Real-Time Detection of the Business Cycle using SETAR Models

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In this paper, we consider a threshold time series model in order to take into account certain stylized facts of the business cycle, such as asymmetries in the phases. Our aim is to point out some thresholds under (over) which a signal of turning point could be given in real-time. First, we introduce the threshold model and we discuss its statistical theoretical and empirical properties. Especially, we recall the classical techniques to estimate the number of regimes, the threshold, the delay and the parameters of the model. Then, we apply this model to the Euro-zone industrial production index to detect in real time, through a dynamic simulation approach, the dates of peaks and troughs for the business cycle.

Keywords: Economic cycle, Real-time detection, Threshold model, Euro-zone IPI.

JEL classification: C32, C51, E32.

1 Introduction

Recently, we witnessed the development of new modern tools in business cycle analysis, mainly based on non-linear parametric modeling. Non-linear models have the great advantage to be flexible enough to take into account certain stylized facts of the economic business cycle, such as asymmetries in the phases. In this respect, much of attention has concentrated on the class of non-linear dynamic models that accommodate the possibility of regime changes.

Especially, Markov-Switching models popularized by Hamilton (1989) have been extensively used in business cycle analysis in order to describe the economic fluctuations. Among the huge amount of empirical studies, we can quote the papers of Sichel (1994), Lahiri and Wang (1994), Potter (1995), Anas and Ferrara (2002a), Chauvet and Piger (2003), Clements and Krolzig (2003) or Ferrara (2003) as regards the US economy and the papers of Krolzig (2001, 2003), Krolzig and Toro (2001) or Guégan (2003), as regards the Euro-zone economy. Generally, the output of these applications is twofold. The authors aim either at dating the turning points of the cycle or at detecting in real-time the current regime of the economy.

However, a clear distinction must be done between dating and detecting the turning points of the cycle. Dating is an *ex post* exercise for which several parametric and non-parametric

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methodologies are available. It turns out that simple non-parametric procedures, such as the famous Bry and Boschan (1971) procedure still used by the Dating Committee of the NBER, are more convenient for this kind of work (see Harding and Pagan, 2001, or Anas and Ferrara, 2002b, for a discussion on this issue). Real-time detection refers mainly to short-term economic analysis, which is not an easy task for practitioners. Indeed, several economic indicators are released on a regularly monthly basis, or even on a daily basis as regards the financial sector, adding volatility to the existing volatility and thus leading to an inflation of the available information set. Moreover, the data are often strongly revised and the diverse statistical methods, such as seasonal adjustment or filtering techniques, lead to edge effects. In this framework, the statistician has a crucial role to play which consists in extracting the right signal to help the short-term economic diagnosis. The too often quoted word "data miner" seems to be here well appropriate. Therefore, the real-time economic analysis asks for methods with strong statistical content.

In this respect, Markov-switching models have shown their interest in real-time business cycle analysis. Besides this well known approach, other parametric models have been proposed in the statistical literature to allow for different regimes. The threshold autoregressive (TAR) models, see Tong (1990), have been used to describe the asymmetry observed in the quartely US real GNP by different authors, such as Tiao and Tsay (1994), Potter (1995) and Proietti (1998) for instance, and using US unemployment monthly data by Hansen (1997). With the TAR model the transition variable is observed: it may be either an exogenous variable, such as a leading index for example, or a linear combination of lagged values of the series. In this latter case, the model is referred to as a self-exciting threshold autoregressive (SETAR) model. This is the main difference with the Markov-Switching model whose parameters of the autoregressive data generating process vary according to the states of the latent Markov chain. These two approaches are complementary because the notion captured under investigation is not exactly the same. Nevertheless, one of the interest of SETAR processes lies on their predictability, see for instance De Goojier and De Bruin (1999) and Clements and Smith (1999, 2000). When dealing with SETAR models, the transition is discrete, but smooth transition is also chosen to study the business cycle by some authors. Thus, we get the so-called STAR model, see for instance Terasvirta and Anderson (1992) and van Dijk, Terasvirta and Franses (2002).

In this paper, we focus on real-time detection of business cycle turning points. Our aim is rather to point out some thresholds under (over) which a signal of turning point could be given in real-time. We prefer the SETAR approach because a threshold model seems to be attractive in terms of business cycle analysis. Here, we propose a prospective approach as an alternative to other approaches, including, for instance, the use of switching models to detect the business cycle. Thus, in the following, we assume that it is possible to adjust a SETAR on the considered data and we do not test this assumption. This way is based on the following intuition: the existence of two (or more) states inside the data and the possible distinction of these states using the structure of the data, without imposing the existence of another series to explain the split from one state to another one.

This paper is split into two parts. In a first step (Section two), we introduce the various threshold models and we discuss their statistical properties. Especially, we recall the classical techniques to estimate the number of regimes, the threshold, the delay and the parameters of the model. In a second step (Section three), we apply these models to the Euro-zone industrial production index to detect in real time the dates of peaks and troughs for the business cycle. By using a dynamic simulation approach, we also provide a measure of performance of our model by comparison to a benchmark dating chronology. Lastly, some conclusions and further research directions are proposed in Section four.

2 Description and inference for SETAR processes

The TAR model introduced in the eighties' has not been widely used in applications until recently, primarily because it was hard in practice to identify the threshold variable and to estimate the associated values and secondly because there was no simple modeling procedure available. Recently some authors have proposed different ways to bypass this problem. In this section we introduce a SETAR model with two regimes and a classical way to estimate its parameters. We assume that it is possible to adjust an AR($p_i, i = 1, 2$) process on each regime. The autoregressive lag p_1 in the first regime may also be different from the lag p_2 in the other regime.

A mean-stationary SETAR $(2, p_1, p_2)$ model can be written in the following form:

$$\begin{aligned} Y_t &= (1 - I(Y_{t-d} > c))(\phi_{0,0} + \sum_{i=1}^{p_1} \phi_{0,i} Y_{t-i} + \sigma_0 \varepsilon_t) && (1) \\ &+ I(Y_{t-d} > c)(\phi_{1,0} + \sum_{i=1}^{p_2} \phi_{1,i} Y_{t-i} + \sigma_1 \varepsilon_t), && (2) \end{aligned}$$

where $I(Y_{t-d} > c) = 1$ if $Y_{t-d} > c$ and zero otherwise. For a given threshold c and the position of the random variable Y_{t-d} with respect to this threshold c, the process $(Y_t)_t$ follows here a AR(p_1) model or an AR(p_2) model. The model parameters are $\phi_{i,j}$, for $i = 0, 1$ and $j = 1, \cdots, p_k$, and $k = 1$ or 2, the threshold c and the delay d. On each state, it is possible to propose more complex stationary models like ARMA(p,q) processes or nonlinear processes (see Guégan, 1994).

The choice of the models in each regime can be done using the procedure proposed by Tsay (1989). As noted above, a major difficulty in applying TAR models is the specification of the threshold variable, which plays a key role in the non-linear structure of the model. Since there is only a finite number of choices for the parameters c and d, the best choice can be done using the Akaike Information Criterion (AIC), see Akaike (1974). This procedure has been developed by Tong (1990) and is used by a large part of the practitioners dealing with this model. We recall now the main steps for estimation theory.

Using some algebraic notations, the model (1) can be rewritten as a regression model. Denote $I_d(c) \equiv I(Y_{t-d} > c)$, $\Phi_0 = [\phi_{0,0}, \cdots, \phi_{0,p}]'$, $\Phi_1 = [\phi_{1,0}, \cdots, \phi_{1,p}]'$ and $\mathbf{Y}'_{\mathbf{t-1}} =$

$[1, Y_{t-1}, \cdots, Y_{t-p}]$, then, we get, for the process $(Y_t)_t$, the following representation:

$$Y_t = (1 - I_d(c))\mathbf{Y}'_{\mathbf{t-1}}\Phi_0 + I_d(c)\mathbf{Y}'_{\mathbf{t-1}}\Phi_1 + ((1 - I_d(c))\sigma_0 + I_d(c)\sigma_1)\varepsilon_t. \quad (3)$$

Now, we assume that we observe a sequence of data $(Y_1, \cdots, Y_n)$ from the model (3). The equation (3) is a regression equation (albeit nonlinear in parameters) and an appropriate estimation method is least squares (LS). Under the auxiliary assumption that the noise $(\varepsilon_t)_t$ is a strong Gaussian white noise, the least squares estimation is equivalent to the maximum likelihood estimation.

Since the regression equation (3) is nonlinear and discontinuous, the easiest method to obtain the LS estimates is to use sequential conditional LS. We will use this approach here. Recall that conditional least squares lead to the minimization of:

$$\sum_{Y_{t-d}<c, t=1}^{n} (Y_t - \phi_{0,0} + \phi_{0,1}Y_{t-1} - \cdots - \phi_{0,p_1}Y_{t-p_1})^2 + \quad (4)$$

$$\sum_{Y_{t-d}>c, t=1}^{n} (Y_t - \phi_{0,0} + \phi_{0,1}Y_{t-1} - \cdots - \phi_{0,p_2}Y_{t-p_2})^2 \;=\; \min, \quad (5)$$

with respect to $\Phi_0, \Phi_1, c, d, p_1, p_2$. Generally, we first assume that the parameters p_1 and p_2 are known.

Chan (1993) proves that, under geometric ergodicity and some other regularity conditions for the process (3), the LS parameter estimates of this process have good properties. The threshold parameter is consistent, tends to the true value at rate n and, suitably normalized, follows asymptotically a Compound Poisson process. The other parameters of the model are $n^{-1/2}$ consistent and are asymptotically distributed. The limitation of the theory of Chan (1993) concerns the construction of confidence intervals for the threshold c. Indeed, if we denote $\hat{c}$ the LS estimate for c, then Chan finds that $(\hat{c} - c)$ converges in distribution to a functional of a Compound Poisson process and, unfortunately, this representation depends upon a host of nuisance parameters, including the marginal distribution of $(Y_t)_t$ and all the regression coefficients. Hence, this theory does not yield a practical method to construct confidence intervals.

The method used here to estimate all the parameters follows Tsay (1989) and Tong (1990). We need to determine the parameters c, d, p_1, p_2. We assume P the maximal possible order of the two subregimes and D the greatest possible delay. The threshold parameter c is chosen by grid-search. The grid points are obtained using the quantiles of the sample under investigation. We use equally spaced quantiles from the 10 (percent) quantiles and ending at the 90 (percent) quantiles. Now, for each fixed pair (d, c_i), $0 < d \leq D$, $i = 1, \cdots s$, the appropriate TAR model is to be identified. The AIC criterion is used for selection of the orders p_1 and p_2. In this context, it becomes:

$$AIC(p_1, p_2, d, c) = \ln(\frac{1}{n}\sum \hat{\epsilon}_t^2) + 2\frac{p_1 + p_2 + 2}{n}, \quad (6)$$

where $\hat{\epsilon}_t$ denotes the residuals.

Finally the model with the parameters p_1^*, p_2^*, d^* and c^* that minimize the AIC criterion can be selected. Since for different d there are different numbers of values that can be used for estimation, the following adjustment should be done. With $n_d = \max(d, P)$ it is:

$$AIC(p_1^*, p_2^*, d^*, c^*) = \min_{p_1,p_2,d,c} \frac{1}{n - n_d} AIC(p_1, p_2, d, c). \tag{7}$$

Different algorithms for the parameters estimation of SETAR models are available, references and details can be found in Ferrara and Guégan (2003). Moreover Tsay (1989) proposes a statistic to test the threshold nonlinearity and specify the threshold variable. This test statistic is derived by simple linear regression and its performance is evaluated by simulation. Hansen (1997) considers a likelihood ratio statistic for testing SETAR hypotheses. A Lagrange Multiplier test is proposed by Proietti (1998). We can remark that no test is exhibited to decide between SETAR models and switching models. This seems very difficult to settle.

3 Empirical results

In this section, our aim is to apply a SETAR model to the Euro-zone Industrial Production Index in order to detect the low phases of the industrial business cycle referred to as the industrial recessions. The application is done in two steps: first we try to find the best SETAR model according to the AIC criterion presented in the previous section and second we use this model to detect the periods of each regime. By comparing the results to reference recession dates, we can assess the ability of the model to reproduce the industrial business cycle features.

3.1 Data description

The analysis is carried out on the IPI series considered in the paper of Anas *et al.* (2003). This series is a proxy of the monthly aggregate Euro-zone IPI for the 12 countries, beginning in January 1970 and ending in December 2002. The data are working day adjusted and seasonally adjusted by using the Tramo-SEATS methodology implemented in the Demetra software. Moreover, the irregular part including outliers has been removed.

The original series $(X_t)_t$ is presented in figure 1 as well as its monthly growth rate $(Y_t)_t$ defined by $Y_t = \log(X_t) - \log(X_{t-1})$. In figure 1, the shaded areas represent the reference industrial recession dates. Several authors have proposed a turning points chronology

Figure 1: Euro12 IPI (top) and its monthly growth rate (bottom), as well as the reference industrial recession periods (shaded areas), from January 1970 to December 2002.

Figure 2: Empirical unconditional distribution of the IPI growth rate, from January 1970 to December 2002.

for the Euro-zone industrial business cycle, by using different statistical techniques and economic arguments. For example, we refer to Anas *et al.* (2003), who propose a classical NBER-based non-parametric approach, and to Artis *et al.* (2003), Krolzig (2003) or Anas and Ferrara (2002b) who apply parametric Markov-Switching models. Generally, the industrial recession dates are more or less similar. In fact, it turns out that the Euro-zone experienced five industrial recessions: in 1974-75 and 1980-81 due to the first and second oil shocks, in 1981-82, in 1992-93, due to the American recession and the Gulf war, and lastly in 2000-2001 because of the global economic slowdown caused itself by the US recession from March 2001 to November 2001. It is noteworthy that, contrary to a common belief among economists, the Asian crisis in 1997-98 has not caused an industrial recession in the whole Euro-zone, but only a slowdown of the production. Finally, we retain as a benchmark for our study the dates proposed by Anas *et al.* (2003) and summarized in the first column in table 2.

To ensure stationarity, we are going to deal with the monthly industrial growth rate $(Y_t)_t$. The unconditional empirical distribution of the IPI growth rate computed by using a non-parametric kernel estimate (with the Epanechnikov kernel) is presented in figure 2. There is a clear evidence of three peaks in the estimated distribution. The lowest peak is due to the negative growth rates during industrial recessions. The intermediate peak seems to be caused by periods of low, but positive, growth rates, experienced for example during the eighties, while the peak corresponding to the highest value is related to periods of fast growth. It is noteworthy that, from 1970 to 2002, periods of low growth rates seem to appear more frequently than periods of high growth rates. Moreover, this empirical distribution is cleary asymmetric (skewness equal to -0.9315) and with heavy tails (excess kurtosis equal to 2.4850). Consequently, the unconditional Gaussian assumption is strongly rejected by a Jarque-Bera test.

3.2 Whole sample modelling

In this subsection we fit various SETAR models to the industrial growth rate series $(Y_t)_t$, that is, we model the speed of the Euro-zone industry. We consider first a two-regime model, the transition variable being successively the lagged series and the lagged differenced series. Then, we consider a multiple regime model by mixing the conditions on these previous series. For each model, we compare the estimated regimes with the reference recession phases in order to assess the ability of the model to reproduce business cycle features (see results in table 2). Only the detailled results related to the third model are presented in this note. Details concerning the other models can be found in Ferrara and Guégan (2003). In the model presented here, we combine the two previous SETAR models in a single model with two transition variables : the lagged growth rate and the acceleration. Therefore, the model possesses four regimes and two thresholds c_1 and c_2. Estimates and their standard errors for the parameters of this model, minimizing the

	Regime 1 $[Y_{t-1} < -0.0015]$ $[Z_{t-10} < -0.0008]$	Regime 2 $[Y_{t-1} < -0.0015]$ $[Z_{t-10} \geq -0.0008]$	Regime 3 $[Y_{t-1} \geq -0.0015]$ $[Z_{t-10} < -0.0008]$	Regime 4 $[Y_{t-1} \geq -0.0015]$ $[Z_{t-10} \geq -0.0008]$
$\hat{\phi}_0$	-0.0040 (0.0010)	-0.0070 (NA)	0.0010 (0.0004)	0.0036 (0.0005)
$\hat{\phi}_1$	1.1454 (0.1408)	1.2803 (NA)	0.7106 (0.0995)	1.3005 (0.0660)
$\hat{\phi}_2$	-0.3712 (0.2076)	-0.0180 (NA)	-0.0750 (0.1216)	-0.7883 (0.0974)
$\hat{\phi}_3$	-0.0359 (0.1408)	-0.0001 (NA)	-0.0331 (0.1003)	-0.3321 (0.0662)
$\hat{\sigma}_\varepsilon$	0.0019	NA	0.0017	0.0012

Table 1: Estimates and standard errors for model described in 4.2.3.

AIC criteria, are given in table 1.

The two thresholds are estimated by using a double loop, but the delays of the model are fixed *a priori* according the two previous estimated models. Both estimated thresholds are negative but very close to zero. The first regime has an empirical unconditional probability of 0.15 and should be considered at a first sight as a period of recession because the estimated recession dates match the reference recession dates. However, the second regime is also meaningful. Indeed, this second regime possesses an unconditional probability of 0.02: only 7 observations over 385 belong to this state. This is the reason why standard errors of estimates are not available in this regime. Although the frequency of this second regime is very low, this regime is persistent and appears in clusters. In fact, this regime is very interesting because it corresponds to the end of a recession phase when the economy is accelerating again. This regime was detected twice: at the end of the 1974-75 recession and at the end of the 1992-93 recession. Thus, the sum of regime 1 and regime 2 corresponds to the industrial recession phase. The third regime can be considered as a slowdown of the industrial production, that is the industry is below its trend growth rate without being in recession. Lastly, when the series is in the high regime, we can deduce that the industrial growth rate is over its trend growth rate. Actually, regime 3 and regime 4 correspond to the high phase of the industrial business cycle. It appears that only three regimes would be sufficient to describe the industrial business cycle. However, we decide to keep four states because they give a deeper understanding of the industrial business cycle features. As regards the dating results, the model provides almost the same results than the first model, the last recession period being not cut into two parts (see table 2). However, this model presents some non-persistent signals of recession.

3.3 Dynamic real-time analysis

In real-time analysis, an economic indicator requires at least two qualities: it must be reliable and must provide a readable signal as soon as possible. Thus, there is a well known trade-off beetwen advance and reliability for the economic indicators. By using

	Reference	Model 1	Model 2	Model 3
Peak	m4 1974	m6 1974	m6 1974	m6 1974
Trough	m5 1975	m5 1975	m3 1975	m6 1975
Peak	-	m3 1977	m12 1976	m3 1977
Trough	-	m6 1977	m7 1977	m7 1977
Peak	m2 1980	m4 1980	m3 1980	m3 1980
Trough	m1 1981	m10 1980	m10 1980	m11 1980
Peak	m10 1981	m5 1982	m6 1982	m6 1982
Trough	m12 1982	m12 1982	m8 1982	m12 1982
Peak	m1 1992	m4 1992	m7 1992	m4 1992
Trough	m5 1993	m5 1993	m1 1993	m6 1993
Peak	-	-	m7 1998	-
Trough	-	-	m11 1998	-
Peak	m12 2000	m2 2001	m1 2001	m2 2001
Trough	m12 2001	m12 2001	m10 2001	m12 2001

Table 2: Reference and estimated dating chronologies stemming from the 3 considered SETAR models.

the previous 4-regime SETAR model, we assess if it is possible to have a clear and timely signal for the turning points of the industrial business cycle in a dynamic analysis. We consider the previous IPI series from January 1970 to December 1999, and we add progressively a monthly data until December 2002. For each step, we re-estimate the model and we classify the series into one of the four regimes. Thus, by using the conclusions of the whole-sample analysis, if the series lies into regime 1 or regime 2, we can conclude that the industry is in a recession phase. We are aware that a *true* real-time analysis should be done by using historically released data (see for instance Chauvet and Piger, 2003) in order to take the revisions and the edge-effects of the statistical treatments of the raw data into account. However, such series are very difficult to find in economic data bases.

The results of the real-time estimated recession period match with the 2001 recession period estimated in the whole-sample analysis. This fact points out the stability of the model. Indeed, we detect a peak in the business cycle in February 2001 and a trough in December 2001. However, it must be noted that a false signal of a change in regime is emitted in August 2001 but it lasts only one month. Knowing that a signal must be persistent to be reliable, we have to propose an *ad hoc* real-time decision rule. Thus, it is advocated to wait for at least two months before sending a signal of a change in regime. We also note that the exit of the recession is very fast, because the series goes directly from regime 1 in December 2001 to regime 4 in January 2002. Moreover, we observe that the series falls into regime three in December 2002.

It is also interesting to consider the evolution of the parameters in a dynamic analysis. In figure 3, the evolution of the thresholds $\hat{c}_1$ and $\hat{c}_2$ is presented. It is striking to observe the change in level of both thresholds during the recession phase. During this phase, thresholds tend to become closer to zero. It is also noteworthy that $\hat{c}_1$ increases slowly from May 2000 to June 2001 but decreases suddently, while, conversely, $\hat{c}_2$ increases sud-

Figure 3: Evolution of the real-time estimated thresholds of the 4-regime SETAR from January 2000 to December 2002.

dently in March 2001 but decreases slowly. This feature indicates perhaps an asymmetry between the start and the end of recession and may be exploited later to get a more advanced signal. Lastly, we note that both thresholds are remarkably stable since the end of the recession. Unfortunately, as noted in section 2, there is no practical way to test a change in the thresholds (see however Tsay, 1989, and Ip *et al.*, 2003).

Conclusion

This paper is an exploratory analysis of the ability of SETAR models to reproduce the business cycle stylised facts. The results are promising. It appears that the model allows to identify the turning points of the industrial cycle and can thus be useful for real-time detection. However, a true real-time analysis should be extended by using historically released data, as used in the recent paper of Chauvet and Piger (2003) as regards the US GDP and employment. Unfortunately, such data are not systematically stored in data bases and are therefore very difficult to get. As another example, business surveys seem to be good candidates for real-time analysis through SETAR models because they are timely released and are not generally revised.

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