

Blind source separation methods applied to synthesized polysomnographic recordings: a comparative study

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Abstract—Blind Source Separation (BSS) problems, under the assumption of static mixture, were extensively explored from the theoretical point of view. Powerful algorithms are now at hand to deal with many concrete BSS applications. Nevertheless, the performances of BSS methods, for a given biomedical application, are rarely investigated. The aim of this paper is to perform quantitative comparisons between various well-known BSS techniques. To do so, synthetic data, reproducing real polysomnographic recordings, are considered.

I. INTRODUCTION

In the last decade Blind Source Separation (BSS) methods have been widely used in the field of biomedical engineering. Noise reduction and useful signal extraction are among the most significant applications of BSS. More particularly, assume that one realization of N -dimensional random vector process $\{x[m]_{m \in \mathbb{N}}\}$ with values in the real field is available, such as:

$$\forall m \in \mathbb{N}, x[m] = \mathbf{A}s[m] + \nu[m] \quad (1)$$

where $s[m] = [s_1[m] \dots s_P[m]]^T$ is a P source vector with statistically independent components, $\nu[m] = [\nu_1[m] \dots \nu_N[m]]^T$ is a N noise vector which is independent from the source vector and $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_P]$ is a $(P \times N)$ matrix, called *mixing matrix*. The BSS problem consists then in finding, only from the data, a $(N \times P)$ matrix \mathbf{W} , called *separator*, such that:

$$\mathbf{y}[m] = \mathbf{W}^T x[m] \quad (2)$$

is an estimate of $s[m]$ to within a *trivial* matrix [5], i.e. a matrix of the form $\mathbf{\Lambda}\mathbf{\Pi}$ where $\mathbf{\Lambda}$ is a diagonal matrix and $\mathbf{\Pi}$ is a permutation matrix.

Up to now, the BSS problem has been extensively explored from the theoretical point of view and many techniques are available to deal with numerous concrete applications. Then a fundamental question needs to be addressed: which BSS method, or class of BSS methods to use for a given biomedical application? In this paper we address this question for a particular problem of electrophysiological signal separation encountered in ambulatory polysomnography [11]. To do so, we have conducted performance comparisons on synthetic data, reproducing real polysomnographic observations, of seven BSS approaches. In the following, these hypotheses are assumed to be fulfilled:

H1) The vector random process $\{s[m]\}_{m \in \mathbb{N}}$ is stationary and ergodic;

H2) For any integer m , the components of $s[m]$ are mutually independent;

H3) At most one source is Gaussian;

H4) For any integer m , the components of $s[m]$ are mutually independent from the components of $\nu[m]$;

H5) The vector random process $\{\nu[m]\}_{m \in \mathbb{N}}$ is stationary, ergodic and Gaussian;

H6) The matrix \mathbf{A} has a full column rank equal to P .

II. DATA GENERATION

The main goal is to obtain synthetic but realistic data for comparing the above BSS methods. To do so, we have used our sleep database (14 patients) [11] and an EEG model [9] for generating background brain activities.

A. Generation of the source and noise data

The simulated sources are denoted by EEGS, EOGRS, EOGLS and ECGS (figure 1(a)). They represent brain activity, rapid eye movements, slow eye movements and cardiac activity, respectively. More precisely, the EEGS source is simulated using the model of Jansen [9] where parameters were selected to derive a cerebral background activity. Note that the statistical distribution of this signal is quasi-Gaussian. This is in agreement with real background EEG data. The other sources are derived from our sleep recordings database presented in [11]. More precisely, the EOGRS source is issued from a band-pass filtering (between 1 Hz and 8 Hz) of the derivation FPZ-CZ of the standard 10-20 system. The EOGLS source corresponds to a low-pass filtering of the classical derivation E1-E2 with the cut-off frequency of 4Hz in order to reduce the effect of EEG and EMG interference. Finally, the ECGS source corresponds to a cardiac signal recorded on patients during their sleep. In reference to the additive noise $\{\nu[m]_{m \in \mathbb{N}}\}$, it is modeled as a spatially correlated Gaussian noise with the spatial correlation equal to 0.5.

B. Generation of the mixing matrix

To derive the mixing matrix associated with the brain and ocular activities, a three concentric sphere head model is used. Four dipoles located at four fronto-parietal positions and a patch of two hundred dipoles (uniformly located in the cortex) characterize the eye movements (rapid and slow) and the background EEG sources, respectively. The EEG recording system contains four electrodes plus one reference electrode: two temporal sensors, in front of the higher part of the ears, denoted by F7m and F8m (where m stands for modified), two frontal sensors, above the eyes, denoted by

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FP1m and FP2m, and the reference electrode CZ located at the top of the head [11]. Then the transfer formula [12] describing the relationship between current dipoles and surface observation is used to obtain the (4×3) mixing matrix \mathbf{A}' . Since the heart contribution to the data is assumed to be non-uniform on all the channels, we decide to add to the (4×3) mixing matrix \mathbf{A}' a fourth column vector \mathbf{a}_4 with different components. Note that the obtained mixing matrix $\mathbf{A} = [\mathbf{A}' \mathbf{a}_4]$ is slightly ill-conditioned.

III. STUDY OF SEVEN CLASSICAL BSS METHODS

We have focused in this section on seven well-know BSS algorithms, namely SOBI [1], TFBSS [7], COM2 [5], JADE [3], COM1 [6], INFOMAX [10] and FastICA [8]. Nevertheless, before presenting these methods, we first formulate the criterion allows for quantitative comparison of the performance given by each BSS method.

A. Performance criterion

In order to compare quantitatively the ability of two separators, \mathbf{W}_1 and \mathbf{W}_2 , to extract a source we used the criterion introduced by Chevalier [4]. Chevalier has shown that the quality of the retrieved source is directly related to the Signal to Interference-plus-Noise Ratio (SINR) of this source after separation. More precisely, the SINR of the p -th source at the i -th output of the separator $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_P]$ is defined by:

$$\text{SINR}_p[\mathbf{w}_i] = \pi_p \frac{|\mathbf{w}_i^H \mathbf{a}_p|^2}{\mathbf{w}_i^H \mathbf{R}_{\nu p} \mathbf{g}_i} \quad (3)$$

where π_p represents the power of the p -th source, \mathbf{w}_i the i -th column of separator \mathbf{W} and $\mathbf{R}_{\nu p}$ is the total noise covariance matrix for the p -th source, corresponding to the data covariance matrix \mathbf{R}_x in the absence of the source P . On the basis of these definitions, the restitution quality of the p -th source at the output of separator \mathbf{W} is evaluated by computing the maximum of $\text{SINR}_p[\mathbf{w}_i]$ with respect to i ($1 \leq i \leq P$). This quantity is denoted by SINRM_p . The performance of a source separator \mathbf{W} is defined by the line vector $\text{SINRM}(\mathbf{W})$ given by:

$$\text{SINRM}(\mathbf{W}) = (\text{SINRM}_1[\mathbf{W}], \dots, \text{SINRM}_P[\mathbf{W}]) \quad (4)$$

in a given context, a separator \mathbf{W}_1 is better than a separator \mathbf{W}_2 for retrieving the source p , if $\text{SINRM}_p[\mathbf{W}_1] > \text{SINRM}_p[\mathbf{W}_2]$.

B. Optimal source separator

The criterion given by (4) allows for a quantification of the source separation performed by BSS algorithms. However, besides this criterion it is necessary to know its upper bound, which is achieved by the optimal source separator, in order to completely evaluate the performance of a given BSS method. It is shown [4] that the optimal source separator corresponds to the separator \mathbf{W}_{SMF} whose columns are the Spatial Matched Filters (SMF) associated with the different sources. It is defined to within a trivial matrix by:

$$\mathbf{W}_{SMF} = \mathbf{R}_x^{-1} \mathbf{A} \quad (5)$$

where \mathbf{R}_x is the covariance of $\{\mathbf{x}[m]_{m \in \mathbb{N}}\}$.

C. Presentation of the considered BSS methods

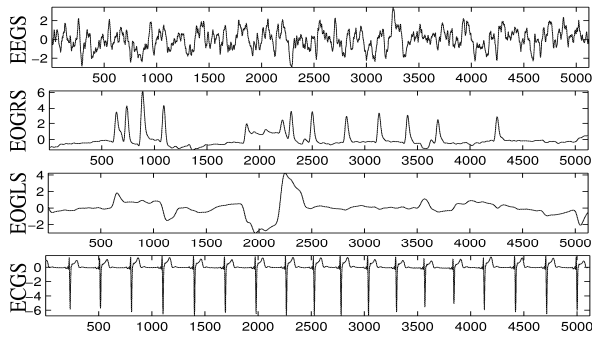
The BSS methods considered in the paper aim at implementing, to within a trivial matrix, an estimate $\hat{\mathbf{W}}_{SMF} = \hat{\mathbf{R}}_x^{-1} \hat{\mathbf{A}}$ of the optimal separator (5) from both an estimate, $\hat{\mathbf{R}}_x$, of the data covariance matrix \mathbf{R}_x common to all the methods, and the estimate, $\hat{\mathbf{A}}$, of \mathbf{A} , computed by each BSS method. More precisely, in order to estimate \mathbf{A} , SOBI (Second Order Blind Identification) [1] exploits a set of delayed covariance matrices of the data. TFBSS (Time-Frequency Blind Source Separation) [7] can be considered as an extension of the SOBI algorithm in order to process non-stationary signals. COM2 (CONtrast MAXimization 2) [5], JADE (Joint Approximated Diagonalization of Eigenmatrices) [3] and COM1 (CONtrast MAXimization 1) [6] rely on the maximization of a contrast criterion based on fourth order cumulants. INFOMAX (INFORMATION MAXimization) [10] maximizes a criterion based on information theory and FastICA (Fast Independent Component Analysis) [8] uses the fixed-point algorithm to maximize a negentropy-based criterion. Note that we have considered two versions of SOBI, denoted by SOBI and SOBIR in the sequel, and two versions of FastICA denoted by FastICA_{DO} and FastICA_{SO}. The difference between SOBI and SOBIR concerns the whitening step. Indeed, while SOBI uses the classical whitening [1], SOBIR exploits the robust whitening [2] which is not affected by a spatially correlated noise provided that this noise is temporally white. Concerning FastICA, DO refers to the Deflation Orthogonal approach (the sources are extracted one by one) while SO refers to the Symmetric Orthogonal approach (the sources are simultaneously extracted).

IV. SIMULATION RESULTS

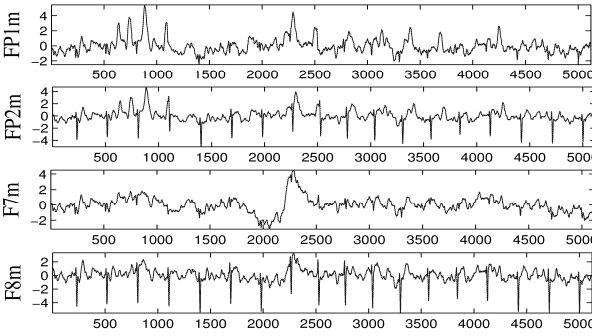
A comparative performance study on synthesized sleep recording data, of the considered seven algorithms is proposed in this section. To do so, two studies are envisaged. For each one, the performance criterion is averaged over two hundred runs. Figures 1(b) and 1(c) illustrate an example of the observations obtained from the generated mixing matrix and the sources presented in figure 1(a) (with a high SNR) and the results of the separation using FastICA_{DO}, of the EEGS, EOGRS, EOGLS and ECGS signals, respectively. Clearly, we observe that cerebral activity (see y_4), and the cardiac activity (see y_2) are well recovered. Regarding eye movements, y_1 and y_3 show rapid eye movements and slow eye movements, respectively. However, a few rapid eye movements also appear on y_3 , which implies that the separation is not perfect.

A. Influence of the data length for a fixed SNR

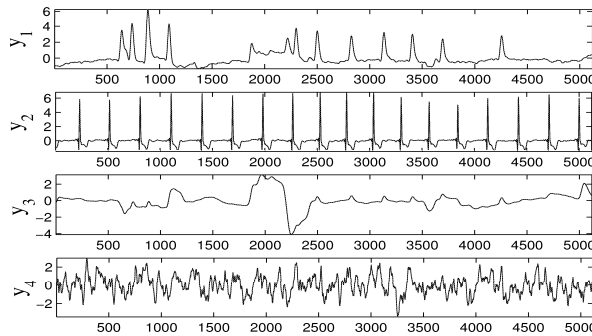
In this experiment, we set the Signal to Noise Ratio (SNR) to 15 dB for each source. The SINRM_p criterion $1 \leq p \leq 4$ at the output of the SOBI, SOBIR, TFBSS, COM2, COM1, JADE, FastICA_{DO}, FastICA_{SO} and INFOMAX, is computed as a function of the number of samples M (with a sampling rate of 256 Hz). Figure 2 shows the separation results of the EEGS, EOGRS, EOGLS and ECGS. COM2, COM1, JADE, FastICA_{DO} and FastICA_{SO} lead to good behaviors,



(a) Synthetic sources.



(b) Observations obtained from the mixing matrix and the synthetic sources with high SNR.



(c) Separation result of signals depicted in figure 1(b).

Fig. 1. An example of 20 secondes portion of original sources, observations and estimated sources

for EEGS, EOGRS, EOGLS and ECGS. Moreover, the ability of these methods to separate sources increases with the data length M . Nevertheless, we noticed that COM1 is slightly less effective for separating EOGLS sources when the data length is low. The INFOMAX algorithm is slightly less effective for separating EEGS, EOGRS and EOGLS, but its performances are equivalent to those obtained by COM2, COM1, JADE, FastICA_{DO} and FastICA_{SO}, in the case of ECGS source. Regarding TFBSS algorithm, it appears that its behavior is slightly lower for all the sources, except for the ECGS source whose extraction is quasi-optimal. Finally, the two versions of SOBI algorithm lead to different results.

Indeed, using the classical whitening, SOBI algorithm is not able to separate the cerebral activity EEGS and has difficulty for separating the three other sources. On the other hand, we note the very good behavior of SOBIR (using the robust spatial whitening) except for the extraction of the EOGRS which will need more than 1500 samples to be well-separated.

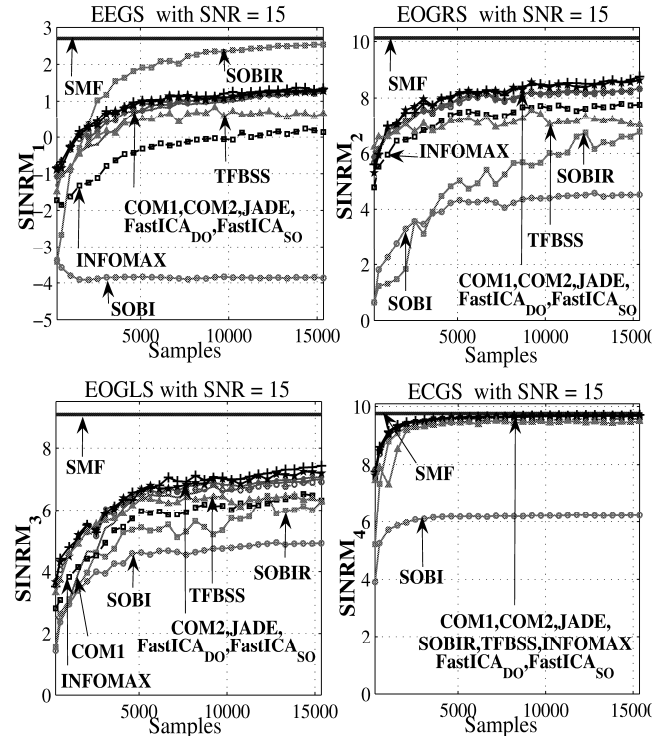


Fig. 2. Variations (in dB) of each source SINRM for the sources as a function of the data length.

B. Influence of the SNR

In this section we study the behavior of the seven BSS methods as a function of the SNR, which is assumed to be the same for each source. The data length has been set to 5120 in agreement with the practical issues in the field of polysomnography. Indeed, in polysomnography epochs of 20 seconds of data are considered. Then for a sampling rate of 256 Hz each epoch contains 5120 samples. Figure 3 shows that, when varying the SNR from -20 dB to 25 dB, COM1, COM2, JADE, TFBSS, FastICA_{DO} and FastICA_{SO} exhibit quasi-optimal performances for all the sources. However, for SNR values higher than 25 dB, their performances deviate from the optimal ones (SMF). More precisely, in the case of EEGS and ECGS the six separators have the same deviation. Regarding the eye movements, it appears that EOGRS is slightly well extracted using COM1, JADE, TFBSS and FastICA_{SO} and EOGLS is well extracted by COM2 and FastICA_{DO}. The INFOMAX algorithm behaves more or less like the six previous methods but its performances slightly decrease when dealing with EOGLS source for an SNR lower than 0 dB and with EEGS for a SNR higher than 30

dB. SOBI and SOBIR present nearly optimal performances for the four sources when the SNR is lower than -5 dB. In addition, the convergence of the SINRM at the output of SOBI and SOBIR is also stopped as soon as the SNR increases beyond 30 dB. Note that for a SNR higher than -5 dB, SOBI and SOBIR provide a poor separation of the ocular sources.

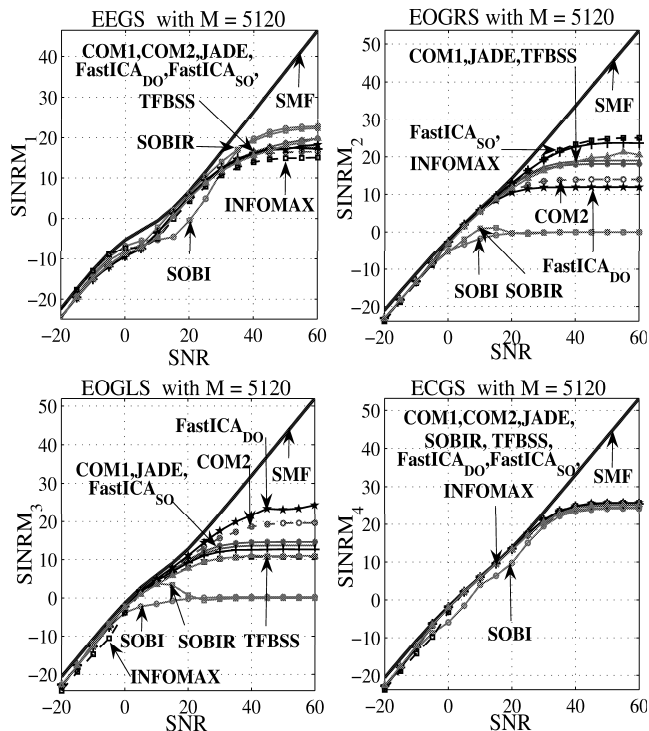


Fig. 3. Variations of each source SINRM (in dB) as a function of a SNR.

V. DISCUSSION AND CONCLUSION

The two studies have shown that, globally, the SINRM of the studied methods are lower than the optimal SMF minus 2 dB, except for the ECG source (it appears on figure 3 provided that a zoom of the figure is performed). This result is due to the fact that the additive noise is spatially correlated and all the studied methods require a prior spatial whitening based on second order statistics. This stage theoretically needs the perfect knowledge of the noise covariance. Now, if we compare the performances provided by each method, it appears that COM2, COM1, JADE, FastICA_{D0} and FastICA_{S0} globally (when all the sources are considered) gives the best results. However, COM1, which requires that all the sources have kurtosis (normalized marginal fourth order cumulants) with the same sign, is slightly affected by the presence of the EOGLS source when only few samples are available. Computer results have shown that the probability of the EOGLS source to have an estimated kurtosis with a different sign is inversely proportional to the number of snapshots. The INFOMAX and TFSS algorithms are slightly less effective for separating the EEGS, EOGRS and EOGLS sources but their performances are still

acceptable comparing to those obtained by other methods. Regarding the SOBI algorithm, its performances are poor for all the sources. One of the reasons could be the fact that the noise is spatially correlated. Indeed, the obtained results show that for EEGS and ECGS sources, SOBIR, which is not affected by the spatially correlated noise, has a better behavior. Another reason is the fact that some sources, especially EOGRS and EOGLS, seem to be poorly temporally correlated. In fact, for these two sources SOBIR also provides lower performances in comparison to those obtained by COM2, COM1, JADE, FastICA_{D0} and FastICA_{S0}. To conclude, the selection of a BSS method should be driven by hypotheses and considerations issued from application objectives such some statistical/physiological prior information on the sources (temporal color, non-gaussianity, sign of kurtosis, ...) and the additive noise.

REFERENCES

- [1] A. BELOUHRANI, K. ABED-MERAIM, J.-F. CARDOSO, and E. MOULINES, "A blind source separation technique using second-order statistics," *IEEE Transactions on Signal Processing*, vol. 45, no. 2, pp. 434–444, February 1997.
- [2] A. BELOUHRANI and A. CICHOCJI, "Robust whitening procedure in blind source separation context," *Electronics Letters*, vol. 23, no. 24, pp. 2050–2051, November 2000.
- [3] J.-F. CARDOSO and A. SOULOUMIAC, "Blind beamforming for non-gaussian signals," *IEE Proceedings-F*, vol. 140, no. 6, pp. 362–370, December 1993.
- [4] P. CHEVALIER, "Optimal separation of independent narrow-band sources : Concept and Performances," *Signal Processing, Elsevier*, vol. 73, pp. 27–47, 1999.
- [5] P. COMON, "Independent component analysis, a new concept?" *Signal Processing, Elsevier*, vol. 36, no. 3, pp. 287–314, April 1994.
- [6] —, "From source separation to blind equalization, contrast-based approaches," in *ICISP 01, International Conference on Image and Signal Processing*, Agadir, Morocco, May 3-5 2001, pp. 20–32.
- [7] C. FEVOTTE and C. DONCARLI, "Two contributions to blind sources separation using time frequency distributions," *IEEE Signal processing Letters*, vol. 11, no. 3, March 2004.
- [8] A. HYVARINEN, J. KARHUNEN, and P. OJA, *Independent component analysis*, ser. Wiley interscience. John Wiley and Sons, 2001.
- [9] B.-H. JANSEN and V.-G. RIT, "Electroencephalogram and visual evoked potential generation in a mathematical model of coupled cortical columns?" *Biological Cybernetics*, vol. 73, pp. 357–366, 1995.
- [10] T. W. LEE, M. GIROLAMI, and T. J. SEJNOWSKI, "Independent component analysis using an extended infomax algorithm for mixed sub-gaussian and super-gaussian sources," *Neural Computation*, vol. 11, no. 2, pp. 417–441, 1999.
- [11] F. POREE, A. KACHENOURA, H. GAUVRIT, C. MORVAN, G. CARRAULT, and L. SENHADJI, "Blind source separation for ambulatory sleep recording," *Transactions on Information Technology in Biomedicine*, vol. 10, no. 2, pp. 293–301, 2006.
- [12] Z. ZHANG, "A fast method to compute surface potentials generated by dipoles within multilayer anisotropic spheres," *Physics in Medicine and Biology*, vol. 40, no. 3, pp. 335–349, March 1995.