

# Political economy of social security with endogenous preferences

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## **Abstract**

In this paper we study the interaction between economic policy and preferences when both are endogenous. Economic policy results from a vote, whereas individual preferences are influenced by specific investment in training and education. The paper focuses on a particular economic policy: the financing of the social security system. Moreover, it considers a specific education investment: parents expect a gift from their children when old and devote resources in order to arouse the altruism of their children. Therefore, preferences of the children are trained in relation to the size of the social security system, which in turn results from the preferences of the median voter. The politico-equilibrium of this economy is compared to the social optimum.

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# 1 Introduction

This paper is a contribution to the literature about the political economy of social security systems. A number of authors have intended to understand why the young (working people), who are more numerous, can be in favor of social security programs that imply a transfer of their own resources to the old (retired people). The literature provides different answers to this question (Galasso and Profeta, 2002). First, young people may benefit from a public transfer to the old if the economy without intergenerational transfer is inefficient (Browning (1975)), or if public transfers introduce some redistribution within cohort of heterogeneous agents (Casamatta et al. (1999)). Second, a social security system may emerge as the equilibrium outcome of a game between generations (Cooley and Soares (1999), Boldrin and A. Rustichini (2000), Grossman and Helpman (1998)). Third, if young agents have altruistic preferences towards the old, a public pension system can more efficiently take the place of private transfers, as private transfers lead to strategic behaviors of parents (Veall (1986), Hansson and Stuart (1989)).

Our contribution is mainly related to this last branch of the literature. We aim at evaluating the political support for a public pension system in a framework where preferences are endogenous. In childhood, preferences of an agent can be influenced by specific investment in training and education from their parents. Assuming economic policy is designed through a simple majority rule voting method, preferences and policies are then jointly determined. Indeed, economic policy results from the preferences of the median voter, which in turn depend on the education that he received during his childhood. But, this education is the consequence of the optimal investment of his parents which depends on the expected economic policy.

More precisely, in our setting, preferences endogeneity is materialized by the fact that the altruism of children toward their parents is endogenous. The individual degree of altruism towards the parent is assumed to be partly random and partly determined by some investment made by the parent in order to arouse the altruism of his children. Of course, preferences of the children are trained in relation to the size of the social security system. For a given expected level of pensions, parents can make two types of investments. On the one hand, they can invest in the altruism of their children, in order to receive some gift in their old age from them. On the other hand, they can invest part of their revenue in savings. Both investments are made strategically: parents know that the gift they will receive is a function of their savings and of the degree of altruism of their offsprings.

The main results are the following. First, we state conditions for the existence of an equilibrium without pension system where parents invest both in savings and in the altruism of their children, so that they consume in their old age the revenue of their savings and the gift received from their children. Starting from such a situation, we then consider the introduction of an exogenous public pension system and show that altruism spendings and savings tend to decrease with the size of the pension system.

We then turn to the definition of optimal policies. We define two types of optimal allocations that we have called *egoistic* and *altruistic*. The egoistic optimum is obtained when the central planner considers the lifetime utility of a young agent without taking into account of the altruism motive, i.e. the fact that its utility increases with the old-age consumption of his parent. On the contrary, the altruistic optimum takes into account the altruism motive. Consequently, with the egoistic objective, altruism spendings are a loss for the economy. We show that the egoistic optimum can be decentralized with the pension system and a tax on gifts. The altruistic optimum needs an additional instrument to be decentralized: a public investment in altruism education.

Finally, we consider that the size of the public pension system is determined through the simple majority rule. Assuming that young agents are more numerous, the majority coalition is formed by the old generation associated with a fraction of the most altruistic young agents. When the dependance ratio is sufficiently high, altruism spendings cancel out at equilibrium. We then study the impact of imperfect foresight and of ageing parameters on the size of the pension system.

Our framework gives several positive justifications for the existence of a public pension system. First, a public pension system enforces cooperation among children (voting equilibrium), while private gifts can lead to a non cooperative equilibrium with free riding. Second, a public pension system brings a benefit with certainty while private gifts depend on the random altruism of the children. Then risk-averse parents will benefit from the inception of a public pension system. Third, the pension system eliminates strategic behavior of the parents, when they tend to undersave in order to take advantage from the altruism of their children. Finally, if altruism spendings are viewed as a loss (as induced by the egoistic optimum), the public pension system could make them useless and then may lead parents to spend no resources in order to increase the altruism of their children.

## 2 The model

### 2.1 The basic setup

We consider an overlapping generation model. In each period  $t$ , a new generation of mass  $N_t$  of individuals living for three periods, is born :  $N_t = nN_{t-1}$ , with  $n \geq 1$ ,  $n$  being the mass of children.

In the first period of life, an individual of generation  $t - 1$  lives with his parent who spends some amount  $\phi_{t-1}$  in order to arouse children's altruism and stimulate their preferences for his own old-age consumption. Typically, those spending consist of all the resources used by the parent in order to display the important part that any child has to play for supporting his old parents, and notably to guarantee some income maintenance during old-age. Those spending are a public good within the family.

Individuals may differ in their natural compassion for their parents. Although their parents have made the same altruism spending, two children may have different degree of altruism, because they differ in their aptitude  $\varepsilon$  to convert altruism spending  $\phi_{t-1}$  in actual altruism. The degree of altruism writes

$$\varepsilon\rho(\phi_{t-1})$$

The parameter  $\varepsilon$  is distributed on the interval  $[0, E]$  according to a distribution  $G$ . The function  $\rho$  satisfies the following properties :

**Assumption 1**  $\rho$  is twice-differentiable, increasing and strictly concave, with  $\rho(0) > 0$  and  $\rho'(0) < +\infty$ .

In particular, this assumption implies that the parent may choose to devote no resources to altruism spendings and that, if he does so, his children will not necessarily be egoistic.

Lifetime utility depends on both consumptions when adult  $c_t$  and old  $d_{t+1}$ .  $d_{t+1}$  is a random variable in period  $t$ . For reasons that will become clearer in the following, we

assume that the utility function is linear with respect to adult consumption. Utility of an agent born in  $t - 1$  is

$$c_t + \beta \mathbb{E}(\ln d_{t+1}) + \varepsilon \rho(\phi_{t-1}) [c_{t-1} + \beta \ln d_t]$$

with  $\beta \geq 0$  and where  $\mathbb{E}(\ln d_{t+1})$  stands for the expected value of old-age utility.

In period  $t$ , this individual becomes a worker and supplies one unit of labor. He allocates net wage  $(1 - \tau_t)w_t$  (where  $\tau_t$  is the contribution rate to the social security system) between consumption  $c_t$ , savings  $s_t$ , altruism spendings  $\phi_t$  (that affects altruism of his own children) and gift to his parent. The budget constraint of an adult writes

$$(1 - \tau_t)w_t = c_t + s_t + g_t + \phi_t.$$

When old, he retires and consumes savings returns, total gift of his  $n$  children  $G_{t+1}$  and pension receipts  $b_{t+1}$

$$d_{t+1} = R_{t+1}s_t + G_{t+1} + b_{t+1}.$$

$G_{t+1}$  is a random variable in  $t$ , because parents ignore the altruism of their children, and therefore the gift that they may receive from them. Consequently,  $d_{t+1}$  is also random. All variables must be non-negative

$$c_t \geq 0, s_t \geq 0, g_t \geq 0, \phi_t \geq 0$$

## 2.2 Non cooperative gift

We assume that, within the family, children behave in a non-cooperative way, and take as given the gifts left by his siblings to their common parent. The optimal gift of a child of type  $\varepsilon$  born in  $t - 1$  maximizes

$$-g_t + \varepsilon \rho(\phi_{t-1}) \beta \ln(R_t s_{t-1} + b_t + g_t + G_t^-)$$

under the non-negativity constraint  $g_t \geq 0$ , where  $G_t^-$  represents the total gifts made by his siblings. The first order condition writes

$$\varepsilon \rho(\phi_{t-1}) \beta (R_t s_{t-1} + b_t + g_t + G_t^-)^{-1} \leq 1 \quad (1)$$

with equality if  $g_t > 0$ . Since this condition only depends on the total gift  $G_t = g_t + G_t^-$ , only the most altruistic child can make a positive gift. Since altruism spendings are a public good within the family, the most altruistic child is the one with highest  $\varepsilon$ , that we denote  $\bar{\varepsilon}$ . His gift corresponds to the total gift and is a function of altruism spendings made by the parent in order to awaken altruism of his own children and of old-age income of the parent

$$G_t = g(\phi_{t-1}, R_t s_{t-1} + b_t, \bar{\varepsilon}) = \max\{0, \beta \bar{\varepsilon} \rho(\phi_{t-1}) - R_t s_{t-1} - b_t\} \quad (2)$$

Here, the possibility of strategic behavior of the parent becomes apparent. By choosing savings  $s_{t-1}$  and altruism spending  $\phi_{t-1}$ , the parent influences the level of the gift that his children will consent to make. One may also notice that the strategic behavior could lead to highly more complicated analysis with a general utility function. Typically, if the utility function were not linear with respect to adult consumption  $c_t$ , gift  $G_t$  could not be set independently of savings  $s_t$  and altruism spending  $\phi_t$ , and thus would depend on the decisions of all future generations.

Equation (2) determines a threshold on the effective parameter  $\varepsilon$  above which families leave a positive gift. This threshold, denoted by  $\psi_t$ , is a function of display spending and old-age income

$$G_t > 0 \Leftrightarrow \bar{\varepsilon} > \frac{R_t s_{t-1} + b_t}{\beta \rho(\phi_{t-1})} \equiv \psi_t.$$

In equation (1), the gift  $G_t$  and the pension  $b_t$  appears as perfect substitutes in the behavior of an agent born in  $t - 1$ : any increase in pension receipts leads the agent to reduce his gift of the same amount, leaving the sum  $b_t + G_t$  unchanged. But, for his parent, who receives both the gift and the pension, there exists an important difference between those two receipts: the gift is risky and depends on the random altruism of children while pension benefits are certain.

### 2.3 Savings and altruism spendings

A parent does not observe *ex ante* the degree of altruism of his children. He only knows that the aptitude to convert altruism spendings into actual altruism of his most altruistic child is distributed according to the cumulative distribution function  $H$ . Appendix 1 shows that  $H$  is defined by:

$$H(x) = G(x)^n$$

An individual born in  $t - 1$  chooses spending  $\phi_t$  and savings  $s_t$  that maximize

$$-(s_t + \phi_t) + \beta \ln(R_{t+1}s_t + b_{t+1}) H(\psi_{t+1}) + \beta \int_{\psi_{t+1}}^E \ln(\bar{\varepsilon} \beta \rho(\phi_t)) dH(\bar{\varepsilon})$$

where

$$\psi_{t+1} = \frac{R_{t+1}s_t + b_{t+1}}{\beta \rho(\phi_t)},$$

under the non-negativity constraints  $s_t \geq 0$ ,  $\phi_t \geq 0$ ,  $c_t \geq 0$ .

First-order conditions write

$$-1 + \frac{\beta R_{t+1} H(\psi_{t+1})}{R_{t+1}s_t + b_{t+1}} \leq 0, \quad = 0 \text{ if } s_t \geq 0 \quad (3)$$

$$-1 + \beta \frac{\rho'(\phi_t)}{\rho(\phi_t)} (1 - H(\psi_{t+1})) \leq 0, \quad = 0 \text{ if } \phi_t \geq 0 \quad (4)$$

Equations (3) and (4) describe the trade-off faced by the parent. First, saving brings him a certain minimum income when old, which is useful only when the altruism of his children is too low. This will occur with probability  $H(\psi_{t+1})$ . Second, altruism spendings allow him to expect higher gift from his most altruistic child. Such an investment may bring positive return with probability  $(1 - H(\psi_{t+1}))$ . Both probabilities are also affected by the two investments.

Assumptions made on technology in the next section imply that savings are positive at equilibrium. Thus, from now on, we focus on the case  $s_t > 0$ . Moreover, we make the following assumption:

**Assumption 2** *The functions  $\rho$  and  $H$  satisfy*

- (i)  $\lim_{\phi \rightarrow +\infty} \frac{\rho(\phi)}{\rho'(\phi)} > \beta$
- (ii)  $H(0) < 1 - \frac{\rho(0)}{\beta \rho'(0)}$ .

(iii)  $H$  is differentiable for  $\psi > 0$ , with a positive derivative.

(iv) For any  $\phi \geq 0$  and  $\psi > 0$  such that  $\beta \frac{\rho'(\phi)}{\rho(\phi)} (1 - H(\psi)) = 1$ , we have

$$\frac{H(\psi)}{\psi} - H'(\psi) > \frac{\beta H(\psi) H'(\psi)}{\left( \frac{\rho(\phi)}{\rho'(\phi)} - \frac{(\rho(\phi))^2 \rho''(\phi)}{[\rho'(\phi)]^3} \right)}$$

The first two statements (i) and (ii) make possible the case  $\phi_t > 0$ . Assumption (iii) is made for sake of simplicity. Point (iv) will be useful in the next lemma in order to state the uniqueness of the consumer optimum.

**Lemma 1** Under assumption 1 and 2, there exists a unique interior solution to the consumer problem if the gross interest rate  $R_{t+1}$  is less than

$$\frac{\rho(0) H^{-1} \left( 1 - \frac{\rho(0)}{\beta \rho'(0)} \right)}{1 - \frac{\rho(0)}{\beta \rho'(0)}}.$$

If  $R_{t+1} > \frac{\rho(0) H^{-1} \left( 1 - \frac{\rho(0)}{\beta \rho'(0)} \right)}{1 - \frac{\rho(0)}{\beta \rho'(0)}}$ , then  $\phi_t = 0$  and saving is the unique solution of

$$\frac{\beta R_{t+1} H \left( \frac{R_{t+1} s_t + b_{t+1}}{\beta \rho(0)} \right)}{R_{t+1} s_t + b_{t+1}} = 1$$

This last equation can also be written as an equation that determines  $\psi_{t+1}$  :

$$\frac{R_{t+1}}{\beta \rho(0)} = \frac{\psi_{t+1}}{H(\psi_{t+1})}$$

The preceding lemma shows that two types of solution to the consumer program may occur. The first one, called interior, corresponds to a positive investment in children altruism  $\phi_t$ . The second one is associated with  $\phi_t = 0$ . This last case happens when the factor of interest  $R_{t+1}$  is high enough. Moreover, the following lemma gives the properties of the consumer program, taking  $R_{t+1}$  as an exogenous parameter.

**Lemma 2** The optimal choice of the consumer  $(s_t, \phi_t)$  is such that:

1.  $\phi_t$  is a decreasing function of  $R_{t+1}$  till it reaches the value 0 for  $R_{t+1} = \frac{\rho(0) H^{-1} \left( 1 - \frac{\rho(0)}{\beta \rho'(0)} \right)}{1 - \frac{\rho(0)}{\beta \rho'(0)}}$ .

Above this threshold, it remains equal to 0.

2.  $\psi_{t+1}$  is increasing with respect to  $R_{t+1}$ .

These results have a simple interpretation. An increase of  $R_{t+1}$  incites people to devote more resources to savings, and less resources to altruism spendings. Thus,  $\phi_t$  decreases with  $R_{t+1}$  and  $\psi_{t+1}$  increases.

### 3 Equilibrium

#### 3.1 Equilibrium conditions

Firms use a standard constant returns technology  $F(K, L)$  and act competitively, so that, in each period  $t$ , gross interest rate  $R_t$  and wage rate  $w_t$  are functions of the capital-labor ratio  $k_t$

$$R_t = F'_K(k_t, 1) \equiv R(k_t) \text{ and } w_t = F'_L(k_t, 1) \equiv w(k_t)$$

Capital stock is equal to savings in preceding period

$$nk_{t+1} = s_t$$

The budget constraint of the pay-as-you-go pension system is

$$b_t = n\tau_t w_t$$

**Assumption 3**  $F'_K(0, 1) = +\infty$ . The revenue of an investment in capital  $R(k)k$  is a non-decreasing function of  $k$ , or  $F''_{KK}(k, 1)k/F'_K(k, 1) \geq -1$ .

Under this assumption, the capital stock must be positive in equilibrium. Thus, the intertemporal equilibrium is characterized by the following conditions (with  $s_t > 0$ )

$$-1 + \frac{\beta R\left(\frac{s_t}{n}\right) H(\psi_{t+1})}{R\left(\frac{s_t}{n}\right) s_t + n\tau_{t+1} w\left(\frac{s_t}{n}\right)} = 0 \quad (5)$$

$$-1 + \beta \frac{\rho'(\phi_t)}{\rho(\phi_t)} (1 - H(\psi_{t+1})) \leq 0, \quad = 0 \text{ if } \phi_t > 0 \quad (6)$$

$$\psi_{t+1} - \frac{R\left(\frac{s_t}{n}\right) s_t + n\tau_{t+1} w\left(\frac{s_t}{n}\right)}{\beta \rho(\phi_t)} = 0. \quad (7)$$

These expressions are three static equations depending on three unknown variables:  $\phi_t$ ,  $\psi_{t+1}$  and  $s_t$ . There is no dynamics in this economy because of the particular utility function considered. Since it is quasi-linear in first period consumption,  $\phi_t$ ,  $\psi_{t+1}$  and  $s_t$  does not depend on the individual income, and are consequently independent of the current capital stock  $s_{t-1}$ .

#### 3.2 Equilibrium without pay-as-you-go pension system

We first state the existence and uniqueness of an interior equilibrium with positive altruism spendings ( $\phi_t > 0$ ) in an economy without pension system ( $\tau_{t+1} = 0$ ). The equilibrium is characterized by the following equations

$$s_t = \beta H(\psi_{t+1}) \quad (8)$$

$$\beta \frac{\rho'(\phi_t)}{\rho(\phi_t)} (1 - H(\psi_{t+1})) = 1 \quad (9)$$

$$\psi_{t+1} = \frac{R\left(\frac{s_t}{n}\right) s_t}{\beta \rho(\phi_t)}. \quad (10)$$

From Lemma 2, equations (8) and (9) define  $\phi_t$  and  $\psi_{t+1}$  as function of  $R_{t+1}$ . From (10), one obtains

$$s_t - \frac{\beta \rho \left( \Phi \left( R \left( \frac{s_t}{n} \right) \right) \right) \Psi \left( R \left( \frac{s_t}{n} \right) \right)}{R \left( \frac{s_t}{n} \right)} = 0 \quad (11)$$

In the appendix, we have shown, that the left-hand side of (11) is increasing with respect to  $s_t$ . Note that with  $\xi(R(0)) = \beta H(0) > 0$ , the LHS is negative for  $s_t = 0$ . The following assumption will allow to obtain the existence of an interior equilibrium.

**Assumption 4** *Let us define  $s^{\max} = \beta - \frac{\rho(0)}{\rho'(0)}$ . We assume that*

$$s^{\max} - \beta H \left( \frac{R \left( \frac{s^{\max}}{n} \right) s^{\max}}{\beta \rho(A(\beta - s^{\max}))} \right) > 0$$

**Proposition 1** *Under assumptions 1-4, there exists a unique interior equilibrium without pension system.*

### 3.3 Equilibrium effect of the pay-as-you-go pension system

Starting from the interior equilibrium without pension system, we study how the introduction of such a system influences the equilibrium.

**Proposition 2** *For any value of the contribution rate  $\tau_{t+1} \geq 0$  such that  $c_t \geq 0$ , there exists a unique equilibrium  $(\phi_t, \psi_{t+1}, s_t)$ . This equilibrium has the following properties:*

1.  $\phi_t$  is a decreasing function of  $\tau_{t+1}$ , till a threshold  $\tau^l$  after which  $\phi_t = 0$ .
2.  $\psi_{t+1}$  is an increasing function of  $\tau_{t+1}$ .
3.  $s_t$  is a decreasing function of  $\tau_{t+1}$ .

*Proof.* Since at equilibrium  $R_{t+1}$  is a decreasing function of the current capital stock  $s_t$ , we obtain from Lemma 2  $\psi$  and  $\phi$  as functions of the current capital stock  $s_t$

$$\psi = \Psi(s_t) \text{ and } \phi = \Phi(s_t)$$

where  $\Psi$  is decreasing and  $\Phi$  is non-decreasing with respect to  $s_t$ . From (5), one may derive the following characterization of the equilibrium capital stock  $s_t$

$$R \left( \frac{s_t}{n} \right) s_t + n\tau_{t+1}w \left( \frac{s_t}{n} \right) - \beta R \left( \frac{s_t}{n} \right) H(\Psi(s_t)) = 0$$

where the LHS is increasing with respect to  $s_t$ . This proves existence and uniqueness of  $s_t$ . Moreover, we deduce from the last equation that savings  $s_t$  are a decreasing function of the contribution rate  $\tau_{t+1}$ . Properties 1 and 2 in Proposition 2 then follow immediatly. ■

## 4 Optimal policies

In this section, we first define the optimal stationary state of our economy, which is a generalization of the standard golden rule of the Diamond (1965)'s model. Then we show that this optimal stationary state can be obtained as an equilibrium for an appropriate choice of some policy instruments.

## 4.1 The optimal stationary state

In our model, the definition of the optimal stationary state is not obvious, and two notions may be introduced. The first one is called *egoistic* and focus on the life-cycle utility of an agent, i.e. the utility that he obtains from his own consumptions. On the contrary, in the *altruistic* stationary state, the social planner takes into account the altruism motive. We successively consider these two optima.

### 4.1.1 Egoistic optimal stationary state

The egoistic optimal stationary state is obtained in maximizing the life-cycle utility of an agent at the stationary state subject to the resource constraint. The program is written

$$\begin{aligned} & \max_{(c,d)} c + \beta \ln d \\ \text{s. t. } & c + \frac{d}{n} + \phi = f(k) - nk \end{aligned}$$

The solution corresponds to the standard optimal golden rule in Diamond (1965):

$$\begin{aligned} k &= \hat{k} \text{ with } \hat{k} \text{ solution of } f'(\hat{k}) = n \\ d &= \beta n \\ c &= f(\hat{k}) - n\hat{k} - \beta \\ \phi &= 0 \end{aligned}$$

We assume that  $f(\hat{k}) - n\hat{k} - \beta > 0$  in order to have an interior solution. Altruism spendings  $\phi$  are obviously equal to zero since it does not appear in the objective function of the social planner.

### 4.1.2 Altruistic optimal stationary state

The altruistic optimal stationary state is obtained by taking into account altruism towards the parents. Young agents have different degree of altruism, corresponding to different values of the aptitude  $\varepsilon$ . Let us assume that the social planner does not observe this variable and only knows the distribution function. Therefore, the objective of the social planner becomes the sum of individual utilities, or equivalently, the average utility. In contrast with the egoistic optimum, the altruistic objective depends on the investment  $\phi$  in altruism that agents have receive in their childhood:

$$c + \beta \ln d + E(\varepsilon)\rho(\phi) (c + \beta \ln d)$$

Thus the program of the social planner becomes:

$$\begin{aligned} & \max_{(c,d)} [1 + E(\varepsilon)\rho(\phi)] (c + \beta \ln d) \\ \text{s. t. } & c + \frac{d}{n} + \phi = f(k) - nk \end{aligned}$$

First-order conditions lead to the following characterization of the solution (assuming that  $f(\hat{k}) - n\hat{k} - \beta - \phi > 0$  in order to have an interior solution):

$$\begin{aligned} k &= \hat{k} \text{ with } \hat{k} \text{ solution of } f'(\hat{k}) = n \\ d &= \beta n \\ 1 + E(\varepsilon)\rho(\phi) &= E(\varepsilon)\rho'(\phi) \left[ f(\hat{k}) - n\hat{k} - \beta - \phi + \beta \ln(\beta n) \right] \\ c &= f(\hat{k}) - n\hat{k} - \beta - \phi \end{aligned}$$

The choice of the optimal value of  $\phi$  results from a completely different trade-off than the private investment in the competitive equilibrium. Indeed, the social planner invests in  $\phi$  because this investment augments the pleasure that agents enjoy from the consumption of their parents. In the equilibrium, parents strategically invest in the altruism of their children, in order to receive a gift from them.

## 4.2 Decentralizing the optimal stationary state

We successively consider the decentralization of the two optimal states, egoistic and altruistic. In the Diamond (1965)'s model, it is well known that this decentralization can be achieved with an intergenerational transfer, for instance a public pension system. In our model, with an endogenous altruism motive, this unique instrument is not sufficient. First, the decentralization of the optimal steady state must leave all agents of the same generation with equal consumptions. This implies uniform gifts within the family. Such a situation cannot be obtained since the degrees of altruism are randomly distributed. The only way to obtain uniform consumption of the young is to make the gifts inoperative for all agents within each family. A tax on the gifts, denoted by  $\eta$  in the following, can be appropriate for this aim.

Making gifts inoperative is useful when the social planner has an egoistic objective. Parents then have no incentives for investing in the altruism of their children and  $\phi$  will immediately be zero. By contrast, for the altruistic optimum, we have seen that investing in children's altruism is welfare enhancing. Thus, inoperative gifts lead the government to be in charge of arousing the altruism of the children. We assume that the government will play this role by investing an amount  $\Phi$  in each family.

### 4.2.1 Decentralizing the egoist optimum

The egoistic optimum can be decentralized using two instruments: an intergenerational transfer implying a tax rate  $\tau$ , and a tax on gifts  $\eta$ . The optimal gift of a child of type  $\varepsilon$  born in  $t - 1$  maximizes

$$-g_t + \varepsilon \rho(\phi_{t-1}) \beta \ln(R_t s_{t-1} + b_t + g_t(1 - \eta) + G_t^-(1 - \eta))$$

Therefore, equation (2) becomes:

$$G_t = \max \{0, (1 - \eta) \beta \varepsilon \rho(\phi_{t-1}) - R_t s_{t-1} - b_t\}$$

and gifts are inoperative at the equilibrium if  $\eta$  is such:

$$\frac{R_t s_{t-1} + b_t}{(1 - \eta) \beta \rho(\phi_{t-1})} \geq E \quad (12)$$

Inoperative gifts imply zero altruism spendings:  $\phi_{t-1} = 0$ . The optimal choice of savings satisfies

$$R_t s_{t-1} + b_t = \beta R_t$$

with  $b_t = \tau n w_t$ . We deduce from this equation the optimal value of  $\tau$  that must satisfy:

$$n \hat{k} + \tau [f(\hat{k}) - n \hat{k}] = \beta \quad (13)$$

Using the expression of (12) at the egoistic optimum, we obtain the constraint that  $\eta$  must satisfy

$$\frac{\beta n}{(1 - \eta) \beta \rho(0)} \geq E$$

or equivalently,

$$1 - \eta \leq \frac{n}{E\rho(0)} \quad (14)$$

**Remark 1** For  $\eta$  and  $\tau$  satisfying (13) and (14), there is no gift at equilibrium, and therefore no income from the tax on gifts  $\eta$ .

For the optimal values  $\tau$  and  $\eta$ , it is straightforward to show that consumptions at the equilibrium correspond to the optimal one.

#### 4.2.2 Decentralizing the altruistic optimum

The altruistic optimum can be decentralized using three instruments: an intergenerational transfer implying a tax rate  $\tau$ , a tax on gifts  $\eta$  and a public investment  $\Phi$  to each family for arousing the altruism of children. The tax on gifts  $\eta$  is needed to cancel out private transfers, which introduce unsuitable heterogeneity between agents. But, if private gifts are no more operative, private investment  $\phi$  in children altruism cancels out, and a public investment  $\Phi$  is required. The optimal value of  $\Phi$  is equal to the value  $\hat{\phi}$  obtained in the altruistic optimum. These spendings is financed through an additional lump-sum tax on the first period income of each parent.

Since gifts are inoperative at equilibrium, the optimal choice of savings remains given by,

$$R_t s_{t-1} + b_t = \beta R_t$$

which defines the optimal value of  $\tau$ :

$$n\hat{k} + \tau \left[ f(\hat{k}) - n\hat{k} \right] = \beta$$

The tax  $\eta$  must be set in order to make gifts inoperative, for the level of altruism education that corresponds to  $\hat{\phi}$ . Therefore, (14) becomes:

$$1 - \eta \leq \frac{n}{E\rho(\hat{\phi})} \quad (15)$$

## 5 Political equilibrium

### 5.1 The preferred tax rate

In period  $t$ , young and old people vote on the contribution rate  $\tau_t$  that determines the current size of the public pay-as-you-go pension system. Savings  $s_{t-1}$  and altruism spending  $\phi_{t-1}$  of the parents have been decided in  $t-1$  with respect to the expected value  $\tau_t^e$  of the contribution rate. Since current capital stock is equal to past savings  $s_{t-1}$ , the result of the voting process does not affect capital stock. Thus, the old prefer the highest possible contribution rate  $\tau_t$ . The young generation is divided between two types of children who will not face the same trade-off with respect to the public pension system. Within each family, the most altruistic child who is the only one to make a positive gift must be distinguished from his siblings. Indirect utility of the most altruistic child in a family is affected by the current contribution rate through the term

$$(1 - \tau_t) w_t - G_t + \bar{\varepsilon} \rho(\phi_{t-1}) \beta \ln(R_t s_{t-1} + n\tau_t w_t + G_t).$$

If  $\bar{\varepsilon} > \frac{R_t s_{t-1} + n\tau_t w_t}{\beta \rho(\phi_{t-1})}$  ( $G_t > 0$ ), this term is equal to

$$(1 - \tau_t) w_t - (\beta \bar{\varepsilon} \rho(\phi_{t-1}) - R_t s_{t-1} - n\tau_t w_t) + \bar{\varepsilon} \rho(\phi_{t-1}) \beta \ln(\beta \bar{\varepsilon} \rho(\phi_{t-1})). \quad (16)$$

If  $G_t = 0$ , then we have

$$(1 - \tau_t) w_t + \bar{\varepsilon} \rho(\phi_{t-1}) \beta \ln(R_t s_{t-1} + n\tau_t w_t). \quad (17)$$

From the individual view point, the gift and the contribution to the pay-as-you-go pension system have not the same marginal utility return. Gifts result from a non-cooperative behavior within the family while the contribution to the pay-as-you-go system imposes to all children to contribute and enforces cooperation. The marginal return of the contribution to the pension system represents  $n$  times the marginal return of an individual gift.

Let us first assume that  $R_t s_{t-1} < n\bar{\varepsilon} \rho(\phi_{t-1}) \beta$ . Since (16) is increasing with respect to  $\tau_t$ , the most altruistic agent prefers a contribution rate that leads to zero gift and that is given by (17). Maximizing (17) gives the optimal value of  $\tau_t$ :

$$R_t s_{t-1} + n\tau_t w_t = n\bar{\varepsilon} \rho(\phi_{t-1}) \beta. \quad (18)$$

Now, if  $R_t s_{t-1} \geq n\bar{\varepsilon} \rho(\phi_{t-1}) \beta$ , this agent prefers a zero gift and a contribution rate equal to zero.

We now turn to children who are not the most altruistic child within their families. When the public pension system does not exist, they do not make any transfer to their parent, but they benefit from the gift made by the most altruistic child of the family. On the contrary, when a public pension system is implemented, they are forced to contribute to a public transfer that implies a utility loss for them. Nevertheless, they may benefit from the fact that the resulting consumption of their parent is now higher than his consumption without pension system. Such a gain increases with their own degree of altruism, and vanishes if they have a very altruistic brother or sister. In fact, if the altruism of the most altruistic child of the family remains operative despite the presence of the pension system, it means that the consumption level of the parent does not depend on  $\tau_t$  and the pension system is only a cost for the less altruistic children.

Let us consider such a child endowed with a degree of altruism  $\varepsilon$ . As long as the most altruistic child of the family makes a gift, his indirect utility term corresponding to (16) is equal to:

$$(1 - \tau_t) w_t + \varepsilon \rho(\phi_{t-1}) \beta \ln(\beta \bar{\varepsilon} \rho(\phi_{t-1})) \quad (19)$$

The pension system only entails a cost for him. If the most altruistic child of the family does not make a gift, the indirect utility term becomes

$$(1 - \tau_t) w_t + \varepsilon \rho(\phi_{t-1}) \beta \ln(R_t s_{t-1} + n\tau_t w_t).$$

If  $R_t s_{t-1} < n\varepsilon \rho(\phi_{t-1}) \beta$ , this expression is maximized when  $\tau_t$  is such that

$$R_t s_{t-1} + n\tau_t w_t = n\varepsilon \rho(\phi_{t-1}) \beta. \quad (20)$$

If  $R_t s_{t-1} \geq n\varepsilon \rho(\phi_{t-1}) \beta$ , the agent prefers a contribution rate equal to zero.

To sum up the case of a child who is not the most altruistic child within his family,

- If  $R_t s_{t-1} \geq n\varepsilon \rho(\phi_{t-1}) \beta$ , his utility decreases with  $\tau_t$  and his preferred tax rate is 0.

- If  $R_t s_{t-1} < n\varepsilon\rho(\phi_{t-1})\beta$  and  $G_t = 0$  for  $\tau_t = 0$ , his utility is unimodal with respect to  $\tau_t$  and his preferred tax rate given by (20).
- If  $R_t s_{t-1} < n\varepsilon\rho(\phi_{t-1})\beta$  and  $G_t > 0$  for  $\tau_t = 0$ , his utility is first decreasing, after increasing and finally decreasing. The preferred tax rate can be either 0 or given by (20).

Two properties appear through these results. First, preferences of some agents can be non single peaked. Second, when agents are in favor of a positive tax rate  $\tau_t$ , its expression only depends on the degree of altruism ( $\bar{\varepsilon}$  or  $\varepsilon$ ): (18) and (20) have the same expressions.

The second remark implies that the median voter theorem cannot be applied in our framework. The tax rate that emerges from a vote is only determined in a particular case where  $n$  is high enough. In this case, the gain associated with cooperation in a voting equilibrium is high, and the most altruistic agents are in favor of a positive tax rate, even if they are not the most altruistic children within their family. Therefore, the majoritarian coalition will be made by the old agents associated with the most altruistic agents among the young.

## 5.2 The Condorcet winner

We admit that all the most altruistic young agents belong to the majoritarian coalition with old agents, even if they are not the most altruistic in their family. We will check ex-post that this property is satisfied at the equilibrium. Under this assumption, the median voter is defined by his degree of altruism  $\varepsilon^m$  such that:

$$\begin{aligned} 1 + n(1 - G(\varepsilon^m)) &= \frac{1 + n}{2} \\ \text{or } G(\varepsilon^m) &= \frac{1}{2n} + \frac{1}{2} \end{aligned} \quad (21)$$

**Proposition 3** *Under the assumption  $n\varepsilon^m \geq E$ , if*

$$\varepsilon^m \rho(\phi_{t-1})\beta \left[ \ln\left(\frac{n\varepsilon^m}{E}\right) - 1 \right] + \frac{R_t s_{t-1}}{n} > 0, \quad (22)$$

*the tax rate  $\tau_t = \tau_t^m$  defined by:*

$$R_t s_{t-1} + n\tau_t^m w_t = n\varepsilon^m \rho(\phi_{t-1})\beta \quad (23)$$

*is the Condorcet winner. In particular, condition (22) is true for  $n$  high enough.*

*Proof.* First consider the case of children who are the most altruistic within their family. Their indirect utility on  $\tau_t$  is unimodal. (23) is the preferred tax rate of such a child endowed with a degree of altruism  $\varepsilon^m$ . Children with a higher degree  $\varepsilon > \varepsilon^m$  are in favor of a higher tax rate and children with a lower degree  $\varepsilon < \varepsilon^m$  are in favor of a lower tax rate.

Second, consider the case of children who are not the most altruistic within their family. Their preferred tax rate is either  $\tau_t^m$  determined by (20) or is 0. Therefore, we

must prove that all children of this type endowed with a degree of altruism  $\varepsilon \geq \varepsilon^m$  prefer  $\tau_t$  determined by (23) to 0. A necessary condition is that the child has not a brother/sister that continues to make a gift with the pension system. This condition will be satisfied only if  $n\varepsilon^m/E \geq 1$ . Indeed if  $E > n\varepsilon^m$ , an agent with a degree of altruism  $\varepsilon > n\varepsilon^m$  would continue to make a gift to his parent despite the existence of the pension system which size depends on  $n\varepsilon^m$ .

Under the assumption  $n\varepsilon^m \geq E$ , consider an agent who is not the most altruistic within his family. The difference between the indirect utilities corresponding to  $\tau_t^m$  and 0 is equal to:

$$-\tau_t^m w_t + \varepsilon \rho(\phi_{t-1}) \beta \ln(n\varepsilon^m \rho(\phi_{t-1}) \beta) - \varepsilon \rho(\phi_{t-1}) \beta \ln(\bar{\varepsilon} \rho(\phi_{t-1}) \beta)$$

This agent will prefer  $\tau_t^m$  to 0 if:

$$-\tau_t^m w_t + \varepsilon \rho(\phi_{t-1}) \beta \ln\left(\frac{n\varepsilon^m}{\bar{\varepsilon}}\right) > 0$$

As  $\bar{\varepsilon} \leq E \leq n\varepsilon^m$  and  $\varepsilon \geq \varepsilon^m$ , a stronger condition is:

$$-\tau_t^m w_t + \varepsilon^m \rho(\phi_{t-1}) \beta \ln\left(\frac{n\varepsilon^m}{E}\right) > 0$$

Using the definition of  $\tau_t^m$  (23), this last condition can be written:

$$\varepsilon^m \rho(\phi_{t-1}) \beta \left[ \ln\left(\frac{n\varepsilon^m}{E}\right) - 1 \right] + \frac{R_t s_{t-1}}{n} > 0$$

A sufficient condition ensuring this inequality is that  $n\varepsilon^m/E > e$ , which is true for  $n$  high enough.

Finally, we have proved that  $\tau_t^m$  given by (23) will be the solution of the voting process. ■

### 5.3 Perfect foresight political equilibrium

In this section, we consider that the median voter is endowed with a degree of altruism defined by equation (21):

$$\varepsilon^m = \chi(n) \equiv G^{-1}\left(\frac{1}{2n} + \frac{1}{2}\right)$$

Let us recall that  $\varepsilon^m$  can be the median voter only if  $n\varepsilon^m \geq E$ . In this case, the Condorcet winner depends only on  $n$  and is not affected by the economic policy or by the current level of capital accumulation. Moreover, we assume that generation  $t-1$  has made perfect foresight on the value of  $\tau_t$ .

An equilibrium is defined by the three equations given in section 3.1 and by the additional equation (23) which gives the preferred value of  $\tau_t$  for the median voter. Thus we obtain a system of four equations with four unknown variables ( $\phi_{t-1}$ ,  $s_{t-1}$ ,  $\psi_t$ ,  $\tau_t$ ):

$$R\left(\frac{s_{t-1}}{n}\right) s_{t-1} + n\tau_t w\left(\frac{s_{t-1}}{n}\right) = \beta R\left(\frac{s_{t-1}}{n}\right) H(\psi_t) \quad (24)$$

$$1 - H(\psi_t) \leq \frac{\rho(\phi_{t-1})}{\beta \rho'(\phi_{t-1})}, = \text{if } \phi_{t-1} > 0 \quad (25)$$

$$R\left(\frac{s_{t-1}}{n}\right) s_{t-1} + n\tau_t w\left(\frac{s_{t-1}}{n}\right) = \beta \rho(\phi_{t-1}) \psi_t \quad (26)$$

$$R\left(\frac{s_{t-1}}{n}\right) s_{t-1} + n\tau_t w\left(\frac{s_{t-1}}{n}\right) = n\varepsilon^m \rho(\phi_{t-1}) \beta \quad (27)$$

From (26) and (27), we obtain that  $\psi_t = n\varepsilon^m \Rightarrow \psi_t > E$ . Thus,  $H(\psi_t) = 1$  and (25) gives  $\phi_{t-1} = 0$ . Using (24) and (27),  $s_{t-1}$  is solution of

$$R\left(\frac{s_{t-1}}{n}\right) = n\varepsilon^m \rho(0)$$

Finally,  $\tau_t$  can be obtained from (24), (26) or (27).

The resulting equilibrium leads to no altruism spending:  $\phi_{t-1} = 0$  and no private gifts. All agents have the same consumptions at equilibrium, as these consumptions are not affected by private random gifts. The equilibrium can therefore be compared with the optimal egoistic stationary equilibrium described in section 4.1.1. The equilibrium level of savings  $s_{t-1}$  is such that:

$$R\left(\frac{s_{t-1}}{n}\right) = n\varepsilon^m \rho(0)$$

when the optimal level  $\hat{s}$  satisfies:

$$R\left(\frac{\hat{s}}{n}\right) = n$$

Therefore, there is under accumulation of capital if  $\varepsilon^m \rho(0) > 1$  and over accumulation if  $\varepsilon^m \rho(0) < 1$ . In the special case where  $\varepsilon^m \rho(0) = 1$ , we have  $s_{t-1} = \hat{s}$ . Moreover, all equilibrium consumptions correspond to their optimal level. Indeed, at equilibrium,  $d_t$  is equal to (from (27)):

$$\begin{aligned} d_t &= R\left(\frac{s_{t-1}}{n}\right) s_t + n\tau_t w \left(\frac{s_{t-1}}{n}\right) \\ &= n\varepsilon^m \rho(0) \beta = n\beta \end{aligned}$$

and  $n\beta$  is the optimal level of  $d$ . If  $d$ ,  $s$  and  $\phi$  are equal to the optimal level,  $c$  also is optimal.

## 5.4 Impact of ageing parameters on the pension system

We study the effect of an increase of  $n$ . The median voter corresponds to a value of  $\varepsilon^m = \chi(n)$  such that  $n\varepsilon^m > E$ . The following lemma states some properties of function  $\chi(n)$ .

**Lemma 3**  $\chi$  satisfies the following properties:

1.  $\chi$  is non indecreasing with  $n$ . When  $n$  tends to be infinite,  $\chi(n)$  tends toward  $G^{-1}(1/2)$ .

2.

$$\frac{d[n\chi(n)]}{dn} = \varepsilon^m \left[ 1 - \frac{1}{2ng(\varepsilon^m)\varepsilon^m} \right]$$

Notice that, following (20),  $n\varepsilon^m = n\chi(n)$  determines the preferred tax rate of the median voter. Result 2 shows that  $n\varepsilon^m$  can either increase or decrease with  $n$  according as  $2n\varepsilon^m g(\varepsilon^m)$  is higher or lower than 1.

The size of the pension system is described by the individual contribution :

$$\tau_t w_t = \frac{n\varepsilon^m \rho(0) \beta}{n} - R\left(\frac{s_{t-1}}{n}\right) \frac{s_{t-1}}{n} \quad (28)$$

with  $s_{t-1}/n$  solution of the equation

$$R\left(\frac{s_{t-1}}{n}\right) = n\varepsilon^m \rho(0) \quad (29)$$

The effect of  $n$  on the contribution  $\tau_t w_t$  is threefold. First, the number of children per family affects the desired old-age consumption of the parent  $n\varepsilon^m \rho(0) \beta$ ; as stated in Lemma 3, this effect can be positive or negative. Second, this desired consumption is divided between a higher number of children; this diminishes the contribution. The net impact of these two first effects is negative. The third effect goes through capital revenue  $R\left(\frac{s_{t-1}}{n}\right) \frac{s_{t-1}}{n}$  and rests on the change in the capital stock induced by  $n$ . By assumption 3, we know that  $R(k)k$  is non decreasing with respect to  $k$ . Moreover, from equation (29), the steady-state capital-labor ratio is decreasing with respect to  $n\varepsilon^m$ . Thus, savings return  $R\left(\frac{s_{t-1}}{n}\right) \frac{s_{t-1}}{n}$  decreases with respect to  $n\varepsilon^m$ .

To sum up all these effects, the contribution  $\tau_t w_t$  is decreasing with  $n$  whenever  $n\varepsilon^m$  is decreasing with  $n$ . If  $n\varepsilon^m$  increases with respect to  $n$ , the resulting effect is ambiguous. For instance, consider a distribution function of  $\varepsilon$  with a high density  $g$  in the neighborhood of  $\varepsilon^m$ . Then, an increase in  $n$  will have a very small negative effect on  $\varepsilon^m$  and a sharp effect on  $n\varepsilon^m$ . The resulting effect on the contribution  $\tau_t w_t$  would then be positive.

The size of the pension system can also be described by the ratio of pension receipts to GDP. With a Cobb-Douglas production function,  $F(K, L) = AK^\alpha L^{1-\alpha}$ , this ratio writes

$$\begin{aligned} \frac{N_{t-1} b_t}{AK_t^\alpha N_t^{1-\alpha}} &= \frac{N_t \tau_t w_t}{AK_t^\alpha N_t^{1-\alpha}} = (1 - \alpha) \tau_t = \varepsilon^m \rho(0) \beta \left(\frac{s_{t-1}}{n}\right)^{-\alpha} - \alpha \\ &= \varepsilon^m \rho(0) \beta \left(\frac{n\varepsilon^m \rho(0)}{\alpha A}\right)^{\frac{\alpha}{1-\alpha}} - \alpha \end{aligned}$$

Thus, the ratio of pension receipts to GDP decreases with respect to  $n$  whenever  $n\varepsilon^m$  decreases with  $n$ . If  $n\varepsilon^m$  increases with  $n$ , the effect is ambiguous. The same discussion about density  $g$  in the neighborhood of  $\varepsilon^m$  allows to state the possibility of a positive effect of  $n$  on the relative size of the pay-as-you-go pension system.

## 5.5 Voting on pension system and altruism spending

We now consider a two-dimensional vote. In addition to the vote on the pension system, we introduce a vote on public spendings devoted to arouse next generation altruism and financed by taxes on the adult generation. In period  $t$ , those spendings, denoted by  $\Phi_t$ , will affect altruism of the adult generation of period  $t + 1$  and are neutral for the old generation in  $t$ . Moreover, the adult generation in  $t$  will vote on public altruism spendings taking into account of their effect on their future pensions. Since the utility function is linear with respect to first-period consumption, there is no interaction between the votes on  $\tau_t$  and  $\Phi_t$ .

Utility of the adult generation in period  $t$  is affected by  $\Phi_t$  through two terms. First, this generation will pay a tax  $\Phi_t$  in period  $t$ . Second, old-age consumption in period  $t + 1$  is  $n\varepsilon^m \rho(\Phi_t) \beta$ . Here, we follow the same line as in the preceding sections:  $n\varepsilon^m \geq E$  which implies that there is no private gift  $G_{t+1} = 0$  and no private altruism spendings  $\phi_t = 0$ . Consequently, the relevant terms in the indirect utility function for the vote on  $\Phi_t$  are

$$-\Phi_t + \beta \ln(n\varepsilon^m \rho(\Phi_t) \beta)$$

which reaches its maximum for  $\Phi_t = \bar{\Phi}$  satisfying (from Assumption 2)

$$\beta \frac{\rho'(\bar{\Phi})}{\rho(\bar{\Phi})} = 1$$

At the political equilibrium in period  $t + 1$ , the individual contribution to the pension system now is

$$\tau_{t+1} w_{t+1} = \frac{n \varepsilon^m \rho(\bar{\Phi}) \beta}{n} - R\left(\frac{s_t}{n}\right) \frac{s_t}{n}$$

with  $s_t/n$  solution of the equation

$$R\left(\frac{s_t}{n}\right) = n \varepsilon^m \rho(\bar{\Phi})$$

The desired old-age consumption  $n \varepsilon^m \rho(\bar{\Phi}) \beta$  is now higher than in the case without voting on  $\Phi_t$ , while capital income  $R\left(\frac{s_t}{n}\right) \frac{s_t}{n}$  is lower. The resulting effect on the individual contribution is positive.

## 6 Appendix

### 6.1 Definition of $H$ with respect to $G$

$G$  is the distribution function of the parameter  $\varepsilon$  among an entire generation, and  $H$ , the distribution function of  $\varepsilon$  among the agents who are the most altruistic among the  $n$  children within each families. When  $n = 1$ , we have in particular:  $H(\varepsilon) = G(\varepsilon)$ . Considering a family with  $n$  children characterized by their innate parameter  $\varepsilon^i$ , we have:

$$\begin{aligned} \Pr [\sup (\varepsilon^i) \leq x] &= \Pr [(\varepsilon^1 \leq x) \cap (\varepsilon^2 \leq x) \dots \cap (\varepsilon^n \leq x)] \\ &= G(x)^n \end{aligned}$$

Therefore, we can write:

$$H(\varepsilon) = G(\varepsilon)^n \tag{30}$$

### 6.2 Concavity

One can rewrite the objective as a function of  $\varrho = \rho(\phi)$  and  $\psi$  and the consumer problem remains to maximize

$$\begin{aligned} W_t(\rho, \psi) &= - \left( \frac{\psi \beta \varrho - b_{t+1}}{R_{t+1}} + \rho^{-1}(\varrho) \right) \\ &\quad + \beta H(\psi) \ln(\psi) + \beta \ln(\beta \varrho) + \beta \int_{\psi}^E \ln(\bar{\varepsilon}) h(\bar{\varepsilon}) d\bar{\varepsilon} \end{aligned}$$

with respect to  $\psi_{t+1}$  and  $\rho_t$ , subject to the following constraints

$$0 \leq \rho^{-1}(\varrho) \leq \frac{\psi \beta \varrho - b_{t+1}}{R_{t+1}} + \rho^{-1}(\varrho) \leq (1 - \tau_t) w_t - g_t$$

First-order derivatives of  $W_t$  are

$$\frac{\partial W_t}{\partial \varrho} = - \left( \frac{\psi \beta}{R_{t+1}} + (\rho^{-1})'(\varrho) \right) + \frac{\beta}{\varrho} \quad (31)$$

$$\frac{\partial W_t}{\partial \psi} = - \frac{\beta \varrho}{R_{t+1}} + \beta \frac{H(\psi)}{\psi} \quad (32)$$

and second-order derivatives are

$$\frac{\partial^2 W_t}{\partial \varrho^2} = - (\rho^{-1})''(\varrho) - \frac{\beta}{\varrho^2}$$

$$\frac{\partial W_t}{\partial \varrho \partial \psi} = - \frac{\beta}{R_{t+1}}$$

$$\frac{\partial^2 W_t}{\partial \psi^2} = \frac{\beta}{\psi} \left[ h(\psi) - \frac{H(\psi)}{\psi} \right]$$

Since  $\rho$  is increasing and strictly concave,  $\rho^{-1}$  is strictly convex and the sign of  $\partial^2 W_t / \partial \varrho^2$  is negative. The determinant of the Hessian matrix of  $W$  writes

$$\left[ (\rho^{-1})''(\varrho) + \frac{\beta}{\varrho^2} \right] \frac{\beta}{\psi} \left[ \frac{H(\psi)}{\psi} - h(\psi) \right] - \left( \frac{\beta}{R_{t+1}} \right)^2$$

and is positive if and only if

$$\frac{1}{\psi} \left[ \frac{H(\psi)}{\psi} - h(\psi) \right] > \frac{\beta}{\beta + \varrho^2 (\rho^{-1})''(\varrho)} \left( \frac{\varrho}{R_{t+1}} \right)^2 \quad (33)$$

where

$$(\rho^{-1})''(\varrho) = \frac{-\rho''((\rho^{-1})(\varrho))}{[\rho'((\rho^{-1})(\varrho))]^3} = \frac{-\rho''(\phi)}{[\rho'(\phi)]^3}$$

### 6.3 Existence of an interior solution

From equations (31) and (32), an interior solution satisfies

$$\beta \frac{\rho'(\rho^{-1}(\varrho))}{\varrho} (1 - H(\psi)) = 1 = \frac{R H(\psi)}{\varrho \psi}.$$

From the first equation, we deduce  $\psi$  as a function of  $\varrho$

$$\psi = H^{-1} \left( 1 - \frac{\varrho}{\beta \rho'(\rho^{-1}(\varrho))} \right) = \Xi(\varrho)$$

which is a decreasing function from  $H^{-1} \left( 1 - \frac{\rho(0)}{\beta \rho'(0)} \right)$  to 0 (since  $\lim_{\phi \rightarrow +\infty} \frac{\rho(\phi)}{\rho'(\phi)} > \beta$ ).

With the second equation, we obtain  $R$  as a function of  $\varrho$

$$R = \frac{\varrho \psi}{H(\psi)} = \frac{\varrho H^{-1} \left( 1 - \frac{\varrho}{\beta \rho'(\rho^{-1}(\varrho))} \right)}{1 - \frac{\varrho}{\beta \rho'(\rho^{-1}(\varrho))}}$$

Thus

$$\frac{dR}{d\varrho} = \frac{\psi}{H(\psi)} + \varrho \left[ \frac{1}{H(\psi)} - \frac{\psi h(\psi)}{[H(\psi)]^2} \right] \frac{d\psi}{d\varrho}$$

where

$$\frac{d\psi}{d\varrho} = \frac{-1}{\beta h(\psi)} \left( \frac{1}{\rho'(\rho^{-1}(\varrho))} - \frac{\varrho \rho''(\rho^{-1}(\varrho))}{[\rho'(\rho^{-1}(\varrho))]^3} \right)$$

Thus

$$\begin{aligned} \frac{dR}{d\varrho} &= \frac{\psi}{H(\psi)} - \frac{\varrho}{\beta} \left[ \frac{1}{H(\psi)} - \frac{\psi h(\psi)}{[H(\psi)]^2} \right] \frac{1}{h(\psi)} \left( \frac{1}{\rho'(\rho^{-1}(\varrho))} - \frac{\varrho \rho''(\rho^{-1}(\varrho))}{[\rho'(\rho^{-1}(\varrho))]^3} \right) \\ &= \frac{\psi}{H(\psi)} \frac{1}{h(\psi)} \left[ h(\psi) - \frac{\varrho}{\beta H(\psi)} \left[ \frac{H(\psi)}{\psi} - h(\psi) \right] \left( \frac{1}{\rho'(\rho^{-1}(\varrho))} - \frac{\varrho \rho''(\rho^{-1}(\varrho))}{[\rho'(\rho^{-1}(\varrho))]^3} \right) \right] \end{aligned}$$

#### 6.4 Effect of $R$ on $\rho$ and $\psi$

Differentiating the first-order conditions  $\frac{\partial W}{\partial \rho}(\rho, \psi, R) = 0$  and  $\frac{\partial W}{\partial \psi}(\rho, \psi, R) = 0$  with respect to  $\rho$ ,  $\psi$  and  $R$ , one obtains

$$\begin{aligned} \frac{\partial^2 W}{\partial \rho^2} d\rho + \frac{\partial^2 W}{\partial \rho \partial \psi} d\psi + \frac{\partial^2 W}{\partial \rho \partial R} dR &= 0 \\ \frac{\partial^2 W}{\partial \rho \partial \psi} d\rho + \frac{\partial^2 W}{\partial \psi^2} d\psi + \frac{\partial^2 W}{\partial \psi \partial R} dR &= 0 \end{aligned}$$

Thus

$$\begin{aligned} \frac{d\rho}{dR} &= \frac{1}{D} \left[ \frac{\partial^2 W}{\partial \rho \partial \psi} \frac{\partial^2 W}{\partial \psi \partial R} - \frac{\partial^2 W}{\partial \rho \partial R} \frac{\partial^2 W}{\partial \psi^2} \right] \\ \frac{d\psi}{dR} &= \frac{-1}{D} \left[ \frac{\partial^2 W}{\partial \rho^2} \frac{\partial^2 W}{\partial \psi \partial R} - \frac{\partial^2 W}{\partial \rho \partial R} \frac{\partial^2 W}{\partial \rho \partial \psi} \right] \end{aligned}$$

where

$$D = \frac{\partial^2 W}{\partial \psi^2} \frac{\partial^2 W}{\partial \rho^2} - \left( \frac{\partial^2 W}{\partial \rho \partial \psi} \right)^2 > 0$$

Thus  $\frac{d\rho}{dR}$  has the same sign as

$$\begin{aligned} \frac{\partial^2 W}{\partial \rho \partial \psi} \frac{\partial^2 W}{\partial \psi \partial R} - \frac{\partial^2 W}{\partial \rho \partial R} \frac{\partial^2 W}{\partial \psi^2} &= -\frac{\beta^2 \rho}{R^3} - \frac{\beta}{\psi} \left[ h(\psi) - \frac{H(\psi)}{\psi} \right] \frac{\psi \beta}{R^2} \\ &= \frac{\beta^2}{R^2} \left[ \frac{H(\psi)}{\psi} - h(\psi) - \frac{\rho}{R} \right] \end{aligned}$$

From the first-order condition, we have

$$\frac{\rho}{R} = \frac{H(\psi)}{\psi}$$

Thus

$$\frac{\partial^2 W}{\partial \rho \partial \psi} \frac{\partial^2 W}{\partial \psi \partial R} - \frac{\partial^2 W}{\partial \rho \partial R} \frac{\partial^2 W}{\partial \psi^2} = -\frac{\beta^2}{R^2} h(\psi) < 0$$

By consequence  $\rho$  is decreasing with respect to  $R$ .

Let us now analyze the effect of  $R$  on  $\psi$ . First, the ratio  $\rho/R$  is decreasing with respect to  $R$ . Second, the concavity of the consumer problem implies that  $\frac{H(\psi)}{\psi}$  is decreasing with respect to  $\psi$ . Thus, from the equality  $\frac{\rho}{R} = \frac{H(\psi)}{\psi}$ , the threshold  $\psi$  is increasing with respect to  $R$ .

Since  $H(\psi)$  is increasing with respect to  $R$  and from the equality  $\frac{\rho\psi}{R} = H(\psi)$ , we deduce that  $\frac{\rho\psi}{R}$  is increasing with respect to  $R$ .

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