INPUT IMPEDANCE & RADIATION PATTERN OF PLANAR EBG ANTENNAS

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ABSTRACT

The radiation pattern and the input impedance of Fabry-Perot and EBG antennas are obtained by respectively plane wave and cylindrical (or spherical) wave expansions. The original procedure applied here leads to simple and rigorous formulas which are very helpful for antenna designers and for understanding the physical behavior of EBG antennas.

I. INTRODUCTION

EBG antennas are generally composed of an EBG material (**E**lectromagnetic **B**and **G**ap), a reflector plane and a primary source that is placed inside the structure (fig. 2c). These antennas are capable to produce highly directive and narrow beam, while they are very compact and low profile, compared to classical directive antennas (horn, parabolic reflector, etc). In this ***poster***, we will first remind the radiation characteristics of these antennas by using a modelling method based on the plane wave expansion of the internal field of Fabry-Perot (FP) cavities and antennas. Then we will propose an original method based on the cylindrical wave expansion allowing to obtain the input impedance of FP or EBG antennas excited by a line source. The method leads to simple and rigorous analytical formulas which are very helpful for the design of EBG antennas and gives a better comprehension of the physical phenomena. The work can be divided into two parts. In the first one, we will apply the classical plane wave method [1, 3, 4] to evaluate the far field, as well as the internal field created by a plane wave source placed inside a Fabry-Pérot cavity. In the second part, the plane wave source is replaced by a cylindrical wave source, in order to evaluate the input impedance more realistically. Finally, to validate the results, they will be confronted with those of a numerical method

II. PLANE WAVE RESPONSE & RADIATION PATTERN OF FABRY-PEROT (FP) ANTENNAS

Let us consider a plane wave source inside a Fabry Perot cavity (fig. 1). The cavity is made of two infinite Frequency Selective Surfaces (FSS) characterized by their transmission (t_1;t_2) and reflexion coefficients (r_1;r_2). By applying the method of the successive reflections inside the cavity, the total transmitted wave T (i.e. the FP response) can be given as a superposition of all partially transmitted fields. We can notice that these partial waves can be considered as waves coming from the successive image points (fig. 1)

$$T_{FP}(\theta)=\frac{E_z^{\,tranamitted}}{E_z^{\,incident}}=t_1\left(1+\sum_{n=1}^{\infty}(r_1r_2)^n\exp(-2jknD\cos(\theta))+\sum_{n=1}^{\infty}r_2(r_1r_2)^n\exp(-2jknD\cos(\theta)-2jkD_2\cos(\theta))\right)=\frac{t_1(1+r_2\exp(-2jkD_2\cos(\theta)))}{1-r_1r_2\exp(-2jkD\cos(\theta))}\quad(1)$$

The radiation pattern $P_{FP}(\theta)$ of the FP antenna can be obtained by the multiplication of the primary source pattern $P_{source}(\theta)$ and the FP response $T_{FP}(\theta)$ (eq. 2, and fig. 2). The reason is that $P_{source}(\theta)$ can be considered as the plane wave expansion of the source far field.

$$P_{FP}(\theta)=P_{source}(\theta)\bullet T_{FP}(\theta)\quad(2)$$

Figure (3) shows the comparison of radiation pattern obtained from the relation (2) with the radiation pattern obtained by a numerical method (FDTD) at f=2.94 GHz. The cavity is composed of a metallic reflector (surface 1) and an FSS (surface 2) which constituted of parallel metallic wires. We observe a good agreement between the two results. The discrepancies come from the fact that the cavity size is infinite for analytical formulas, but finite for the FDTD simulation (L_2=1m).

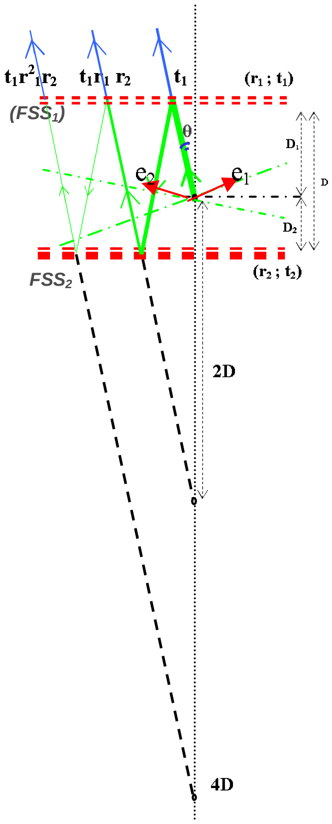


Fig.1. Method of the successive reflections for studying the radiation pattern

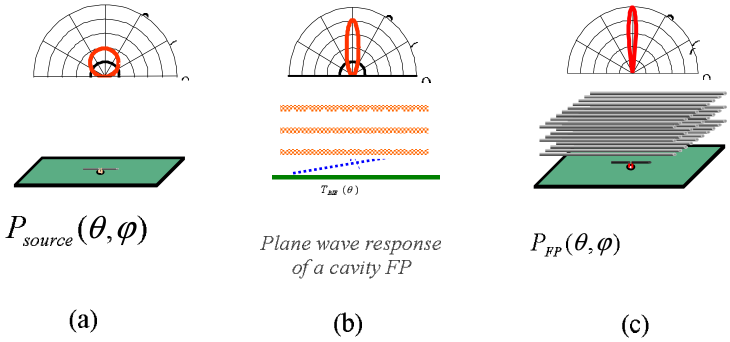


Fig. 2. $P_{FP}(\theta)$ of the FP antenna can be obtained by the multiplication of the primary source pattern $P_{source}(\theta)$ and the FP response $T_{FP}(\theta)$

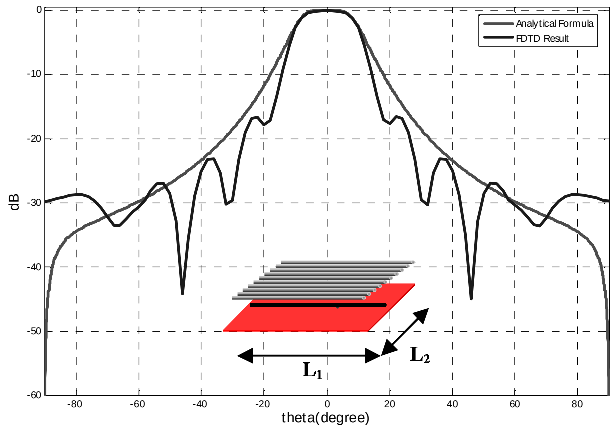


Fig.3. Radiation pattern of a infinitely long current source inside the cavity with $D_1=D_2=24mm$; $L_2=1m$; surface of metallic wires with a/Pt=5% ; Pt=12mm, a: diameter of wires, Pt: distance between wires

III. INPUT IMPEDANCE FOR EXCITING PLANE WAVE SOURCE

The total electric and magnetic field inside the FP cavity can again be evaluated by the same procedure as for the outside field (superposition of the reflected waves):

$$E^{sup}{}_z = E_z^{inc}\left(1+\sum_{n=1}^{\infty}(r_1r_2)^n \exp(-2jknD\cos(\theta))+\sum_{n=1}^{\infty} r_2(r_1r_2)^n \exp(-2jknD\cos(\theta)-2jkD_2\cos(\theta))\right)=\frac{(1+r_2\exp(-2jkD_2\cos\theta))}{1-r_1r_2\exp(-2jkD\cos\theta)}E_z^{inc} \qquad (3)$$

$$\overrightarrow{H}^{sup} = \sum_{n=0}^{\infty}(r_1r_2)^n\exp(-2jknD\cos(\theta))H^{inc}\overrightarrow{e_1}+r_2\sum_{n=0}^{\infty}(r_1r_2)^n\exp(-2jknD\cos(\theta)-2jkD_2\cos(\theta))H^{inc}\overrightarrow{e_2} \qquad (4)$$

$$\overrightarrow{H^{sup}}=\left(\frac{1}{1-r_1r_2\exp(-2jkD\cos(\theta))}\vec{e_1}+\left(\frac{1+r_2\exp(-2jkD_2\cos(\theta))}{1-r_1r_2\exp(-2jkD\cos(\theta))}\right)\vec{e_2}\right)H^{inc}$$

$$H_x^{sup}=\frac{\sin\theta(1+r_2\exp(-2jkD_2\cos\theta))}{1-r_1r_2\exp(-2jkD\cos\theta)}H_x^{inc} \qquad H_Y^{sup}=\frac{\cos\theta(1-r_2\exp(-2jkD_2\cos\theta))}{1-r_1r_2\exp(-2jkD\cos\theta)}H_y^{inc} \qquad (5)$$

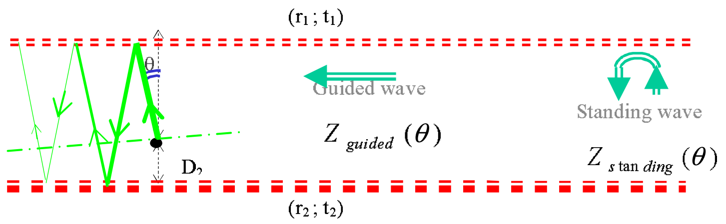


Fig. 4. Guided & standing wave inside the cavity

The magnetic field H has obviously two components which can be projected on x and y axes. The resulting Hx and Hy given in relation (5) lead to two distinct impedance expressions called here Z_{guide} and $Z_{standing}$ corresponding to the guided and standing waves inside the FP cavity.

$$Z_{guided}(\theta)=\frac{E_z^{\sup}}{H_x^{\sup}}=\frac{\eta}{\sin\theta} \tag{6}$$

$$Z_{standing}(\theta)=\frac{E_z^{\sup}}{H_y^{\sup}}=\eta\frac{(1+r_2\exp(-2jkD_2\cos\theta))}{\cos\theta(1-r_2\exp(-2jkD_2\cos\theta))} \tag{7}$$

η is the intrinsic impedance of plane wave in free space.

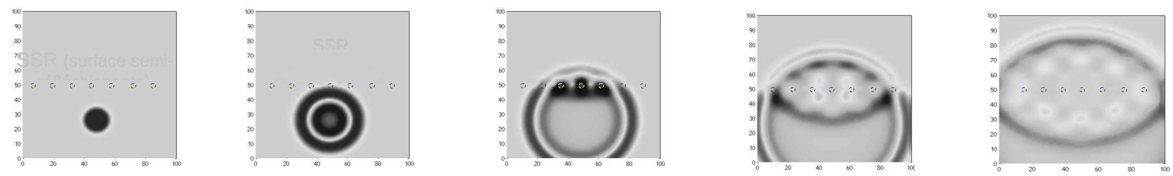
It should be noted here that the expressions (6 and 7) are the impedances of an ideal plane wave source and not the input impedances of a real primary source. In the following section, we consider a line source as primary source and apply the method of successive reflections to extract the expression of the real input impedance for the line source.

IV. CYLINDRICAL WAVE RESPONSE

Let us consider a line source (infinitely long current source) placed inside the FP cavity. A simulation by the FDTD method enables us to visualize the interaction between a cylindrical wave and the two FSS of the cavity.
In these images (fig. 5), the incident cylindrical wave is centred on the primary source. After having met the FSS, it appears two waves: the transmitted wave remains centred on the primary source and the reflected one is centred on the image of the primary source through the FSS. We should notice that the two waves are always cylindrical waves. By using the method of successive reflexions applied in the first part for the plane waves and illustrated in fig. (6) for cylindrical waves, we obtain the analytical expression of the electrical field E and the magnetic field H inside the cavity:

$$E=k^2\left(H_0^{(2)}(0^+)+r_1\sum_{n=0}^{\infty}(r_1r_2)^nH_0^{(2)}(2k(D_1+nD))+r_2\sum_{n=0}^{\infty}(r_1r_2)^nH_0^{(2)}(2k(D_2+nD))+2\sum_{n=1}^{\infty}(r_1r_2)^nH_0^{(2)}(2knD)\right) \tag{8}$$

$$H=k^2\left(\frac{H_1^{(2)}(0^+)}{j\eta}\right) \tag{9}$$



Incident cylindrical wave $H_0^{(2)}$ | Incident cylindrical wave $H_0^{(2)}$ | Incident cylindrical wave arrives at the FSS | Reflected & transmitted cylindrical wave | Reflected & transmitted wave

Fig. 5. Interaction between a cylindrical wave and the FSS of the cavity

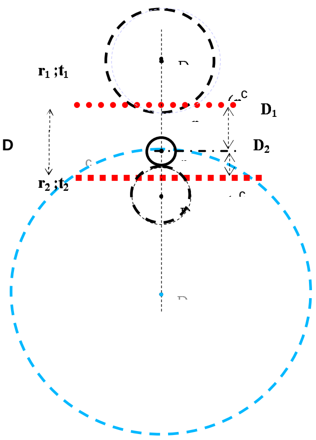


Fig.6. Images successive of a cylindrical wave source are also cylindrical wave sources

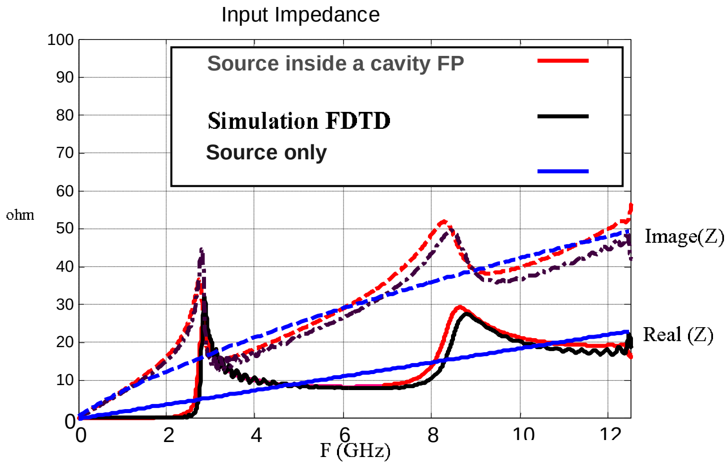


Fig.7. Input impedance of a infinitely long current source placed inside a cavity

$H_0^{(2)}$, $H_1^{(2)}$ are Hankel functions of order zero and 1 consecutively, and of the second kind (output waves) [5].

The input impedance of the exciting (line) source becomes simply the ratio of electrical field and magnetic field at the source point (D_1):

$$Z = \frac{E}{H} = Z_{source} + \frac{j\eta}{H_1^{(2)}(0^+)}\left(r_1 \sum_{n=0}^{\infty} (r_1 r_2)^n H_0^{(2)}(2k(D_1 + nD)) + r_2 \sum_{n=0}^{\infty} (r_1 r_2)^n H_0^{(2)}(2k(D_2 + nD)) + 2\sum_{n=1}^{\infty} (r_1 r_2)^n H_0^{(2)}(2knD) \right) \quad (10)$$

Figure (7) shows that there is a very good agreement between the relation (10) and the input impedance calculated by the FDTD method.

It should be noted that our results can be applied into a multilayer EBG structure by using a simple recursive method developed in [3, 4]

V. CONCLUSION

We showed that the classical ray theory can allow to obtain the radiation pattern of FP and EBG antenna, but it leads to expressions for the input impedance that are only valid for idealized plane wave sources. For a realistic line source, we proposed an original method based on the successive reflections of cylindrical waves inside the cavity. We are currently developing the generalisation of this method to point source (spherical waves) and arbitrary primary sources. All these results can be applied to multilayer EBG antennas by a recursive method. The elaborated expressions constitute a precious tool for antenna designers and engineers and allow a better understanding of the physical phenomena associated with EBG antennas.

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