Negative Slope Coefficient: A Measure to Characterize Genetic Programming Fitness Landscapes

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Abstract. Negative slope coefficient has been recently introduced and empirically proven a suitable hardness indicator for some well known genetic programming benchmarks, such as the even parity problem, the binomial-3 and the artificial ant on the Santa Fe trail. Nevertheless, the original definition of this measure contains several limitations. This paper points out some of those limitations, presents a new and more relevant definition of the negative slope coefficient and empirically shows the suitability of this new definition as a hardness measure for some genetic programming benchmarks, including the multiplexer, the intertwined spirals problem and the royal trees.

1 Introduction

What makes a problem easy or hard for Evolutionary Algorithms? A first effort to answer this challenging question has been done by Goldberg and coworkers (e.g., see [3, 5]) in the field of Genetic Algorithms (GAs). Their approach consisted in constructing functions that should *a priori* be easy or hard for GAs to solve. These ideas have been followed by many others (e.g. [12, 4]) and have been the source of many hypotheses as to what makes a problem easy or difficult for GAs. One concept that underlies many of these approaches is the notion of *fitness landscape* [15]. A fitness landscape is a plot where the points in the horizontal subspace represent the different individual genotypes in a search space and the points in the vertical direction represent their fitness [9]. Individual genotypes are usually placed on the horizontal subspace according to a certain neighborhood structure. If genotypes can be visualized in two dimensions, the plot can be seen as a three-dimensional surface, which may contain peaks and valleys. The task of finding the best solution to the problem is equivalent to finding the highest peak (for maximization problems). The problem solver is seen as a short-sighted explorer searching for it. The fitness landscape plot can be helpful to understand the difficulty of a problem, i.e. the ability of a searcher to find the optimal solution for that problem (see for instance [17] for a deep analysis). Nevertheless, fitness landscapes are impossible to be plotted in practice, given the generally huge size of the space of solutions and the multi-dimentionality and complexity of the possible neighborhood structures. For this reason, in the last few years researchers have been looking for an algebraic measure able to capture some of the interesting properties of fitness landscapes. Early attemps

P. Collet et al. (Eds.): EuroGP 2006, LNCS 3905, pp. 178–189, 2006.

are represented by [22, 11, 7]. A signicant contribution to this field has been given by Jones [6] with the introduction of an hardness measure for GAs called *fitness distance correlation* (*fdc*). This measure has been extended to tree-based Genetic Programming (GP) and proven a suitable hardness indicator in [17, 16, 19, 20, 2]. Nevertheless, these contributions have also shown that *fdc* has some flaws, the most important one being the fact that *fdc* is not predictive, i.e. the optimal solution (or solutions) must be known beforehand, which is almost unrealistic in applied search and optimization problems. Thus, it is important to investigate other approaches based on quantities that can be measured without any explicit knowledge of the genotype of optimal solutions. Preliminary results of this enquiry can be found in [18], where a new measure called *negative slope coefficient* (*nsc*) has been introduced.

This paper aims at extending and generalizing the study of *nsc* for tree-based GP. It is structured as follows: section 2 introduces the concept of *fitness cloud* on which *nsc* is based. Section 3 takes up the original definition of the *nsc* as it has been presented in [18] and section 4 points out its main limitations. Section 5 proposes a new method for calculating the *nsc* and shows some experimental results pointing out that this method enables to overcome some drawbacks of the original definition of the *nsc*. Finally, section 6 offers our conclusions and hints for future research.

2 Fitness Clouds

Evolvability is a feature that is intuitively related, although not exactly identical, to problem difficulty. It has been defined as the capability of genetic operators to improve fitness quality [1]. The most natural way to study evolvability is, probably, to plot the fitness values of individuals against the fitness values of their neighbors, where a neighbor is obtained by applying one step of a genetic operator to the individual. Such a plot has been first introduced for binary landscapes by Vérel and coworkers [21] and called by them *fitness cloud*. In this paper, the genetic operator used to generate fitness clouds is standard subtree mutation [8], i.e. mutation obtained by replacing a subtree of the selected individual with a randomly generated tree.

2.1 Definition

Let $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ be the whole search space of a GP problem and let $V(\gamma_j) = \{v_1^j, v_2^j, \ldots, v_{m_j}^j\}$ be the set of all the neighbors of individual $\gamma_j, \forall j \in [1,n]$. Now let f be the fitness function of the problem at hand. The following set of points can be defined: $S = \{(f(\gamma_j), f(v_k^j)),\ \forall j \in [1,n],\ \forall k \in [1,m_j]\}$. The graphical representation of S on a bidimentional plane, or fitness cloud, is the scatterplot of the fitness of all the individuals belonging to the search space against the fitness of all their neighbors. The main idea is that the shape of this scatterplot can give an indication of the evolvability of the genetic operators used and thus some hints about the difficulty of the problem at hand. The fitness cloud also implicitly gives some insight on the genotype to phenotype map: the set of genotypes that all have equal fitness is a *neutral set*. Such a set corresponds to one abscissa in the fitness/fitness plane; according to this abscissa, a vertical slice from the cloud represents the set of fitnesses that could be reached from this set of neutrality.

For a given offspring fitness value $\tilde{f}$, an horizontal slice represents all the fitnesses from which one can reach $\tilde{f}$.

2.2 Sampling Methodology

In general, the sizes of the search space and of the neighborhoods do not allow one to consider all the possible individuals. Thus, samples are needed. Since selection used by GP is likely to eliminate bad individuals from the population, importance sampling techniques must to be used. As in [17, 18], also in this work the well-known *Metropolis-Hastings* technique [10] is used to sample the search space and the k-tournament selection algorithm [8] (with $k = 10$) is used to sample neighborhoods (see [17] for a detailed motivation of these choices). Using random samples would assume that the space is relatively uniform, e.g. that the measurement in one region of space (the sampled space) applies to all regions of space (or at least a large enough percentage of the space). The choice of important samplings has also been done to limitate this drawback. The terminology of section 2.1 is thus updated: from now on, Γ represents a sample of individuals obtained with the Metropolis-Hastings technique and, for each γ_j belonging to Γ, $V(\gamma_j)$ is a subset of its neighbors, obtained by the applying tournament selection.

3 Negative Slope Coefficient

The fitness cloud can be of help in determining some characteristics of the fitness landscape related to evolvability and problem difficulty. But the mere observation of the scatterplot is not sufficient to quantify these features. In [17, 18] an algebraic measure called *negative slope coefficient* (*nsc*) has been introduced. It can be calculated as follows: the abscissas of a scatterplot can be partitioned into k segments $\{I_1, I_2, \ldots, I_k\}$ of the same length. From those segments, one can deduce the set $\{J_1, J_2, \ldots, J_k\}$, where each J_i contains all the ordinates corresponding to the abscissas contained in I_i. Let $M_1, M_2, \ldots, M_k$ be the averages of the abscissa values contained inside the segments $I_1, I_2, \ldots, I_k$ and let $N_1, N_2, \ldots, N_k$ be the averages of the ordinate values in $J_1, J_2, \ldots, J_k$. Then, the set of segments $\{S_1, S_2, \ldots, S_{k-1}\}$ can be defined, where each S_i connects the point (M_i, N_i) to the point (M_{i+1}, N_{i+1}). For each one of these segments S_i, the *slope* P_i is defined as $P_i = (N_{i+1} - N_i)/(M_{i+1} - M_i)$. Finally, the negative slope coefficient is defined as $nsc = \sum_{i=1}^{k-1} min\ (0, P_i)$. The hypothesis proposed in [18] is that *ncs* should classify problems in the following way: if $nsc = 0$, the problem is easy; if $nsc < 0$ the problem is difficult and the value of *nsc* quantifies this difficulty: the smaller its value, the more difficult the problem. The idea behind this hypothesis is that the presence of a segment with negative slope indicates a bad evolvability for individuals having fitness values contained in that segment (see [17] for a detailed discussion on this issue). Results shown in [18], using the previous definition, are encouraging, nevertheless the technique used to partition the fitness cloud into segments was totally arbitrary: fitness clouds were partitioned into a certain number of bins of the same size; the number of segments and their size was chosen in an arbitrary way (10 segments of the same size in [18]). This may have some undesirable consequences: for instance, segments containing too few points may be generated (and thus the averages calculated on their abscissas and ordinates may lack statistical significance). In the next section, we present some experiments showing that *nsc* calculated by partitioning the fitness cloud into a fixed number of segments can give wrong indications about the difficulty of some GP problems.

4 Limitations of the Original Definition

The *nsc* has been tested as a measure of problem hardness on a set of well-known GP benchmarks, namely the multiplexer problem [8], the intertwined spirals problem [8] and the royal trees [14]. These benchmarks have been chosen because *nsc* has never been tested on them before and because they are problems of different nature and often showing different behaviors. Below, we briefly describe these three benchmarks (see [8] and [14] for a more detailed description) and the parameters used in the experiments.

k-Multiplexer. The problem is to design a Boolean function with k inputs and one output. The first x of the k inputs can be considered as address lines. They describe the binary representation of an integer number. This integer chooses one of the 2^x remaining inputs. The correct output for the multiplexer is the input on the line specified by the address lines. The terminals are the k inputs to the function. The non-terminals are the boolean operators AND, OR, NOT, IF. The raw fitness is calculated by counting the number of correct outputs returned by the boolean function over all the possible 2^k inputs (fitness cases). Subtracting 2^k to raw fitness makes this problem a maximization one and dividing the result by 2^k allows to normalize all fitness values into range $[0,1]$. Empirical results show that the difficulty of the multiplexer problem increases as the number of inputs k increases.

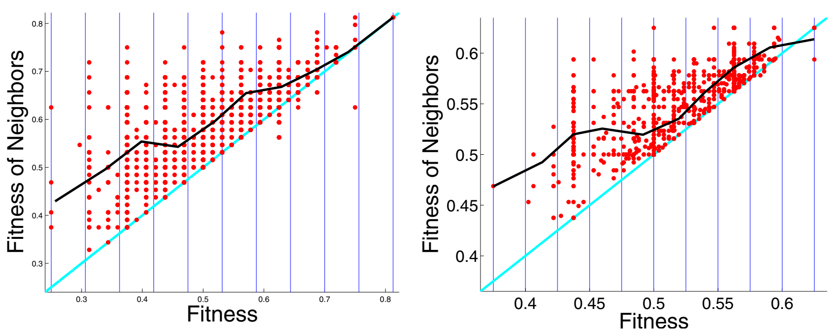
Intertwined Spirals. The goal of this problem is to find a program to classify a given point in the $x-y$ plane as belonging to one of two well defined spirals. These two spirals are defined by a set of 194 given points. The functions and terminals set used to build this function are: $\mathcal{F} = \{+,-,*,//,IFLTE,SIN,COS\}$ and $\mathcal{T} = \{X,Y,\mathcal{R}\}$, where $\mathcal{R}$ is an ephemeral random floating point constant ranging between -1 and 1 and $//$ is a protected division returning 1 instead of an error if the denominator is 0. In order to calculate fitness, individuals are evaluated and mapped into 1 if they return a positive value and into -1 otherwise. The fitness is calculated by subtracting 194 to the number of correctly classified points, among the 194 points which define the spirals. In this way, the problem is transformed into a maximization one and values are normalized into the range $[0,1]$ by dividing them by 194. Empirical results show that it is difficult for GP to find a perfect solution to this problem.

Royal Trees. The language used to code individuals is composed by a set of functions A,B,C, etc. with increasing arity (i.e. $arity(A)=1$, $arity(B)=2$, and so on) and a single terminal X (i.e. $arity(X)=0$). Royal trees are based on the concept of "perfect tree", which is a tree in which all the links are "perfect links". A link is perfect if it joins a node of arity n (at level l in the tree) with a node of arity $n-1$ (at level $l-1$). The fitness of each tree is calculated by assigning a *bonus* to each perfect link and a *penalty* to each non-perfect link. The global optimum is the perfect tree having the node with the maximum arity as root. In [14], Punch and coworkers have empirically shown that the difficulty of royal trees increases as the maximum arity allowed for the tree nodes increases.

Performance Measure and GP Parameters. Once a measure of hardness and the way to compute it have been chosen, the problem remains of finding a means to validate the prediction of the measure with respect to the problem instance and the algorithm. The easiest way is to use a *performance* measure [13]. For the purposes of the present work, performance is defined as being the proportion of the runs for which the global optimum has been found in less than 500 generations over 100 runs. Even if this definition is informal and prone to criticism, good or bad performance values correspond to our intuition of what "easy" or "hard" means in practice. All GP runs executed to calculate performance values have used the same set of GP parameters used in [17, 18]: generational GP, population size of 200 individuals, standard GP mutation used as the sole genetic operator with a rate of 95%, tournament selection of size 10, ramped half-and-half initialization, maximum depth of individuals equal to 10, elitism (i.e. survival of the best individual into the newly generated population).

4.1 Experimental Results

Empirical results for the royal trees are not shown here for lack of space. Nevertheless, they can be found in [17] and they confirm the consistency of the *nsc* as an hardness indicator. Results for the multiplexer problem are summarized by figures 1(a) and 1(b) and by table 1. Figure 1(a) as well as the first row in table 1 concern the 6 multiplexer problem. For this instance of the problem, performance (p) of GP using only standard mutation as a genetic operator is equal to 0.58 (which means that a global optimum has been found in 58 runs over 100 before generation 500). Given that p is larger than 0.5, this problem can be considered as an "easy" one for GP (even though, being the value of p so close to 0.5, the term "uncertain" would probably be more appropriate). On the



(a) (b)

Fig. 1. Multiplexer problem, fitness cloud and segments. (a): 6 multiplexer. (b): 11 multiplexer.

Table 1. Multiplexer. Indicators related to scatterplots of figure 1.

scatterplot	problem	p	*nsc*
Fig. 1(a)	6 multiplexer	0.58	-0.16
Fig. 1(b)	11 multiplexer	0	-0.24

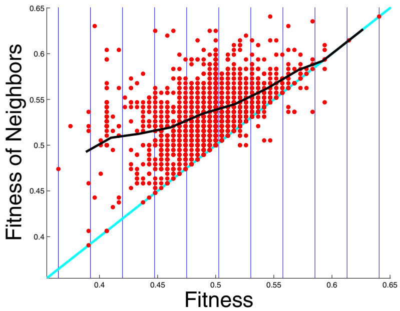


Fig. 2. Intertwined Spirals Problem. Fitness cloud with segments.

Table 2. Intertwined Spirals Problem. Indicators related to the scatterplot of figure 2.

scatterplot	problem	*p*	*nsc*
Fig. 2	intertwined spirals	0	0

other hand, the value of the *nsc* is equal to -0.16, which means that, according to the *nsc* measure, the 6 multiplexer problem should be difficult to solve. Thus, the *nsc* does *not* give the correct indication about the hardness of this problem.

Figure 1(b) and the second row of table 1 concern the 11 multiplexer problem. For this instance of the problem, no global optimum has been found by GP using mutation over the 100 runs performed. Thus the problem is clearly difficult and, consistently, *nsc* has a negative value.

The intertwined spirals problem is difficult to solve by GP. In fact, no optimum has been found over 100 runs. On the other hand, as shown by figure 2 and table 2, the *nsc* value is equal to zero, which means that, according to the *nsc* measure, the problem should be easy. While the lack of reliability of the *nsc* reported in the case of the 6 multiplexer problem could eventually be considered as “marginal”, since GP performance in that case is very close to 0.5 (and thus hardness is very difficult to measure), the wrong indication on the intertwined spirals problem leaves no room for doubts: the *nsc*, as it has been used in [18] and until now in this paper, cannot be used as a general hardness indicator for GP. Thus, the next sections are dedicated to the definition of a new technique for partitioning fitness clouds into segments, in order to assure a higher statistical significance to the set of points belonging to each segment. Successively, some new empirical results are shown, confirming that the new partitioning technique allows to overcome the *nsc* drawbacks presented here.

5 Size Driven Bisection

One of the most natural ways to automatically partition a fitness cloud into a set of segments consists in applying the well-known *bisection* algorithm: as a first step, the fitness cloud may be partitioned into two segments, each one containing the same number of points; then the algorithm may be recursively applied to each one of these two segments until at least one of the segments contains a smaller number of points than a prefixed threshold. This technique has been tested and it has a major drawback: let the *size* of a segment be defined as the difference between the rightmost and the leftmost abscissas of that segment. After a few number of steps, very small bins (i.e. bins with a very small size) may be generated (see [17], at page 162, for a practical case where this problem arises). In such cases, if small segments have negative slopes, the *nsc* may take very large values (around 10^6 in the example shown in [17], page 162). This is clearly unacceptable, even in consideration of the fact that all the points included inside that segments give more or less the same information. In other words, a segment of such a small size is clearly not a significant one. Such a pathologic situation manifests very often if the bisection strategy is applied [17]. Thus, a different strategy is needed.

In an informal way, one may say that the arbitrary criteria used in [18] and in section 4 took into account the size of the segments, but not the number of points they contain, while the bisection algorithm does exactly the opposite. In this section, a technique which takes into account *both* the size of the segments and the number of points they contain, called *size driven bisection* for brevity, is proposed. The starting point of this algorithm is the same as for bisection: the fitness cloud is partitioned into two segments, each of which contains the same number of points. After that, instead of recursively applying bisection to both these segments, only the segment with larger size is further partitioned. Partition is done, once again, by bisection, i.e. the segment is partitioned into two bins, each one containing the same number of points. The algorithm is iterated until one of the two following conditions is satisfied: either a segment contains a smaller number of points than a prefixed threshold, or a segment has become smaller than a prefixed minimum size. In this paper, as a first approximation, 50 has been chosen as a threshold for the number of points belonging to a bin, and the 5% of the distance between the leftmost and the rightmost points in the fitness cloud has been chosen as the minimum size of a segment.

5.1 Experimental Results

Empirical results for the multiplexer problem using size driven bisection are summarized by figures 3 and table 3. Figure 3(a) and figure 3(b) as well as the first row in table 3 concern the 6 multiplexer problem. This time, the value of the *nsc* is equal to 0 and this value is consistent with the fact that GP performance is > 0.5. Nevertheless, figure 3(b) (containing standard deviations of the same points as in figure 3(a)) shows that the slope of some segments is not statistically significant. Our interpretation of it is that, since the GP performance value is near 0.5, it is difficult to state if the problem is “easy” or “hard”. We could informally say that it is “moderatedly easy” or maybe “uncertain” and the standard deviations of some segments seem to confirm this “uncertainty”.

Figures 3(c) and 3(d) and the second row of table 3 concern the 11 multiplexer problem. The *nsc* calculated with the size driven bisection has a negative value and, this time, standard deviations leave few chances to segment slopes to change. In conclusion, *nsc* using size driven bisection seems a reasonable hardness indicator for the multiplexer problem.

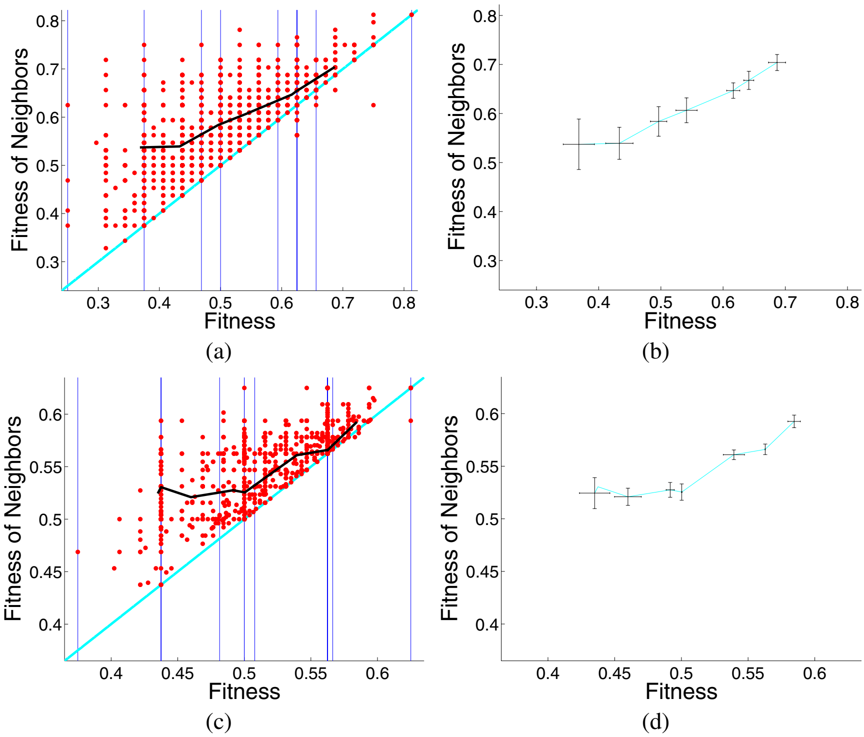


Fig. 3. Multiplexer problem. (a): 6 multiplexer, fitness cloud and segments. (b): 6 multiplexer, segments with standard deviations. (c): 11 multiplexer, fitness cloud and segments. (d): 11 multiplexer, segments with standard deviations.

Table 3. Multiplexer. Indicators related to scatterplots of figure 3.

scatterplot	problem	*p*	*nsc*
Fig. 3(a) and (b)	6 multiplexer	0.58	0
Fig. 3(c) and (d)	11 multiplexer	0	-0.21

Figure 4 shows the scatterplot and segments with their standard deviations for the intertwined spirals problem. Table 4 shows that the value of the *nsc* calculated using size driven bisection is negative. Standard deviations confirm that all the segment slopes are statistically significant. We conclude that *nsc* gives reasonable indications on the hardness of this problem too.

Finally, figures 5(a), 5(b) and 5(c) show the scatterplot and segments for the royal tree problem in the case where $\mathcal{F} = \{A,B\}$, $\mathcal{F} = \{A,B,C\}$ $\mathcal{F} = \{A,B,C,D\}$. In these cases, performance values (p) confirm that the problem is easy for GP. The first three rows of table 5 show that *nsc* is equal to zero in all these cases. Furthermore (see figures 6(a), 6(b) and 6(c)) standard deviations confirm that all the segment slopes are statistically significant. On the other hand, if E, F and G nodes are added to the set

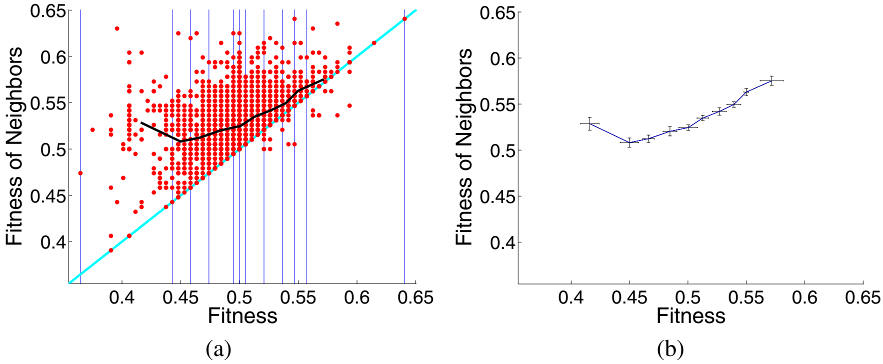


Fig. 4. Intertwined Spirals Problem. (a) : Fitness cloud with segments. (b) : Segments with standard deviations.

Table 4. Intertwined Spirals. Indicators related to scatterplots of figure 4.

scatterplot	problem	*p*	*nsc*
Fig. 4(a) and (b)	intertw. spirals	0	-0.41

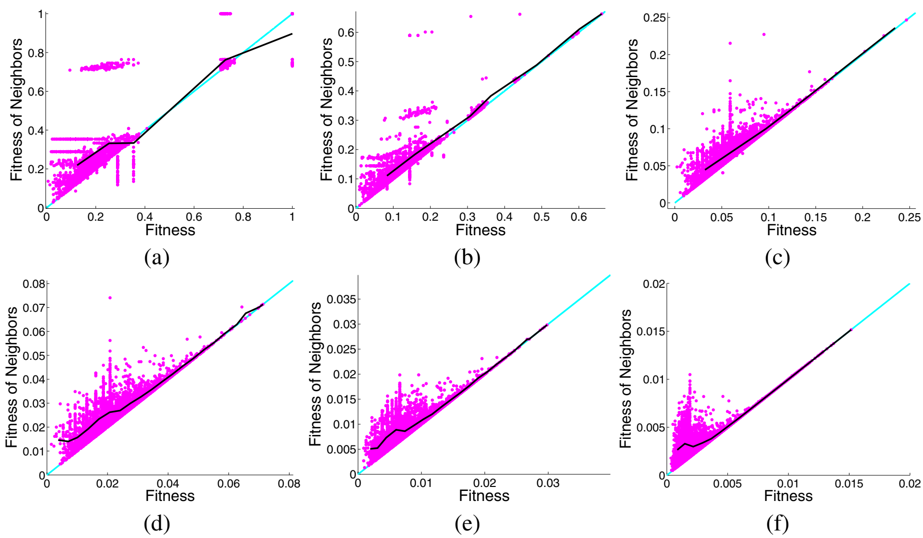


Fig. 5. Royal tree problem. Fitness clouds with segments. (a): $\mathcal{F} = \{A,B\}$. (b): $\mathcal{F} = \{A,B,C\}$. (c): $\mathcal{F} = \{A,B,C,D\}$. (d): $\mathcal{F} = \{A,B,C,D,E\}$. (e): $\mathcal{F} = \{A,B,C,D,E,F\}$. (f): $\mathcal{F} = \{A,B,C,D,E,F,G\}$. Note the change in the axis scale as the $\mathcal{F}$ set increases in size.

of possible function nodes, the problem becomes difficult (since performance is equal to zero, as shown by the last three rows of table 5). Consistently, the *nsc* becomes more and more negative for these instances of the problem. Figures 5(d), 5(e) and 5(f) show scatterplots and segments for these three instances and figures 6(d), 6(e) and 6(f) show

Table 5. Royal trees. Some data related to scatterplots of figure 5.

scatterplot	root	p	*nsc*
Fig. 5(a)	B	1	0
Fig. 5(b)	C	0.91	0
Fig. 5(c)	D	0.72	0
Fig. 5(d)	E	0	-0.17
Fig. 5(e)	F	0	-0.21
Fig. 5(f)	G	0	-0.32

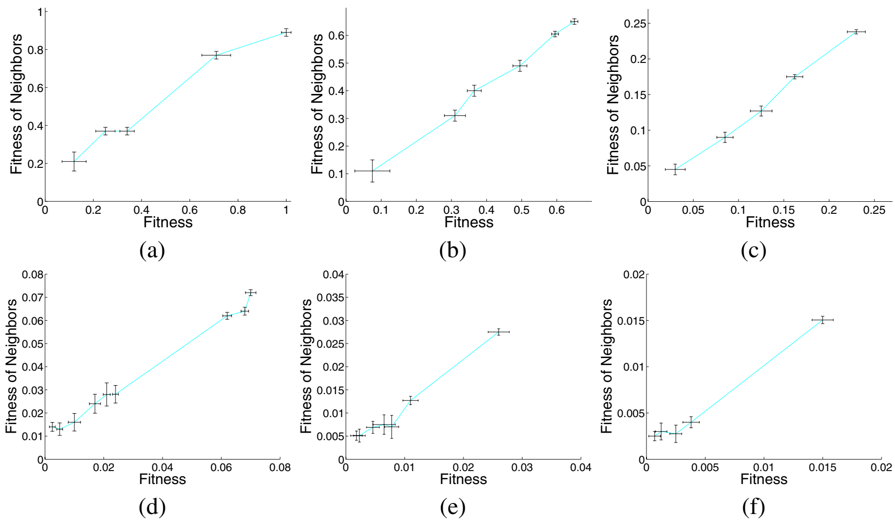


Fig. 6. Royal tree problem. Segments with standard deviations. (a): $\mathcal{F} = \{A,B\}$. (b): $\mathcal{F} = \{A,B,C\}$. (c): $\mathcal{F} = \{A,B,C,D\}$. (d): $\mathcal{F} = \{A,B,C,D,E\}$. (e): $\mathcal{F} = \{A,B,C,D,E,F\}$. (f): $\mathcal{F} = \{A,B,C,D,E,F,G\}$. Note the change in the axis scale as the $\mathcal{F}$ set increases in size.

the respective standard deviations. Once again, *nsc* values correctly indicate the relative order of GP problem hardness.

6 Conclusions and Future Work

The negative slope coefficient (*nsc*) is *not* a reliable measure of problem hardness if the fitness cloud is partitioned into segments in an arbitrary way. In this paper, a new way of partitioning fitness clouds, called size driven bisection, has been presented. It represents a suitable tradeoff between the size of segments and the density of points in the partitions. The *nsc* using size driven bisection gave reliable indications about the hardness of two instances of the multiplexer problem, the intertwined spirals problem and six instances of the royal trees problem. Furthermore, the *nsc* using size driven bisection gave reliable indications on some other typical GP benchmarks (such as the artificial ant on the Santa Fe trail, the even parity and one particular instance of the symbolic regression) and on trap functions (these results have not been shown here for lack of space. They are presented in [17]). These results encourage us to continue the study of *nsc*, with further tests on more “real life” GP problems. This paper leaves some open problems: first of all, it is based on statistical samplings of the search space and thus counterexamples can surely be built for this measure. Furthermore, and even more importantly, no technique has been found yet to normalize *nsc* values into a given range, in order to enable comparisons between the difficulties of two or more problems of different nature. Only the hardness of different instances of the same problem can be calculated using *nsc*, as defined here.

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