltage. On the other hand, while the duration of the response is generally lower than 200ms , the duration of the creep is higher than 2min . Here, we analyze the amplitude of the creep for a given voltage ($U = 15\text{V}$) but with two working temperatures. The results are shown in the Fig. 8. While the creep is insignificant with $T = 20^{\circ}\text{C}$, it is very relevant for $T = 30^{\circ}\text{C}$.

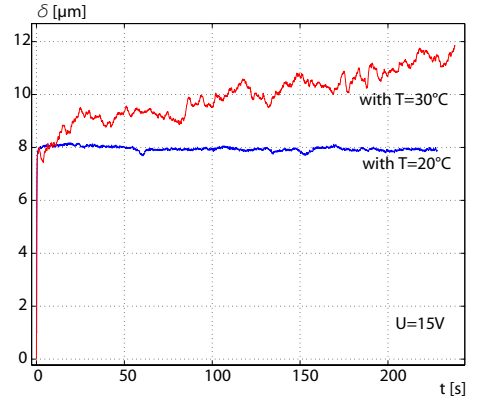


Fig. 8. Effect of the temperature on the creep phenomenon.

In this part, we have seen the effects of the temperature change on unimorph piezoelectric cantilevers. These effects are significant and it is necessary to reject them when high accuracy is required, such as in micromanipulation and microassembly. The aim of the following sections is the synthesis of a robust controller to reject the thermal effects.

III. MODELLING

In order to synthesize a controller, we give a behavioral model in this section. Let the deflection δ be the output variable while the voltage U the applied force F and the amount of temperature $\Delta T = T - T_0$ ($T_0 = 20^\circ C$ is the initial temperature) be the excitations (Fig. 9).

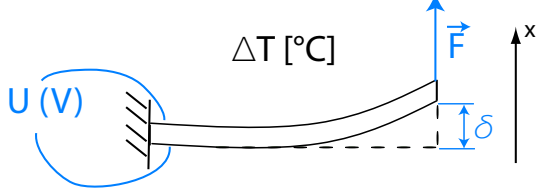


Fig. 9. The unimorph piezoelectric cantilever under three external excitations.

According to *Smits and Choi* [8], the relation between the deflection and the three excitations in the static mode is affine:

$$\delta = d_p \cdot U + s_p \cdot F + c_p \cdot \Delta T \quad (1)$$

with the piezoelectric constant $d_p > 0$, the elastic constant $s_p > 0$ and the thermoelastic constant $c_p > 0$.

However, the previous experiments (see Fig. 5) show that the thermal working domain leads to a nonlinear characteristic. Then, we have:

$$\delta = d_p \cdot U + s_p \cdot F + f(\Delta T) \quad (2)$$

Where $f(\Delta T)$ is a nonlinear operator.

Let $D(s)$ (where s is the Laplace variable and $D(0) = 1$) be the dynamic part of the transfer function relating the voltage and the deflection. It has been shown that $D(s)$ also represents the dynamic part of the transfer function relating the force and the deflection [12]. Let $g(\Delta T)$ an operator relating the temperature and the deflection. It takes into account the static nonlinear operator $f(\Delta T)$ and the dynamic part between the temperature and the deflection. So, we have:

$$\delta = (d_p \cdot U + s_p \cdot F) \cdot D(s) + g(\Delta T) \quad (3)$$

The last equation is equivalent to:

$$\delta = d_p \cdot D(s) \cdot \left(U + \frac{s_p}{d_p} \cdot F \right) + g(\Delta T) \quad (4)$$

In the (4), the input control is the voltage U . The nominal model $G = d_p \cdot D(s)$ is subjected to an input mechanical disturbance and an output thermal disturbance (Fig. 10). During the controller synthesis, we use the signals b_1 and b_2 to represent these disturbances.

The different parameters have been identified. we have:

$$\begin{cases} d_p = 0.533[\mu m/V] \\ s_p = 1.33[\mu m/mN] \end{cases} \quad (5)$$

and

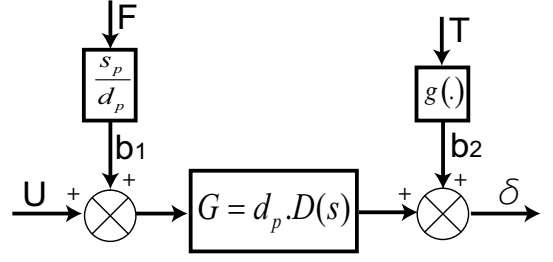


Fig. 10. The bloc-scheme of the piezoelectric cantilever.

$$D(s) = \frac{-0.084 \cdot (s - 2.4 \times 10^4) \cdot (s + 10^4) \cdot (s^2 + 7075 \cdot s + 3.5 \times 10^7)}{(s^2 + 6669 \cdot s + 2.4 \times 10^7) \cdot (s^2 + 93 \cdot s + 2.9 \times 10^7)} \quad (6)$$

IV. H_∞ CONTROL OF THE DEFLECTION

Because of the robustness offered by the H_∞ control, a H_∞ controller has been synthesized and implemented. The objective is to reject the mechanical and thermal disturbances in preserving tracking performances. As the unimorph cantilever does not require high voltage, the minimization of the latter will not be taken into account. We introduce three weighting transfer functions W_1 , W_2 and W_3 (Fig. 11). In this figure, δ_c represents the reference deflection.

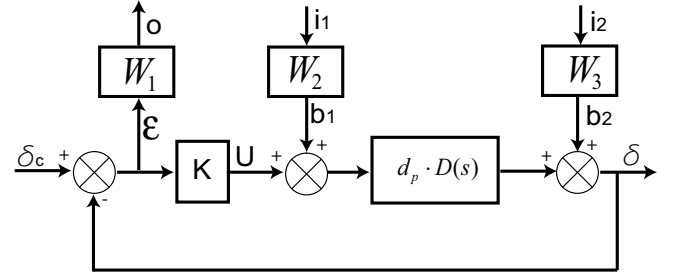


Fig. 11. The closed-loop scheme with the weighting transfer functions.

A. Standard form

Let $P(s)$ be the plant equivalent to the nominal system $G(s)$ augmented by the weighting functions. Fig. 12 represents the corresponding standard-scheme.

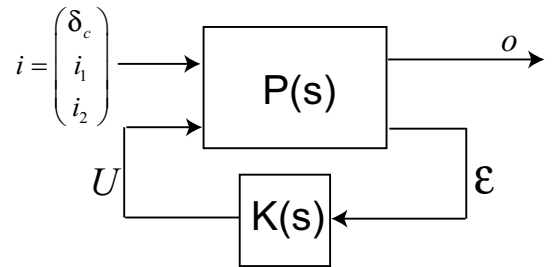


Fig. 12. The standard form.

The standard H_∞ problem consists in finding an optimal value $\gamma > 0$ and a controller $K(s)$ stabilizing the closed-loop scheme of the Fig. 12 and guaranteeing the following inequality [13]:

$$\|F_l(P(s), K(s))\|_\infty < \gamma \quad (7)$$

where $F_l(\cdot, \cdot)$ is the lower Linear Fractional Transformation and is defined by $F_l(P(s), K(s)) = \frac{o(s)}{i(s)}$.

From the Fig. 11, we have:

$$o = W_1 \cdot S \cdot \delta_c - W_1 \cdot S \cdot G \cdot W_2 \cdot i_1 - W_1 \cdot S \cdot W_3 \cdot i_2 \quad (8)$$

where $S = (1 + K \cdot G)^{-1}$ is the sensitivity function.

Using the condition (7) and the (8), we infer:

$$\left\| \begin{array}{c} W_1 \cdot S \\ W_1 \cdot S \cdot W_3 \\ W_1 \cdot S \cdot G \cdot W_2 \end{array} \right\|_\infty < \gamma \Rightarrow \begin{array}{l} |S| < \frac{\gamma}{|W_1|} \\ |S| < \frac{\gamma}{|W_1 \cdot W_3|} \\ |S \cdot G| < \frac{\gamma}{|W_1 \cdot W_2|} \end{array} \quad (9)$$

To solve the problem (9), we use the Glover-Doyle algorithm which is based on the Riccati equations [14][15]. The issued controller K is robust in the fact that it ensures the stability and the performances even if the nominal plant G has uncertainty relative to the real plant. The wanted performances are introduced through the weighting functions.

B. Choice of the weighting functions

The choice of the weighting functions are derived from the specifications. The latter have been chosen from needs in micromanipulation in our laboratory.

1) *Choice of W_1* : the transfer function $\frac{1}{W_1}$ is chosen from the specifications on the tracking performances. These specifications are:

- the maximal response time is inferior to $10ms$,
- the overshoot is null,
- and the maximal static error is inferior to 5%.

For that, we chose:

$$W_1 = \frac{s + 300}{s + 1.5} \quad (10)$$

2) *Choice of W_2* : here, the specifications relative to the mechanical disturbance rejection are used. These are as follow:

- the disturbance rejection time is inferior to $10ms$,
- and the maximal static error is inferior to $\frac{\delta}{F}(s = 0) = \frac{1\mu m}{13mN}$.

From the second specification, we have:

$$\frac{\delta}{b_1}(s = 0) = 0.03 \left[\frac{\mu m}{V} \right] \quad (11)$$

To complete the specifications, let us choose:

$$\frac{1}{W_1 \cdot W_2} = 6 \times 10^{-6} \cdot \frac{1}{W_1} \quad (12)$$

where $\frac{1}{W_1 \cdot W_2}(s = 0) = 0.03 \left[\frac{\mu m}{V} \right]$.

We automatically infer:

$$W_2 = 6 \times 10^6 \quad (13)$$

3) *Choice of W_3* : in this part, we use the specifications relative to the thermal disturbance rejection. These are as follow:

- the maximal disturbance rejection time is inferior to $10ms$,
- and the maximal static error is inferior to $\frac{\delta}{\Delta T}(s = 0) = \frac{1.68\mu m}{10^\circ C}$.

According to the Fig. 5, the relation between $b_2[\mu m]$ and the temperature $\Delta T[^\circ C]$ is nonlinear. However, in order to simplify the choice of the weighting function, we suppose it to be linear. That is not impeding because the temperature is considered as a disturbance. However, a high enough value of slope must be chosen. We use:

$$\frac{b_2}{\Delta T} = 3 \left[\frac{\mu m}{^\circ C} \right] \quad (14)$$

Thus, from the second specification and the (14), we have:

$$\frac{\delta}{b_2}(s = 0) = 0.03 \quad (15)$$

To complete the specifications, let us choose:

$$\frac{1}{W_1 \cdot W_3} = 6 \cdot \frac{1}{W_1} \quad (16)$$

Then, we obtain:

$$W_3 = 0.17 \quad (17)$$

C. Calculation of the controller

The computed controller has an order of 9. Such order is relatively high and may lead to time consuming in the computer so that computation errors may be provided. So, the controller order has been reduced using the balanced realization technique [16]. Finally, we obtain:

$$\left\{ \begin{array}{l} K = \frac{1093128 \cdot (s+1.5) \cdot (s^2+939 \cdot s+1.2 \times 10^7)}{(s+2.4 \times 10^4) \cdot (s^2+3 \cdot s+2.3)} \\ \gamma_{opt} = 1.024 \end{array} \right. \quad (18)$$

V. EXPERIMENTAL RESULTS

The controller has been implemented in a PC-DSPACE setup through the Simulink-Matlab software. The first experiment consists in analyzing the transient performance of the closed-loop system for different values of temperature. For that, we apply a step reference with $10\mu m$ of amplitude. According to the results (Fig. 13), the wanted performances (response time and overshoot) are maintained whatever the working temperature is (here, we use the two extremal temperatures $T = 20^\circ C$ and $T = 30^\circ C$, i.e. $\Delta T = 10^\circ C$).

In the second experiment, we analyze the static performance of the closed-loop system under the thermal

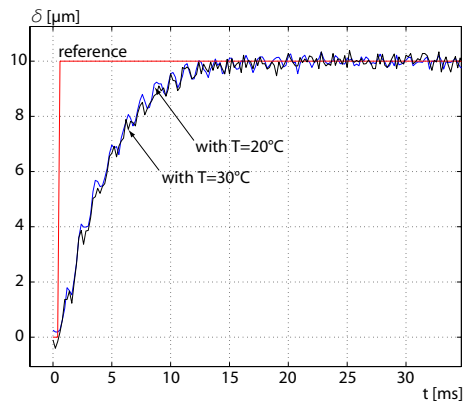


Fig. 13. Response analysis of the closed-loop system.

disturbance. A $10\mu\text{m}$ step amplitude of reference is applied and the temperature varies from $T = 20^\circ\text{C}$ to $T = 30^\circ\text{C}$. The results are presented in the Fig. 14. It can be deduced that the static performance is maintained even if the working temperature varies. As the compensation is done through the voltage U (Fig. 14-b), the saturation of the setup amplifier and the maximal applicable voltage of the cantilever only limit the achievement of such a performance.

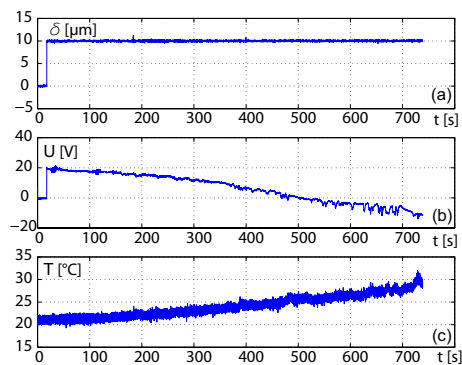


Fig. 14. Static performance analysis of the closed-loop system.

The last experiment consists in a frequential domain analysis with the two extremal working temperatures. Such analysis combines the transient and the static parts analyses and is more precise. The analysis concerns the complementary sensitivity function. A sine reference δ_c is applied while the output δ is measured. The corresponding magnitude is shown in the Fig. 15. The results confirm that the implemented H_∞ controller maintain the whole performances whatever the temperature is.

VI. CONCLUSION

In micromanipulation, the working temperature variations may decrease indeniably the micromanipulation and microassembly performances when they are not taken into account. This paper has described the analysis of the thermal effect on a unimorph piezoelectric cantilever dedicated to micromanipulation and proposed a

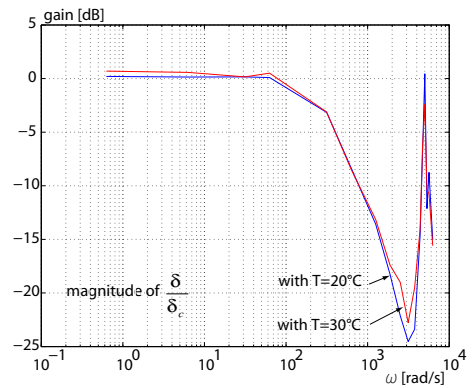


Fig. 15. Frequential performance analysis of the closed-loop system.

H_∞ controller to reject it and maintain performances. On the one hand, the open-loop analyses have shown that the temperature mainly influences the thermal expansion and fewly on the piezoelectric constant. On the other hand, while the response is insensitive with the temperature, the creep is highly impacted. Finally, the implementation of a H_∞ controller has made possible the rejection of the thermal disturbance and the achievement of performances required in micromanipulation.

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