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Abstract

In this paper, the effect of network-induced delay introduced into the control loop is modelled as time-varying disturbance. Based on this model, a fault isolation filter (FIF) for fault detection of networked control systems (NCSs) with multiple faults is then parameterized. Directional residual generations decoupling from the disturbance ensure the treatment of multiple faults appearing simultaneously or sequentially. The remaining degrees of freedom in the design of the filter's gain are used to satisfy an H_∞ disturbance attenuation and poles assignment constraints in the frame of Markov jump linear systems (MJLS). The sufficient existence conditions of the free parameters are formulated as a convex optimization problem over a couple of linear matrix inequalities (LMIs). An illustrative example is given to show the efficiency of the proposed method for NCS.

1 Introduction

Networked Control Systems (NCSs) may represent a class of spatially distributed systems wherein the plants, sensors, controllers and actuators are assembled feedback loops through a shared communication network. Because of the inherent complexity of such systems, the control issues of NCS have attracted most attention of many researchers with taking into account network-induced delays and/or packet losses, and so on. For instance, the stability and stabilization problems of NCS were investigated in [8, 15, 1, 30, 12] for network-induced delays, [13, 20] for packet losses, and [9, 27, 29, 11] for network-induced delays and packet losses. We refer the readers to the survey [23] for more information about the state-of-the-art of NCS.

On the other hand, due to an increasing complexity of dynamical systems, as well as the need for reliability, safety and efficient operation, the model-based fault detection and isolation (FDI) has been becoming an important subject in modern control theory and practice, see e.g. [25, 3, 4, 2, 14] and the references therein. The procedures of model-based FDI problem are i) generations of residuals which are ideally close to zero under fault-free conditions, minimally sensitive to noises and disturbances and maximally sensitive to faults, ii) residual evaluation, namely design of decision rules based on these residuals. When involving the model-based FDI of NCS, the aforementioned problems will be more complex than those of traditional point-to-point systems because of the network-induced effects. Whereas there are ample significant results on the control issues of NCS, the studies related to FDI of NCS are just at the starting point, which motivates the current research of this paper.

In this paper, we show our attention on the design of a robust fault isolation filter for networked control systems with network-induced delays and multiple faults. Similar to [8], sensor-to-controller network-induced delay and controller-to-actuator network-induced delay are lumped together as a single delay, which is assumed to be random and governed by a Markov chain. Then, the effect of network-induced delays introduced into the control loop is regarded as time-varying disturbance. Based on this model, a fault isolation filter for fault detection of such networked control systems is then parameterized in order to generate the residuals having directional properties in response to a particular fault and decoupling from the disturbance. The remaining degrees of freedom in the design of the filter's gain are used to satisfy an H_∞ disturbance attenuation and poles assignment constraints in the frame of Markov jump linear systems (MJLS). The sufficient existence conditions of the free parameters are formulated as a convex optimization problem over a couple of linear matrix inequalities (LMIs), which can be very efficiently solved by interior-point methods.

The rest of this paper is organized as follows. In Section 2, the networked control systems model taking into account network-induced time-delays and multiple faults is described and main assumptions are presented. Parameterized solutions to the gains of filter and projector, and a sufficient condition for the free parameter satisfying two constrained conditions are developed in Section 3. An illustrative example is presented in Section 4 to show the effectiveness of the result.The paper is concluded in Section 5.

Notations: In what follows, if not explicitly stated, ma-

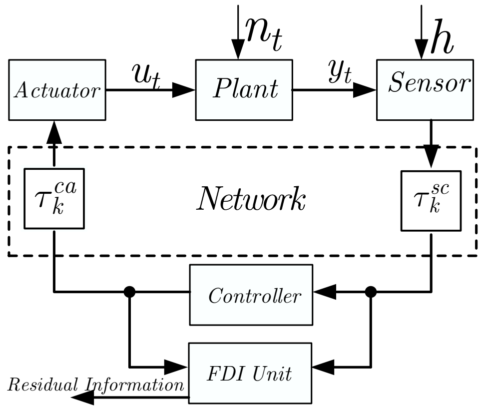

Figure 1. A structure of an NCS with FDI unit

trices are assumed to have compatible dimensions. $\mathbb{R}$ and $\mathbb{Z}_+$ denote the set of real numbers and the set of nonnegative integer numbers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ are, respectively, the n-dimensional Euclidean space and the set of $n \times m$ real matrices. The notations $A > 0$ and $A < 0$ are used to denote a positive and negative definite matrix, respectively. A^T denotes the transpose of matrix or vector A. A^{-1} and A^+ represent the inverse and pseudo-inverse of A, respectively. $\mathrm{diag}(a_1, \ldots, a_n)$ refers to an $n \times n$ diagonal matrix with a_i as its ith diagonal entry. $\|A\|$ denotes the Euclidean norm for vectors or the spectral norms of matrices A. $\mathrm{rank}(A)$ stands for the rank operator of matrix A. $\lambda_i(A)$ represents the ith eigenvalue of matrix A.

2 Problem description and preliminaries

The networked control system structure discussed in this paper is shown in Fig. 1. The plant, sensor, controller and actuator are spatially distributed and closed over network. We suppose that the sensor is time-driven with an identical sampling period h. By event-triggered controller or actuator, we mean that calculation of the new control or actuator signal is started as soon as the new control or actuator information arrives. Similar to [8], the distributed network-induced delays can be lumped together as a single time-delay $\tau_k = \tau_k^{sc} + \tau_k^{ca} \in [0, h)$, where τ_k^{sc} and τ_k^{ca} are delays of sensor-to-controller and controller-to-actuator, respectively.

Suppose that an LTI dynamical model with multiple faults is described as following:

$$\begin{cases} \dot{x}_t = \underline{A}x_t + \underline{B}u_t + \underline{F}n_t \\ y_t = \underline{C}x_t \end{cases} \quad t \in \mathbb{R} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the state vector, $y_t \in \mathbb{R}^m$ the output vector, $u_t \in \mathbb{R}^p$ the input vector. $\underline{F} = \begin{bmatrix} \underline{f}_1 & \cdots & \underline{f}_i & \cdots & \underline{f}_q \end{bmatrix}$ is the fault distribution matrix, and $n_t \in \mathbb{R}^q$ is the fault vector.

Taking into account the network-induced delay and the control input over a sampling interval $[kh,\ (k+1)h]$:

$$u_t = \begin{cases} u_{k-1}, & t \in [kh,\ kh + \tau_k]; \\ u_k, & t \in [kh + \tau_k,\ (k+1)h]. \end{cases} \tag{2}$$

Integration the plant (1) over such interval will then give:

$$\begin{cases} x_{k+1} = Ax_k + B_{0,\tau_k}u_k + B_{1,\tau_k}u_{k-1} + Fn_k \\ y_k \quad\ = Cx_k \end{cases}, k \in \mathbb{Z}_+ \tag{3}$$

where

$$A = \mathrm{e}^{\underline{A}h}, C = \underline{C}, B_{0,\tau_k} = \int_0^{h-\tau_k} \mathrm{e}^{\underline{A}s}\mathrm{d}s\underline{B},$$

$$B_{1,\tau_k} = \int_{h-\tau_k}^{h} \mathrm{e}^{\underline{A}s}\mathrm{d}s\underline{B}, F = \int_0^h \mathrm{e}^{\underline{A}s}\mathrm{d}s\underline{F}.$$

According to the property of definite integral, Equation (3) is rewritten as:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Fn_k + B_{1,\tau_k}\Delta u_k, \\ y_k = Cx_k, \end{cases} \tag{4}$$

where $B = \int_0^h \mathrm{e}^{\underline{A}s}\mathrm{d}s\underline{B}$, $\Delta u_k = u_{k-1} - u_k$, $F = \begin{bmatrix} f_1 & \ldots & f_i & \ldots & f_q \end{bmatrix}$.

Remark 1 *Different from the sampled-data system without taking into account the network-induced delay, there exists a time-varying term $B_{1,\tau_k}\Delta u_k$ in the state evolution equation of system (4). Therefore, the traditional strategy of detecting and isolating multiple faults must be reevaluated for the NCS.*

Remark 2 *Inspired by the principles of Delta-Modulation which was introduced in [6] for NCS to generally decrease bits to specify the control signal, we propose to send the decrements of the control signals rather than sending them directly. The actual input signal u_k is readily reconstructed by discrete-time integration as $u_k = u_{k-1} - \Delta u_k = u_0 - \sum_{i=1}^{k} \Delta u_i$, where u_0 is the initial control signal and assumed to be known in advance. Because the FDI scheme considered in this paper is open-loop, the term Bu_k in the dynamics equation (4) will not be important and can be omitted when computing the residual. Therefore, we only focus the attention on the property of Δu_k.*

In the sequel, we further impose two communication schemes on the design as follows:

S1 The data from the controller to each actuator is restricted to belong to a small and fixed finite set of scalars, $\mathbb{U}$.

S2 The inputs of the plant $(\Delta u_i)_k$ $(i = 1, \ldots, p)$ can be lumped together into one network packet and transmitted at the same time. Between updates, all plant inputs are held at their previous values.

Communication schemes 1 and 2 can be summarized by a simple finite-set constraint on the decrements $\Delta u_k = u_{k-1} - u_k$. More precisely, at every time instant k, the vector

$$\Delta u_k \triangleq \begin{bmatrix} (\Delta u_1)_k & \cdots & (\Delta u_p)_k \end{bmatrix}^T \quad (5)$$

is restricted to belong to the set $\mathbb{V}$, which is defined as

$$\mathbb{V} \triangleq \{\eta \in \mathbb{R}^p : \eta = \begin{bmatrix} \upsilon & \cdots & \upsilon \end{bmatrix}^T, \text{ for some } \upsilon \in \mathbb{U}\}.$$

Remark 3 *The communication scheme S2 is suitable for networks with large packets size, e.g. Ethernet which can hold a maximum of 1500 bytes of data in a single packet, see e.g. [21, 22].*

Remark 4 *Utilizing a receding horizon scheme as deployed in model predictive control, see e.g. [6, 28], the optimal Δu_k can be obtained which is not within the scope of this paper. When implementing the design of FDI, Δu_k is assumed to be known.*

Without loss of generality, we assume that the statistical behavior of network-induced delay τ_k is random and governed by the Markov chain

$$\theta_k \in \mathcal{S} = \{1, 2, \ldots, s\}, \ \forall k \in \mathbb{Z}_+ \quad (6)$$

with the transition probabilities λ_{ij} denoting as $\lambda_{ij} = \mathbf{Pr}[\theta_{k+1} = j|\theta_k = i]$, $\lambda_{ij} \geq 0$ and $\sum_{j=1}^{s} \lambda_{ij} = 1$ for any $i \in \mathcal{S}$. For sake of simplifying notations, we denote B_{1,τ_k} as B_{1,θ_k} and Δu_k as w_k. Then, the model of networked control system considered in this paper is replaced the state space system (4) with the following particular Markov jump linear system:

$$\begin{cases} x_{k+1} &= Ax_k + Bu_k + Fn_k + B_{1,\theta_k} w_k, \\ y_k &= Cx_k, \end{cases} \quad (7)$$

The following filter is presented as the residual generator of NCS (7):

$$\begin{cases} \hat{x}_{k+1} &= A\hat{x}_k + Bu_k + K(y_k - C\hat{x}_k), \\ \alpha_k &= L(y_k - C\hat{x}_k), \end{cases} \quad (8)$$

where $\hat{x}_k$ is the state of the filter, α_k the residual generator or the fault indicator. Filter gain $K \in \mathbb{R}^{n\times m}$ and projector $L \in \mathbb{R}^{q\times m}$ are unknown matrices to be found for the solution of the fault detection and isolation problem.

The objectives of this paper are i) to construct a fault isolation filter as (8) for fault detection and isolation of multiple faults in networked control system, ii) to design a free parameter ensuring that the energy ratio between useful and disturbance signal w_k defined on the fault indicators is maximized, iii) to locate the closed-loop poles within a prescribed region in the complex plane in order that the residual dynamical has the given transient properties.

It should be noted that the particular case we have focused on ensures that the definitions of fault detectability indexes and matrices introduced in [10] still hold true for system (7). We end this section by recalling these definitions which will be essential for the proofs in the next section.

Definition 1 *The networked control system (7) is said to have fault detectability indexes $\rho = \{\rho_1, \rho_2, \ldots, \rho_q\}$ if $\rho_i = \min\{\nu : CA^{\nu-1} f_i \neq 0, \nu = 1, 2, \ldots\}$.*

Definition 2 *If the networked control system (7) has finite fault detectability indexes, the fault detectability matrix Ψ is defined as:*

$$\Psi = CD \quad (9)$$

with

$$D = \begin{bmatrix} A^{\rho_1-1} f_1 & \cdots & A^{\rho_i-1} f_i & \cdots & A^{\rho_q-1} f_q \end{bmatrix}.$$

3 Robust FIF for NCS

In this section, we will firstly give the parameterized solutions of filter gain and projector. Under two constrained conditions, a robust FIF for NCS will then be designed from reduced gains describing the available degrees of freedom.

From (4) and (8), the state estimation error $e_k = x_k - \hat{x}_k$ and the output of the filter α_k propagate as

$$\begin{cases} e_{k+1} = (A - KC)e_k + Fn_k + B_{1,\theta_k} w_k \\ \alpha_k \quad = LCe_k \end{cases}, k \in \mathbb{Z}_+, \theta_k \in \mathcal{S} \quad (10)$$

Let $G_{n\alpha}(z)$ be the transfer function from n_k to the output residual α_k. Then the following theorem is presented to design K and L such that

$$\begin{aligned} G_{n\alpha}(z) &= LC(zI - (A - KC))^{-1} F \\ &= \text{diag}\{z^{-\rho_1}, \ldots, z^{-\rho_q}\}, \end{aligned} \quad (11)$$

which ensures the isolation of multiple faults.

Theorem 1 *Under the condition* $\text{rank}(\Psi) = q$*, the solutions of (11) can be parameterized as $K = \omega\Pi + \bar{K}_{\theta_k}\Sigma, L = \Pi$, with $\Sigma = \beta(I - \Psi\Pi), \Pi = \Psi^+, \omega = AD$ and $\Psi = CD$, where $\bar{K}_{\theta_k} \in \mathbb{R}^{n\times m-q}$ is the free parameters to be designed, Ψ^+ is the pseudo-inverse of Ψ and β is an arbitrary matrix chosen so that* $\text{rank}(\Sigma) = m - q$.

Proof. From (11), we have

$$\begin{aligned} G_{n\alpha}(z) &= LC(zI - (A - KC))^{-1} F \\ &= \sum_{k\geq 0} LC(A - KC)^k F z^{-k-1} \\ &= \sum_{k\geq 0} z^{-k-1} \begin{bmatrix} \cdots & LC(A - KC)^k f_i & \cdots \end{bmatrix} \end{aligned} \quad (12)$$

where

$$\sum_{k\geq 0} z^{-k-1}LC(A-KC)^k f_i$$
$$= z^{-1}LCf_i + z^{-2}LC(A-KC)f_i + \cdots$$
$$= LCA^{\rho_i-1}f_i z^{-\rho_i} + \sum_{k\geq 0} LC(A-KC)^{k+1}A^{\rho_i-1}f_i z^{-k-1-\rho_i} \quad (13)$$

Substituting (13) into (12), we have

$$G_{n\alpha}(z) = [\ \cdots \quad LCA^{\rho_i-1}f_i z^{-\rho_i} + \sum_{k\geq 0} LC(A-KC)^{k+1}A^{\rho_i-1}f_i z^{-k-1-\rho_i} \quad \cdots\ ] \quad (14)$$

If the filter gain K satisfies the following algebraic constraint

$$(A-KC)[\ \cdots \quad A^{\rho_i-1}f_i \quad \cdots\ ] = 0 \Longleftrightarrow (A-KC)D = 0,$$

then (14) gives

$$G_{n\alpha}(z) = L\Psi\text{diag}\{z^{-\rho_1}, \ldots, z^{-\rho_q}\}, \quad (15)$$

and (11) is satisfied under $L\Psi = I$. By the generalized inverse of matrix [18], K and L can be parameterized as the statements of this theorem. The proof is complete.

Remark 5 *It's should be noted that the free parameter $\bar{K}_{\theta_k}$ is independent of the multiple faults isolation because any $\bar{K}_{\theta_k}$ ensures (11). So, the filter gain K is independent of $\bar{K}_{\theta_k}$ and hence independent of the Markov chain $\theta_k \in \mathcal{S}$ when the theorem is proven. Otherwise, K governed by Markov chain θ_k will bring about the complexity of analysis in the case where the filter (8) and the error system (10) are hybrid. However, the introduction of $\bar{K}_{\theta_k}$ gives an extra degree of freedom to satisfy some other design requirements.*

From Theorem 1, the FIF (8) is rewritten from the free parameter $\bar{K}_{\theta_k}$ as :

$$\begin{cases} \hat{x}_{k+1} &= A\hat{x}_k + Bu_k + \omega\alpha_k + \bar{K}_{\theta_k}\Sigma(y_k - C\hat{x}), \\ \alpha_k &= \Pi(y_k - C\hat{x}_k), \end{cases} \quad (16)$$

where α_k is a deadbeat filter of fault n_k and given by:

$$\alpha_k = \breve{\alpha}_k + \begin{bmatrix} n^1_{k-\rho_1} & \cdots & n^i_{k-\rho_i} & \cdots & n^q_{k-\rho_q} \end{bmatrix}^T \quad (17)$$

where $\breve{\alpha}_k$ is the faults indicator signal without faults and propagates from the fault-free state estimation error $\bar{e}_k = \tilde{x}_k - \hat{x}_k$ as:

$$\begin{cases} \bar{e}_{k+1} &= (\bar{A} - \bar{K}_{\theta_k}\bar{C})\bar{e}_k + B_{1,\theta_k}w_k, \\ \breve{\alpha}_k &= \Pi C\bar{e}_k, \end{cases} \quad (18)$$

where $\bar{A} = A - \omega\Pi C$, $\bar{C} = \Sigma C$ and $\tilde{x}_k$ is the fault-free state. The transfer function from w_k to $\breve{\alpha}_k$ is then given by:

$$G_{w\breve{\alpha}}(z) = \Pi C(zI - (\bar{A} - \bar{K}_{\theta_k}\bar{C}))^{-1}B_{1,\theta_k}. \quad (19)$$

Let $\hat{\alpha}_k$ be the faults indicator signal without disturbances. From Equation (11), the transfer function $G_{n\hat{\alpha}}(z)$ from fault n to fault indicator $\hat{\alpha}_k$ is a pure delay and

$$\|G_{n\hat{\alpha}}(z)\|_\infty := \sup_{\theta_0\in\mathcal{S}} \sup_{0\neq n\in\ell_2} \frac{\|\hat{\alpha}\|_2}{\|n\|_2} = 1, \quad (20)$$

where $\|s\|_{\ell_2} = (\sum_{k=0}^{\infty} \|s_k\|)^{1/2}$ is the ℓ_2 norm of the signal s_k.

Rest of three aims of this paper are then to design the free parameters $\bar{K}_{\theta_k}$ satisfying the constraints as:

C1 the H_∞-norm of transfer function $G_{w\breve{\alpha}}(z)$ is less than a prescribed scalar $\gamma > 0$, namely

$$\|G_{w\breve{\alpha}}(z)\|_\infty := \sup_{\theta_0\in\mathcal{S}} \sup_{0\neq w\in\ell_2} \frac{\|\breve{\alpha}\|_2}{\|w\|_2} < \gamma \quad (21)$$

C2 the poles of $(\bar{A} - \bar{K}_{\theta_k}\bar{C})$ are inside discs $D_{\theta_k}(\xi_{\theta_k}, \delta_{\theta_k})$ in the complex plane with the center $\xi_{\theta_k} + \mathrm{j}0$ and $-1 < -\xi_{\theta_k} + \delta_{\theta_k} < 1$, namely $\lambda_i\Big(\frac{\bar{A}-\bar{K}_{\theta_k}\bar{C}-\xi_{\theta_k}I}{\delta_{\theta_k}}\Big) \subset D(0,1), i = 1, \ldots, n,\ \forall\theta_k \in \mathcal{S}$.

Remark 6 *By locating the closed-loop poles in a prescribed region in the complex plan, the constraint C2 ensures that the error system (18) has the given transient properties. This scheme is borrowed from the robust control of linear system with poles constraints, see e.g. [7, 24, 26].*

Before proceeding further, we recall the notation of second-moment stability (SMS) and some related results. Consider the following stochastic system $\mathcal{P}$:

$$(\mathcal{P}): \quad \begin{bmatrix} x_{k+1} \\ e_k \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{\theta_k} & \mathcal{B}_{\theta_k} \\ \mathcal{C}_{\theta_k} & \mathcal{D}_{\theta_k} \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} \quad (22)$$

where $x_k \in \mathbb{R}^{n_x}$ is the state, $w_k \in \mathbb{R}^{n_w}$ is the disturbance vector and $e_k \in \mathbb{R}^{n_e}$ is the error vector. The notation of θ_k is given as (6).

Definition 3 ([19]) *The system $\mathcal{P}$ is weakly controllable if for every initial state/mode, (x_0, θ_0), and any final state/mode, (x_f, θ_f), there exists a finite time T_c and an input $d_{c,k}$ such that $\mathbf{Pr}[x_{T_c} = x_f$ and $\theta_{T_c} = \theta_f] > 0$.*

Lemma 1 ([19]) *System $\mathcal{P}$ is SMS if and only if there exist matrices $P_i > 0$ that satisfy $\mathcal{A}_i^T\bar{P}_i\mathcal{A}_i - P_i < 0\ \forall i = \theta_k \in \mathcal{S}$ where $\bar{P}_i := \sum_{j=1}^{s}\lambda_{ij}P_j$.*

Lemma 2 (Bounded Real Lemma) *Assume the system $\mathcal{P}$ is weakly controllable and let $x_0 = 0$. The following conditions are equivalent:*

i) $\mathcal{P}$ is SMS and the H_∞-norm denoted as

$$\|\mathcal{P}\|_\infty := \sup_{\theta_0\in\mathcal{S}} \sup_{0\neq w\in\ell_2} \frac{\|e\|_2}{\|w\|_2} \quad (23)$$

satisfies $\|\mathcal{P}\|_\infty < \gamma$.

ii) there exist matrices $P_i > 0$ that satisfy

$$\begin{bmatrix} \mathcal{A}_i & \mathcal{B}_i \\ \mathcal{C}_i & \mathcal{D}_i \end{bmatrix}^T \begin{bmatrix} \bar{P}_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \mathcal{A}_i & \mathcal{B}_i \\ \mathcal{C}_i & \mathcal{D}_i \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0 \quad (24)$$

where $\bar{P}_i = \sum_{j=1}^{s} \lambda_{ij} P_j$ and $i = \theta_k \in \mathcal{S}$.

iii) there exist matrices $P_i > 0$ and G_i that satisfy

$$\begin{bmatrix} G_i\mathcal{A}_i & G_i\mathcal{B}_i \\ \mathcal{C}_i & \mathcal{D}_i \end{bmatrix}^T \begin{bmatrix} \bar{P}_i - G_i - G_i^T & 0 \\ 0 & I \end{bmatrix}^{-1} \times \begin{bmatrix} G_i\mathcal{A}_i & G_i\mathcal{B}_i \\ \mathcal{C}_i & \mathcal{D}_i \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0 \quad (25)$$

where $\bar{P}_i = \sum_{j=1}^{s} \lambda_{ij} P_j$ and $i = \theta_k \in \mathcal{S}$.

Proof. The equivalence of i) and ii) has been proven in [19]. By the so-called slack variable method which was introduced in [16, 17], the equivalences of i) and iii), ii) and iii) will be proven.

Remark 7 *Due to the slack variables G_i, the condition iii) gives potential advantages over condition ii), especially for the design of the free parameters $\bar{K}_i$ in the following theorem.*

Theorem 2 *For given discs $D_i(\xi_i, \delta_i)$, if there exist matrices $P_i = P_i^T > 0$, G_i and Y_i for prescribed scalars $\gamma > 0$, $-1 < -\xi_i + \delta_i < 1$, $\forall i = \theta_k \in \mathcal{S}$ such that*

$$\begin{bmatrix} -P_i & 0 & \bar{A}^T G_i^T - \bar{C}^T Y_i^T & C^T \Pi^T \\ 0 & -\gamma^2 I & B_{1,i}^T G_i^T & 0 \\ G_i\bar{A} - Y_i\bar{C} & G_i B_{1,i} & \bar{P}_i - G_i - G_i^T & 0 \\ \Pi C & 0 & 0 & -I \end{bmatrix} < 0, \quad (26)$$

$$\begin{bmatrix} -\delta_i^2 P_i & \bar{A}^T G_i^T - \bar{C}^T Y_i^T - \xi_i G_i^T \\ G_i\bar{A} - Y_i\bar{C} - \xi_i G_i & P_i - G_i - G_i^T \end{bmatrix} < 0, \quad (27)$$

where $\bar{A} = A - \omega\Pi C$, $\bar{C} = \Sigma C$, then the free parameters are designed as $\bar{K}_i = G_i^{-1} Y_i$ ensuring the SMS of the error system (18) and the constraints C1 and C2. At the minimal possible value of γ leading to a solution $P_i = P_i^T > 0$, G_i and Y_i, the energy ratio between useful and disturbance signal defined on the fault indicators will be maximized.

Proof. If there exist matrices $P_i = P_i^T > 0$, G_i and $Y_i = G_i\bar{K}_i$ such that (26) holds, we obtain:

$$\begin{bmatrix} -P_i & 0 & \mathcal{A}_i^T G_i^T & \mathcal{C}_i^T \\ 0 & -\gamma^2 I & \mathcal{B}_i^T G_i & 0 \\ G_i\mathcal{A}_i & G_i\mathcal{B}_i & \bar{P}_i - G_i - G_i^T & 0 \\ \mathcal{C}_i & 0 & 0 & -I \end{bmatrix} < 0, \quad (28)$$

where $\mathcal{A}_i = \bar{A} - \bar{K}_i\bar{C}$, $\mathcal{B}_i = B_{1,i}$, $\mathcal{C}_i = \Pi\bar{C}$. By Schur complement, we obtain that (28) implies (25). So, the free parameter $\bar{K}_i = G_i^{-1} Y_i$ ensures the SMS of the error system (18) and the constraint C1.

Furthermore, if there exist matrices $P_i = P_i^T > 0$, G_i and $Y_i = G_i\bar{K}_i$ such that (27) holds, pre-multiplying and post-multiplying $\left[\delta_i^{-1} I, -\delta_i^{-1}(\bar{A} - \bar{K}_i\bar{C} - \xi_i I)^T\right]$ and its transpose to (27) will yield:

$$\left(\frac{\bar{A} - \bar{K}_i\bar{C} - \xi_i I}{\delta_i}\right)^T P_i \left(\frac{\bar{A} - \bar{K}_i\bar{C} - \xi_i I}{\delta_i}\right) - P_i < 0, \quad (29)$$

which immediately implies (27) satisfying the constraint C2. In summary, the free parameters are designed as $\bar{K}_i = G_i^{-1} Y_i$ ensuring the SMS of the error system (18) and the constraints C1 and C2. The proof is complete.

Remark 8 *Given discs $D_i(\xi_i, \delta_i)$, $i = \theta_k \in \mathcal{S}$, the search problem of the lowest possible value of γ can be formulated as the following convex optimization problem:*

$$\mathcal{OP}: \min_{P_i = P_i^T > 0, G_i, Y_i} \gamma \quad \text{s.t. LMI (26), (27)} \quad (30)$$

which can be effectively solved by the existing Matlab LMI toolbox [5].

4 Illustrative Example

In this section, we will present an example to illustrate the design approach proposed in this paper. We borrow the modified example from [10] described by (7), where the parameters are as follows:

$$A = \begin{bmatrix} 0.2 & 1 & 0 & 0.2 \\ 0 & 0.5 & 1 & 0.4 \\ 0 & 0 & 0.8 & 1 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ -1 & 1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix},$$

$$F = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and sampling time $h = 0.1sec$. Therefore, the parameters of original continuous time LTI system (1) can be obtained by the Matlab command d2c, for example, $[\underline{A}, \underline{B}] = \text{d2c}(A, B, h)$. The transition probability matrix of (6) is

$$\Lambda = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.3 & 0.47 & 0.23 \\ 0.26 & 0.10 & 0.64 \end{bmatrix},$$

and the corresponding random network-induced delays are $\tau_1 = 0.06sec$, $\tau_2 = 0.07sec$, $\tau_3 = 0.08sec$. According to Theorem 1, we have $\Psi = CF$ and $D = F$ since $\text{rank}(CF) = q$ $(\rho_1 = 1,\ \rho_2 = 1)$. The parametrization of the FIF's gain K is given by

$$\Sigma = \begin{bmatrix} -0.333 & 0.333 & 0.333 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} 0.6667 & 0.3333 & 0.3333 \\ -0.3333 & -0.6667 & 0.3333 \end{bmatrix}.$$

Furthermore, the discs $D_i(\xi_i, \delta_i)$ $(i = 1, 2, 3)$ are given as $D_1(0, 0.3)$, $D_2(-0.2, 0.4)$, $D_3(0.2, 0.3)$. According to

Table 1. RMS of the fault estimation errors

	$(\alpha_1)_k$	$(\alpha_1^\sharp)_k$	$(\alpha_2)_k$	$(\alpha_2^\sharp)_k$
RMS	0.7368	0.8005	0.5378	0.6787

Theorem 2, the minimal possible value of γ is found to be 1.5854 and the free parameters $\bar{K}_i$ are constructed as:

$$\bar{K}_1 = \begin{bmatrix} 0.7900 \\ -0.9002 \\ -1.7999 \\ -0.3003 \end{bmatrix}, \bar{K}_2 = \begin{bmatrix} 0.7834 \\ -0.7293 \\ -1.9742 \\ -0.4672 \end{bmatrix},$$

$$\bar{K}_3 = \begin{bmatrix} 0.8038 \\ -1.0827 \\ -1.6090 \\ -0.1293 \end{bmatrix}.$$

The fault associated to the first column of the matrix F occurs at time instant $r_1 = 50$ with $n_k^1 = 10\sin(0.1k)$. The fault associated to the second column occurs at time instant $r_2 = 120$ with $n_k^2 = 5$. Fig.2 gives the innovation sequences of Markov chain mode, the network-induced time-delay governed by such Markov chain and the control decrements $\Delta u_k = [(\Delta u_1)_k,\ (\Delta u_2)_k]^T$. It should be noted that the finite scalar set $\mathbb{U}$ of the communication scheme S1 is given as $\mathbb{U} = \{-1,\ 0,\ 1\}$ when implementing simulation. Fig.3 shows the innovation sequences of residual $\alpha_k = [(\alpha_1)_k,\ (\alpha_2)_k]^T$ and $\alpha_k^\sharp = \left[(\alpha_1^\sharp)_k,\ (\alpha_2^\sharp)_k\right]^T$, where (a) and (b), (c) and (d) correspond to the residual obtained by the FIF (16) with the free parameters $\bar{K}_i$ $(i = 1, 2, 3)$ and the FIF (16) without the free parameters, respectively. In order to compare the obtained results by different methods, we propose the root mean square (RMS) of the fault estimation error as a performance index. This error for the scalar variable n_i with respect to its estimate α_i for $\mathbf{N_s}$ simulation steps is defined as

$$\mathrm{RMS}_i \triangleq \sqrt{\frac{\sum_{j=1}^{\mathbf{N_s}} \left[n_i^j - \alpha_i^j\right]^2}{\mathbf{N_s}}} \tag{31}$$

where n_i^j is the value of the variable n_i in the j-th step, α_i^j is the estimate of n_i^j , and $i = 1, 2$. From Fig.3 and Tab.1 which shows the RMS of the fault estimation errors, we can conclude that the FIF (16) with the free parameters $\bar{K}_i$ $(i = 1, 2, 3)$ works better than the FIF (16) without the free parameters does.

5 Conclusion

This paper modelled the effect of network-induced delay introducing into the control loop as time-varying disturbance. Based on this model, a fault isolation filter for fault detection of networked control systems with multiple faults was then parameterized. Directional residual generations decoupling from the disturbance ensured the treatment of multiple faults appearing simultaneously or sequentially. The remaining degrees of freedom in the design of the filter's gain were used to satisfy an H_∞ disturbance attenuation and poles assignment constraints within the framework of H_∞ control for Markov jump linear systems. The sufficient existence conditions of the free parameters were formulated as a convex optimization problem over a couple of linear matrix inequalities. An Illustrative example was then given to show the efficiency of the proposed method for NCS. In contrast with the fault isolation filter without free parameters, it can be concluded that the proposed fault isolation filter works better for NCS.

Throughout the paper, only the network-induced delays shorter than one sampling period were taken into account in the model of the networked control system. Since the inherent complexity is concerned with the model of networked control system, much work can be carried out especially for the case where the network-induced delays are longer than one sampling period. On the other hand, the control decrement sequences in this paper are assumed to be known in advance. If the network scheduling algorithm is adopted to optimize the sequences, the fault estimation performance will be expected to increase. Study of integrating network scheduling and fault detection and isolation problem for networked control systems is also encouraged.

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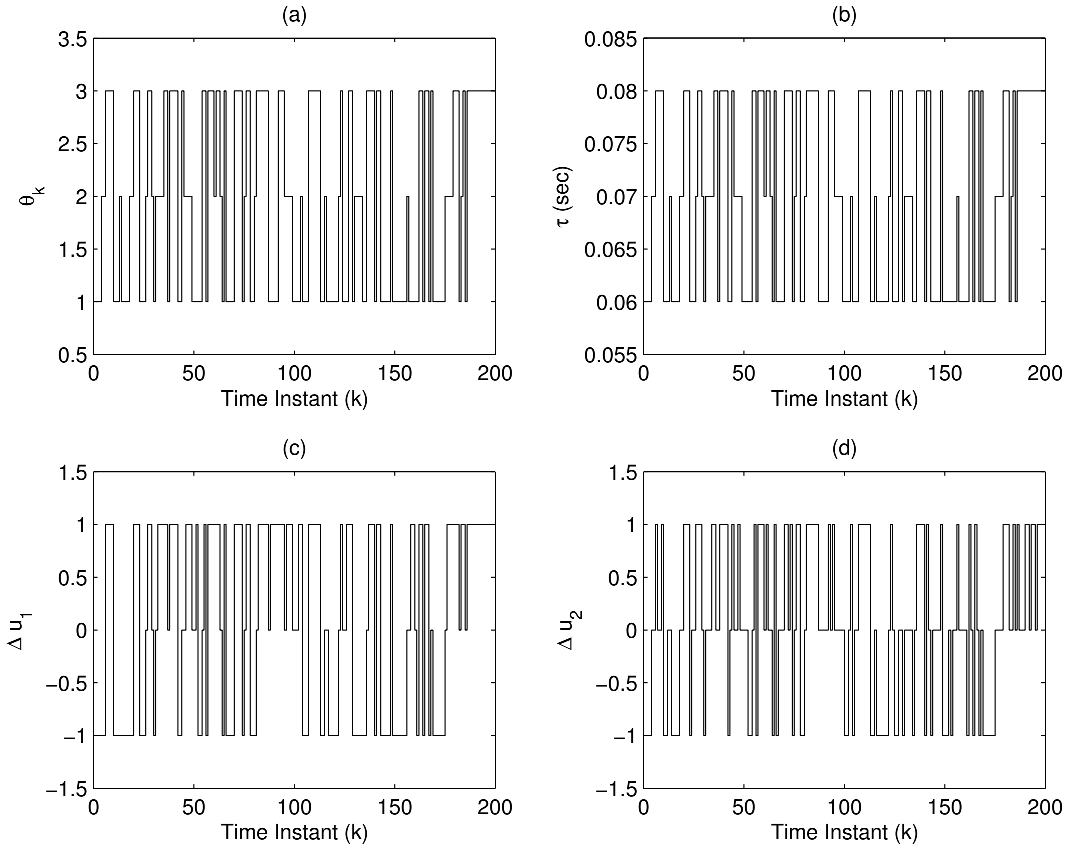

Figure 2. Innovation sequences of (a) Markov chain mode θ_k (b) the network-induced time delay τ_k (c) the control decrements $(\Delta u_1)_k$ (d) the control decrement $(\Delta u_2)_k$

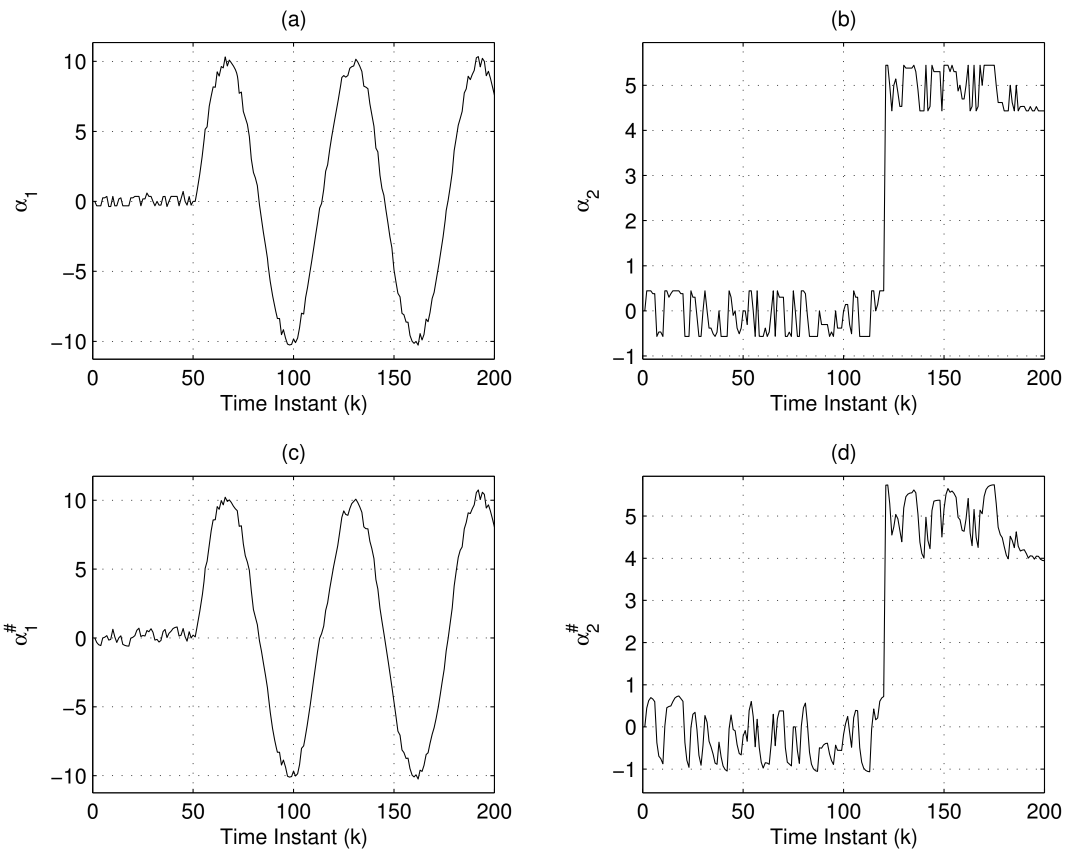

Figure 3. Innovation sequences of residual α_k and $\alpha_k^\sharp$

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