

# One way to design the control law of a mini-UAV

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**This paper deals with a method used to design the control law of the  $\mu$ Drone MAV. This vehicle uses six propellers to fly and the dynamic model approximation for the motion is a MIMO linear time-invariant system. As we want to design a linear regulator, it is necessary to build a robust feedback control law. The LQ state feedback regulator design is applied to a standard model, taking into account some perturbations. This is why the model is augmented with a perturbation vector and an observable subsystem is extracted in order to build a state estimator whose gain is the solution of a LQ problem. The subsystem is then decomposed into a controllable set and an uncontrollable one. The use of an asymptotic rejection strategy of the influence of uncontrollable modes gives the possibility to find a state feedback applied only to the controllable ones. Here again feedback matrix is chosen as the solution of a LQ problem. To compute the weighting matrices of quadratic criterions we use a "partial observability gramian". The great advantage of this method is due to the use of only three scalars to synthesize the control law.**

## Nomenclature

$x, y, z$	=	centre of gravity coordinates of the MAV
$\alpha$	=	roll angle
$\beta$	=	pitch angle
$\lambda$	=	yaw angle
$u_i$	=	actuator control input for solid $S_i$
$F_i$	=	thrust produced by the $i$ th actuator
$M_i$	=	moment produced by the $i$ th actuator
$\tau_i$	=	time constant of the $i$ th actuator
$k_i$	=	DC gain of the $i$ th actuator
$\underline{u}$	=	$[u_1 \ u_2 \ u_3 \ u_4 \ u_5 \ u_6]^T$ input vector
$\underline{y}$	=	$[\dot{x} \ \dot{y} \ z \ \alpha \ \beta \ \lambda]^T$ output vector
$\underline{y}_r$	=	reference vector
$\underline{x}$	=	state vector of initial linear MAV model $\in R^{18}$
$A, B, C$	=	matrices of initial state-space model

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$\underline{x}_p$  = perturbation vector

## I. Introduction

THIS paper deals with a method used to design the control law of the  $\mu$ Drone. This MAV uses six propellers to fly and it is not obvious to control the six brushless motors simultaneously. In order to build a dynamical model of the MAV, we worked on the assumption that it can be described with seven solids in interaction. After a linearization of the equations in the vicinity of a horizontal trim, the model is a multi-input, multi-output (MIMO) linear time-invariant dynamic system. This approximation can be used only if it is possible to build a robust feedback control law.

The interest of the present paper is to provide a simple method to reach the needed robustness, using the LQ state-feedback regulator design applied to a “standard model”, tacking into account some perturbations. This method is detailed in Ref. 1 and very recently in Ref. 2.

The paper is organized as follows. Section II gives some details on the MAV model and the control design is explained in section III. Section IV gives complements about the choice of adjustment parameters.

## II. Model of the $\mu$ Drone

The  $\mu$ Drone is a “gyro dyne” with six propellers. We worked on the assumption that it can be described with seven solids in interaction.

Solid  $S_0$  is the body.

Solids  $S_1$  and  $S_2$  are made up of a fix pitch propeller, the motor shaft and the blades holder. The two propellers are counter-rotating and provide the lift of the MAV.

Solids  $S_3$  and  $S_4$  include a variable pitch propeller, the motor shaft and the variable pitch system. These propellers provide the trim stabilization.

Solids  $S_5$  and  $S_6$  are like the two precedents and provide MAV propulsion.

The detailed description of this mechanical system modeling can be found in Ref. 3.

Let  $F_i$  denotes the thrust produced by the  $i$ th actuator and  $M_i$  the associated moment. If  $u_i$  denotes the control input of the actuator, we assume that

$$(1) \quad \begin{aligned} \tau_i \frac{dF_i}{dt} + F_i(t) &= k_i \cdot u_i(t) \\ M_i(t) &= k_i \cdot F_i(t) \end{aligned}$$

With this assumption and after linearization of the mechanical model, the MIMO linear time-invariant system, denoted DRO, is an approximation of the MAV behavior<sup>4,5</sup>. DRO is a system with 6 inputs, 6 outputs, and state-vector  $\underline{x}$  is of size 18.

$$(2) \quad \begin{aligned} \frac{d\underline{x}}{dt} &= A \cdot \underline{x} + B \cdot \underline{u} \\ \underline{y} &= C \cdot \underline{x} \end{aligned}$$

The eigenvalues of the  $A$  matrix are so defined: 10 are zero, 6 are real negative and 2 are purely imaginary. The rank of controllability matrix is 2 and that of observability matrix is 4.

## III. Control design of the MAV

In order to build a “standard model” we introduce a perturbation vector  $\underline{x}_p$  and we work on the assumption that each perturbation is constant. It is then possible to give a simple model for these perturbations

$$(3) \quad \frac{d\underline{x}_p}{dt} = A_p \cdot \underline{x}_p$$

where  $A_p$  is a square matrix of zeros. In our application  $\underline{x}_p \in R^2$  and the perturbations are moments acting on roll and pitch rates. The initial system DRO is augmented and becomes

$$(4) \quad \begin{aligned} \frac{d\underline{x}_e}{dt} &= A_e \cdot \underline{x}_e + B_e \cdot \underline{u} \\ \underline{y} &= C_e \cdot \underline{x}_e \end{aligned} \quad \text{with} \quad A_e = \begin{bmatrix} A & A_{12} \\ O & A_p \end{bmatrix}, B_e = \begin{bmatrix} B \\ O \end{bmatrix}, C_e = [C \quad O]$$

As we want to elaborate the control law with a state-feedback, it is necessary to build a state estimator. The first step is to compute the observability staircase form of  $(A_e, B_e, C_e)$  and extract an observable subsystem

$$(5) \quad \begin{aligned} \frac{d\underline{x}_o}{dt} &= A_{eo} \cdot \underline{x}_o + B_{eo} \cdot \underline{u} \\ \underline{y} &= C_{eo} \cdot \underline{x}_o \end{aligned}$$

It is then possible to build a state estimator

$$(6) \quad \frac{d\hat{\underline{x}}_o}{dt} = A_{eo} \cdot \hat{\underline{x}}_o + B_{eo} \cdot \underline{u} + K_o \cdot (\underline{y} - C_{eo} \cdot \hat{\underline{x}}_o)$$

The gain matrix  $K_o$  is chosen as the solution of the LQ problem

$$\text{find } \underline{v} = -KK_o \cdot \underline{x}_o \text{ that minimize } J_o = \int_0^\infty (\underline{x}_o^T \cdot Q_o \cdot \underline{x}_o + \underline{v}^T \cdot R_o \cdot \underline{v}) dt \quad \text{for} \quad \frac{d\underline{x}_o}{dt} = A_{eo}^T \cdot \underline{x}_o + C_{eo}^T \cdot \underline{v}$$

and define a positive scalar  $k_o$  such as

$$(7) \quad K_o = k_o \cdot KK_o^T$$

At this point, the problem is to make a good choice for the weighting matrices  $Q_o$  and  $R_o$  of the quadratic criterion  $J_o$ . In order to adjust easily  $Q_o$ , we use a “partial observability gramian” defined as

$$(8) \quad G_o(T_{oo}) = \int_0^{T_{oo}} \exp(A_{eo}^T \cdot t) \cdot C_{eo}^T \cdot C_{eo} \cdot \exp(A_{eo} \cdot t) \cdot dt$$

where  $T_{oo} = k_o \cdot T_o$  with  $T_o$  positive, called the filtering horizon and  $k_o$  the shape factor.

As the pair  $(A_{eo}, C_{eo})$  is observable,  $G_o(T_{oo})$  is asymmetric positive matrix, and it is possible to choose

$$(9) \quad \begin{aligned} Q_o &= [T_{oo} \cdot G_o(T_{oo})]^{-1} \\ R_o &= I \end{aligned} \quad \text{with } I \text{ the identity matrix.}$$

The state estimator is then

$$(10) \quad \frac{d\hat{\underline{x}}_o}{dt} = (A_{eo} - K_o \cdot C_{eo}) \cdot \hat{\underline{x}}_o + [B_{eo} \quad K_o] \cdot \begin{bmatrix} \underline{u} \\ \underline{y} \end{bmatrix}$$

Now let us compute the controllability staircase form of  $(A_{eo}, B_{eo}, C_{eo})$  with the similarity transformation matrix  $T_{oc}$

$$(11) \quad \frac{d}{dt} \begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix} = \begin{bmatrix} A_{eonc} & O \\ A_{eo21} & A_{eoc} \end{bmatrix} \begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix} + \begin{bmatrix} O \\ B_{eoc} \end{bmatrix} \underline{u} \quad \text{with} \quad \begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix} = T_{oc} \cdot \underline{x}_o$$

$$\underline{y} = \begin{bmatrix} C_{eonc} & C_{eoc} \end{bmatrix} \begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix}$$

As  $A_{eo21}$  is not a matrix of zeros, uncontrollable modes  $\underline{x}_{nc}$  perturb controllable modes  $\underline{x}_c$ . In order to reject the influence of uncontrollable modes it is possible to use a linear transformation

$$(12) \quad \begin{aligned} \underline{u}(t) &= \underline{v}(t) - G_a \cdot \underline{x}_{nc}(t) \\ \tilde{\underline{x}}_c(t) &= \underline{x}_c(t) + T_a \cdot \underline{x}_{nc}(t) \end{aligned} \quad \text{where } G_a \text{ and } T_a \text{ are constant matrices, leading to}$$

$$(13) \quad \begin{aligned} \frac{d\tilde{\underline{x}}_c}{dt} &= A_{eoc} \cdot \tilde{\underline{x}}_c + [A_{eo21} - A_{eoc} \cdot T_a + T_a \cdot A_{eonc} - B_{eoc} \cdot G_a] \underline{x}_{nc} + B_{eoc} \cdot \underline{v} \\ \underline{y} &= C_{eoc} \cdot \tilde{\underline{x}}_c + [C_{eonc} - C_{eoc} \cdot T_a] \underline{x}_{nc} \end{aligned}$$

Now, it is often possible to find  $G_a$  and  $T_a$  such as

$$(14) \quad \begin{aligned} [A_{eo21} - A_{eoc} \cdot T_a + T_a \cdot A_{eonc} - B_{eoc} \cdot G_a] &= [O] \\ [C_{eonc} - C_{eoc} \cdot T_a] &= [O] \end{aligned}$$

With these conditions equations (11) become

$$(15) \quad \begin{aligned} \frac{d\tilde{\underline{x}}_c}{dt} &= A_{eoc} \cdot \tilde{\underline{x}}_c + B_{eoc} \cdot \underline{v} \\ \frac{d\underline{x}_{nc}}{dt} &= A_{eonc} \cdot \underline{x}_{nc} \quad \text{where the pair } (A_{eoc}, B_{eoc}) \text{ is controllable} \\ \underline{y} &= C_{eoc} \cdot \tilde{\underline{x}}_c \end{aligned}$$

Once more, it is possible to compute the solution of the LQ problem

$$\text{find } \underline{v} = -K_c \cdot \tilde{\underline{x}}_c \text{ which minimize } J_c = \int_0^\infty (\tilde{\underline{x}}_c^T \cdot Q_c \cdot \tilde{\underline{x}}_c + \underline{v}^T \cdot R_c \cdot \underline{v}) dt \text{ for } \frac{d\tilde{\underline{x}}_c}{dt} = A_{eoc} \cdot \tilde{\underline{x}}_c + B_{eoc} \cdot \underline{v}$$

In order to adjust weighting matrices  $Q_c$  and  $R_c$  of the quadratic criterion  $J_c$ , define a positive scalar  $k_c$ , a positive control horizon  $T_c$  such as

$$(16) \quad T_c = T_o / k_c$$

and the "partial observability gramian"

$$(17) \quad G_{oc}(T_c) = \int_0^{T_c} \exp(A_{eoc}^T \cdot t) \cdot C_{eoc}^T \cdot C_{eoc} \cdot \exp(A_{eoc} \cdot t) \cdot dt$$

It is then possible to choose

$$(18) \quad \begin{aligned} R_c &= T_c \cdot B_{eoc}^T \cdot G_{oc}(T_c) \cdot B_{eoc} \\ Q_c &= C_{eoc}^T \cdot C_{eoc} \end{aligned}$$

Now, it is possible to elaborate the feedback control

$$(20) \quad \underline{v} = -K_c \cdot \tilde{\underline{x}}_c + K_r \cdot \underline{y}_r$$

where  $K_r$  is a constant matrix and  $\underline{y}_r$  the reference vector.

In order to determine  $K_r$  matrix we use equations (15) in which vector  $\underline{v}$  is replaced by expression (20)

$$(21) \quad \begin{aligned} \frac{d\tilde{\underline{x}}_c}{dt} &= (A_{eoc} - B_{eoc} \cdot K_c) \cdot \tilde{\underline{x}}_c + B_{eoc} \cdot K_r \cdot \underline{y}_r \\ \underline{y} &= C_{eoc} \cdot \tilde{\underline{x}}_c \end{aligned}$$

In permanent rate we want  $\underline{y} = \underline{y}_r$ . As  $(A_{eoc} - B_{eoc} \cdot K_c)$  is a stability matrix, it is invertible and

$$(22) \quad \underline{y} = -C_{eoc} \cdot (A_{eoc} - B_{eoc} \cdot K_c)^{-1} \cdot B_{eoc} \cdot K_r \cdot \underline{y}_r$$

Then, the choice

$$(23) \quad K_r = -[C_{eoc} \cdot (A_{eoc} - B_{eoc} \cdot K_c)^{-1} \cdot B_{eoc}]^{-1}$$

leads to the solution. In our application, this last matrix is not invertible; it is the reason why we use the expression of the pseudo inverse and it appears that it is not possible to choose reference  $\alpha_r$  except for the zero value.

Now the use of expression (12) leads to the control vector

$$\underline{u} = \underline{v} - G_a \cdot \underline{x}_{nc} = -K_c \cdot \tilde{\underline{x}}_c + K_r \cdot \underline{y}_r - G_a \cdot \underline{x}_{nc}$$

which gives

$$(24) \quad \underline{u} = -[K_c \cdot T_a + G_a \quad K_c] \begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix} + K_r \cdot \underline{y}_r$$

but with the transformation matrix  $T_{oc}$  in (11) it is possible to express  $\underline{u}$  as a feedback control

$$(25) \quad \underline{u} = -[K_c \cdot T_a + G_a \quad K_c] T_{oc} \cdot \underline{x}_o + K_r \cdot \underline{y}_r$$

According to separation principle and using state estimator expression (10), it follows easily the regulator equations

$$(26) \quad \begin{aligned} \frac{d\underline{x}_o}{dt} &= A_{reg} \cdot \underline{x}_o + B_{reg} \cdot \begin{bmatrix} \underline{y}_r \\ \underline{y} \end{bmatrix} \\ \underline{u} &= C_{reg} \cdot \underline{x}_o + D_{reg} \cdot \begin{bmatrix} \underline{y}_r \\ \underline{y} \end{bmatrix} \end{aligned}$$

with

$$\begin{aligned} A_{reg} &= A_{eo} - K_o \cdot C_{eo} - B_{eo} \cdot [K_c \cdot T_a + G_a \quad K_c] T_{oc} \\ B_{reg} &= [B_{eo} \cdot K_r \quad K_o] \\ C_{reg} &= -[K_c \cdot T_a + G_a \quad K_c] T_{oc} \\ D_{reg} &= [K_r \quad O] \end{aligned}$$

#### IV. Choice of the adjustment parameters

Among the main objectives, the stability of state matrix estimator  $A_{reg}$  in (26) is essential. It is clear that parameters  $T_o$ ,  $k_o$  and  $k_c$  will influence stability, acting on  $K_o$  and  $K_c$  matrices. The first step is to assign a fixed value to  $k_o$  and  $k_c$  parameters; often 5 is a good value to start. Now the filtering horizon  $T_o$  is adjusted

so that  $A_{reg}$  is a stability matrix; it is often effective to start with the slowest time constant value of the initial model.

In order to quantify the robustness stability of the feedback, the regulator, without reference, can be converted to a transfer function matrix. Starting with

$$(27) \quad \frac{d \underline{x}_o}{dt} = A_{reg} \cdot \underline{x}_o + K_o \cdot \underline{y}$$

$$\underline{u} = C_{reg} \cdot \underline{x}_o$$

we obtain

$$(28) \quad \underline{u}(s) = -K(s) \cdot \underline{y}(s)$$

where

$$(29) \quad K(s) = KK_c \cdot (sI - A_{eo} + B_{eo} \cdot KK_c + K_o \cdot C_{eo})^{-1} \cdot K_o$$

$$KK_c = [K_c \cdot T_a + G_a \quad K_c] T_{oc}$$

(It is interesting to notice the symmetry of expression (29) in relation to matrix gains  $KK_c$  and  $K_o$ ).

Similarly, the initial model of the MAV is

$$(30) \quad \underline{y}(s) = DRO(s) \cdot \underline{u}(s)$$

Now define

$$(31) \quad L_u(s) = K(s) \cdot DRO(s) \quad \text{the input loop transfer}$$

$$(32) \quad L_y(s) = DRO(s) \cdot K(s) \quad \text{the output loop transfer}$$

$$(33) \quad M_{st} = \min \left\{ \frac{1}{\max |L_u(j\omega)|}, \frac{1}{\max |L_y(j\omega)|} \right\} \quad \text{the static margin}$$

$$(34) \quad M_{dyn} = \min \left\{ \frac{1}{\max |\omega \cdot L_u(j\omega)|}, \frac{1}{\max |\omega \cdot L_y(j\omega)|} \right\} \quad \text{the dynamic margin}$$

Then, when  $A_{reg}$  is a stability matrix, the stability of the global feedback is robust in relation to any relative error such as

$$(35) \quad \left| \frac{\Delta DRO(j\omega)}{DRO(j\omega)} \right| < M_{st} \quad \text{it is why } M_{st} \text{ is expressed in \%}$$

Similarly, the stability of the feedback is robust in relation to any relative error such as

$$(36) \quad \left| \frac{\Delta DRO(j\omega)}{DRO(j\omega)} \right| < \omega \cdot M_{dyn}$$

Notice that:  $M_{dyn} \leq M_{Td}$  where  $M_{Td}$  is the time delay margin.

Now it is not difficult to compute static and dynamic margin for different values of  $T_o$ ,  $k_o$  and  $k_c$ . It then may be possible to find a triplet for which the stability margins and the dynamic performances are fulfilled.

The reason why this strategy is often efficient is due to the natural robustness of a LQ problem solution and to the notion of loop transfer recovery (LTR).

In our application the following results are obtained

$$M_{st} = 42.2 \%, \quad M_{dyn} = 32.7 \text{ ms.}$$

## References

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