

# Hyperfine Quenching of the $4s4p\ ^3P_0$ Level in Zn-like Ions

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**Abstract.** In this paper, we used the multiconfiguration Dirac-Fock method to compute with high precision the influence of the hyperfine interaction on the  $[Ar]3d^{10}4s4p\ ^3P_0$  level lifetime in Zn-like ions for stable and some quasi-stable isotopes of nonzero nuclear spin between  $Z = 30$  and  $Z = 92$ . The influence of this interaction on the  $[Ar]3d^{10}4s4p\ ^3P_1 - [Ar]3d^{10}4s4p\ ^3P_0$  separation energy is also calculated for the same ions.

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## 1 Introduction

It has been found before that the hyperfine interaction plays a fundamental role in the lifetimes and energy separations of the  $^3P_0$  and  $^3P_1$  levels of the configurations  $1s2p$  in He-like [1, 2, 3, 4, 5, 6, 7, 8, 9],  $[He]2s2p$  in Be-like [10, 11, 12], and  $[Ne]3s3p$  in Mg-like [11, 13] ions, and also in  $3d^4J = 4$  level in the Ti-like ions [14].

In the He-like ions, in the region  $Z \approx 45$ , these two levels undergo a level crossing [15] and are nearly degenerate due to the electron-electron magnetic interaction, which leads to a strong influence of the hyperfine interaction on the energy splitting and on the  $^3P_0$  lifetime for isotopes with nonzero nuclear spin. In Be-like, Mg-like and Zn-like ions a level crossing of the  $^3P_0$  and  $^3P_1$  levels has not been found [16], but hyperfine interaction still has strong influence on the lifetime of the  $^3P_0$  metastable level and on the energy splitting, for isotopes with nonzero nuclear spin.

Until recently, laboratory measurements of atomic metastable states hyperfine quenching have been performed only for He-like systems, for  $Z = 28$  to  $Z = 79$  [3, 4, 5, 8, 17, 18, 19, 20]. Hyperfine-induced transition lines in Be-like systems have been found in the planetary nebula NGC3918 [12]. Measured values of transition probabilities were found to agree with computed values [11, 21].

Divalent atoms are being investigated, both theoretically and experimentally, in order to investigate the possible use of the hyperfine quenched  $^3P_0$  metastable state for ultraprecise optical clocks and trapping experiments [22].

Recently, dielectronic recombination rate coefficients were measured for three isotopes of Zn-like  $Pt^{48+}$  in the

Heidelberg heavy-ion storage ring TSR [23]. It was suggested that hyperfine quenching of the  $4s4p\ ^3P_0$  in isotopes with non-zero nuclear spin could explain the differences detected in the observed spectra.

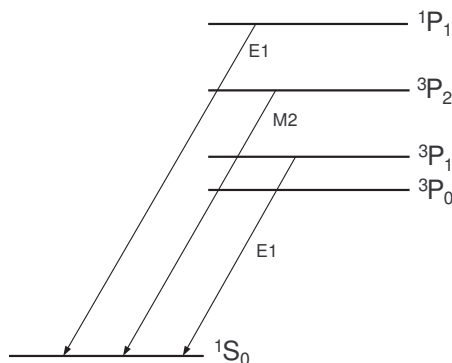
In this paper we extend our previous calculations [2, 10, 13] to the influence of the hyperfine interaction on the  $1s^22s^22p^63s^23p^63d^{10}4s4p$  levels in Zn-like ions.

In Fig. 1 we show the energy level scheme, not to scale, of these ions. The  $^3P_0$  level is a metastable level; one-photon transitions from this level to the ground state are forbidden, and multiphoton transitions have been found to be negligible in similar systems, so the same behavior can be expected for Zn-like ions. Therefore, in first approximation, we will consider the lifetime of this level as infinite. The energy separation between the  $4\ ^3P_0$  and  $4\ ^3P_1$  levels is small for  $Z$  values around the neutral Zn atom and increases very rapidly with  $Z$ . Hyperfine interaction is expected to have a strong influence on the energy splitting and on the  $4\ ^3P_0$  lifetime for isotopes with nonzero nuclear spin.

The different steps of this calculation are described in Refs. [2, 24]. Here we will emphasize only the fundamental topics of the theory and the characteristic features of Zn-like systems.

In this work we used the multi-configuration Dirac-Fock code of Desclaux and Indelicato [25, 26, 27] to evaluate, completely *ab initio* the  $4s4p$  fine structure energies and transition probabilities.

Several terms, such as the nonrelativistic ( $J$  independent) contribution to the correlation, are the same for all levels. In this calculation Breit interaction is included, as well as radiative corrections, using the method described,



**Fig. 1.** Energy level and transition scheme for Zn-like ions (not to scale).

for instance, in Ref. [28] and references therein. The MCDF method, in principle, allows for precise calculations because it can include most of the correlation relatively easily, i. e., with a small number of configurations. Here, correlation is important in the determination of transition energies to the ground state, which are used in the calculation of transition probabilities. We found that the largest effect is obtained by using  $[\text{Ar}]3d^{10}4s^2$  and  $4p^2$  as the configuration set for the ground state. For the excited states we included all configuration state functions (CSF) originated from  $[\text{Ar}]3d^{10}4s4p$ ,  $4p4d$ , and  $4d4f$  (which is usually defined as intrashell correlation), because in second-order perturbation theory the dominant energy difference denominators correspond to these configurations. Correlation originating from interaction with the Ar-like core has been neglected. In particular we included, in a test calculation, spin-polarization from  $s$  subshells and found a negligible influence in both energies and transition probabilities. We note that the fine structure energy separation,  $\Delta E_{0;fs} = E_{3P_1} - E_{3P_0}$ , which is the important parameter in the calculation of the hyperfine quenching, is not very sensitive to correlation, similarly to what we found for systems with smaller number of electrons [10, 13]. The same set of CSF have been used for energy, transition probabilities, and for the calculation of the hyperfine matrix elements.

All energy calculations are done in the Coulomb gauge for the retarded part of the electron-electron interaction, to avoid spurious contributions (see for example Refs. [29, 31]). The lifetime calculations are all done using exact relativistic formulas. The length gauge has been used for all transition probabilities.

## 2 Relativistic calculation of hyperfine contribution to fine structure splitting and to transition probabilities

In the case of a nucleus with nonzero spin, the hyperfine interaction between the nucleus and the electrons must be taken into account. The correspondent Hamiltonian can

be written as

$$H_{\text{hfs}} = \sum_k \mathbf{M}^{(k)} \cdot \mathbf{T}^{(k)}, \quad (1)$$

where  $\mathbf{M}^{(k)}$  and  $\mathbf{T}^{(k)}$  are spherical tensors of rank  $k$ , representing, respectively, the nuclear and the atomic parts of the interaction. As in the case of He-like, Be-like and Mg-like ions, the only sizable contribution from Eq. (1) is the magnetic dipole term ( $k = 1$ ). The hyperfine interaction mixes states with the same  $F = J + I$  values. In our case, we are interested in the  $^3P_0$  level, so  $J = 0$  and  $F = I$ . The contribution of this interaction for the total energy has been evaluated through the diagonalization of the matrix:

$$H_{\text{tot}} = \begin{bmatrix} E_0 + \frac{1}{2}i\Gamma_0 + W_{0,0} & W_{0,1} & W_{0,2} \\ W_{1,0} & E_1 + \frac{1}{2}i\Gamma_1 + W_{1,1} & W_{1,2} \\ W_{2,0} & W_{2,1} & E_2 + \frac{1}{2}i\Gamma_2 + W_{2,2} \end{bmatrix} \quad (2)$$

Here,  $E_f$  is the unperturbed level energy and  $\Gamma_f$  is the radiative width of the unperturbed level ( $f = 0, 1, 2$  stands, respectively, for  $^3P_0, ^3P_1, ^1P_1$ ). In reality there is a fourth level,  $^3P_2$ , which we included in all calculations but was found unnecessary, because the influence of this level in the  $^3P_0$  lifetime is negligible. This can be explained by the large energy separation between the  $^3P_2$  and  $^3P_0$  levels and also because the probability of the allowed M2 transition  $4s4p\ ^3P_2 \rightarrow 4s^2\ ^1S_0$  is many orders of magnitude smaller than those of the E1 transitions  $4s4p\ ^3P_1 \rightarrow 4s^2\ ^1S_0$  and  $4s4p\ ^1P_1 \rightarrow 4s^2\ ^1S_0$ . Also, the magnetic dipole hyperfine matrix element between the  $^3P_2$  and  $^3P_0$  levels is very small. This also leads to a negligible influence of the  $^3P_2$  level on the  $^3P_1 - ^3P_0$  separation energy. This is consistent with the results of Plante e Johnson [32], who found that the magnetic quadrupole term of the hyperfine interaction affects the  $^3P_2$  level only at high  $Z$ . The influence of the  $^1P_1$  level, however, must be taken into account, specially for light nuclei, because the large energy separation between  $^1P_1$  and  $^3P_0$  levels is compensated by the much shorter lifetime of the  $^1P_1$  level. The

hyperfine matrix element

$$W_{f,f'} = W_{f',f} = \langle [\text{Ar}] 3d^{10}4s4p f | H_{hfs} | [\text{Ar}] 3d^{10}4s4p f' \rangle$$

may be written as

$$W_{J_1, J_2} = \langle I, J_1, F, M_F | \mathbf{M}^{(1)} \cdot \mathbf{T}^{(1)} | I, J_2, F, M_F \rangle, \quad (3)$$

where  $I$  is the nuclear spin and  $F$  the total angular momentum of the atom, and may be put in the form:

$$W_{J_1, J_2} = (-1)^{I+J_1+F} \begin{Bmatrix} I & J_1 & F \\ J_2 & I & 1 \end{Bmatrix} \times \langle I || \mathbf{M}^{(1)} || I \rangle \langle J_1 || \mathbf{T}^{(1)} || J_2 \rangle. \quad (4)$$

The  $6j$  symbol leads to  $W_{0,0} = 0$ . Also the nuclear magnetic moment  $\mu_I$  in units of the nuclear magneton  $\mu_N$  may be defined by

$$\mu_I \mu_N = \langle I || \mathbf{M}^{(1)} || I \rangle \begin{pmatrix} I & 1 & I \\ -I & 0 & I \end{pmatrix}, \quad (5)$$

with  $\mu_N = eh/2\pi m_p c$ .

The electronic matrix elements were evaluated on the basis set  $|^3P_0\rangle$ ,  $|^3P_1\rangle$ ,  $|^1P_1\rangle$  with all intrashell correlation included.

The final result is then obtained by a diagonalization of the  $3 \times 3$  matrix in Eq. (2), the real part of each eigenvalue being the energy of the correspondent level and the imaginary part its lifetime.

### 3 Results and discussion

The MCDF method has been used to evaluate the influence of the hyperfine interaction on the  $[\text{Ar}]3d^{10}4s4p\ ^3P_0$ ,  $^3P_1$  and  $^1P_1$  levels for all  $Z$  values between 30 and 92 and for all stable and some quasi-stable isotopes of nonzero nuclear spin. Contributions from other levels have been found to be negligible. A detailed list of the contributions to the theoretical  $^3P_0$ ,  $^3P_1$  and  $^1P_1$  level energies is presented in Table 1, for  $Z = 36, 54$ , and  $82$ .

In Table 2 we present, for all possible values of the nuclear spin  $I$ ,  $Z$ , and the mass number  $A$ , the diagonal and off-diagonal hyperfine matrix elements  $W_{i,j}$ , and the nuclear magnetic moment,  $\mu_I$  [37], in nuclear magneton units. The indexes 0, 1, and 2 in the hyperfine matrix elements stand for  $^3P_0$ ,  $^3P_1$ , and  $^1P_1$ , respectively.

In Table 3 are presented, for all possible values of  $I$ ,  $Z$ , and  $A$ , the unperturbed separation energies  $\Delta E_0 = E_{^3P_1} - E_{^3P_0}$ , the hyperfine-affected separation energy  $\Delta E_{\text{hf}}$ , the  $4s4p\ ^3P_1$  and  $4s4p\ ^1P_1$  levels lifetime values,  $\tau_1$  and  $\tau_2$ , respectively, which are not affected, within the precision shown, by the hyperfine interaction, and finally the perturbed lifetime  $\tau_0$  of the  $4s4p\ ^3P_0$  level. As we pointed out before, the unperturbed value of  $\tau_0$  is assumed to be infinite.

In Fig. 2 is plotted the difference  $E = \Delta E_{\text{hf}} - \Delta E_0$  as a function of  $Z$  for the different possible values of nuclear spin. The influence of the hyperfine interaction on this

energy is shown to increase slowly with  $Z$ , the increase becoming more rapid for  $Z \gtrsim 60$ .

In Fig. 3 is plotted the perturbed  $4s4p\ ^3P_0$  level lifetime  $\tau_0$ , for the different nuclear spin values  $I$ , as a function of  $Z$ . One can easily conclude that the opening of a new channel for the decay of the  $4s4p\ ^3P_0$  level has a dramatic effect on its lifetime.

After submitting this paper, it was brought to our attention the work by Liu *et al* [38], which contains independent calculations of the hyperfine quenched lifetime of the  $^3P_0$  level in several Zn-like ions. Our results for this lifetime are three times higher than the values found by Liu *et al* for  $Z = 30$  and 1.5 higher at  $Z = 47$ . To look for the origin of this discrepancy we used the  $^3P_1 \rightarrow ^1S_0$  and  $^1P_1 \rightarrow ^1S_0$  transition energies and probabilities of Liu *et al* to diagonalize the matrix in eq. 2 and obtained results very close to our own calculations. As we do not know the values of the hyperfine matrix elements calculated by Liu *et al*, the reasons for this discrepancy remain unknown.

One of the most interesting practical implication of these calculations comes from the relationship between the  $4s4p\ ^3P_0 - 4s4p\ ^3P_1$  levels energy separation and the  $4s4p\ ^3P_0$  level lifetime (Eq. 2). As referred in [13] this energy separation can be estimated from a measurement of the hyperfine-quenched  $4s4p\ ^3P_0$  lifetime of Zn-like ions with nuclear spin  $I \neq 0$ . This method has been demonstrated for heliumlike  $\text{Ni}^{26+}$  [4],  $\text{Ag}^{45+}$  [3],  $\text{Gd}^{62+}$  [17] and  $\text{Au}^{77+}$  [8]. In the Zn-like ions case, as in the corresponding Be-like and Mg-like ions, the situations is different, because, even for the highest  $Z$  values, the lifetimes involved are much longer than in heavy heliumlike ions. However, measurements of Be-like hyperfine quenched transition rates have been performed from astrophysical sources [12] and the hyperfine quenching of the Zn-like  $^{195}\text{Pt}^{48+}$  ion was observed in the TSR heavy-ion storage-ring [23].

The continuous progress in storage rings, ion sources and ion traps leads us to believe that lifetimes between 0.1 s and 10  $\mu\text{s}$  could be measured, with some accuracy, by directly looking at the light emitted by the ions as a function of time, after the trap has been loaded. Very good vacuum inside the trap is needed, of course, if long lifetimes are to be measured. It remains to be demonstrated that this method is experimentally feasible. Such experiments would be able to provide, for different isotopes, the unperturbed energy separation, because nuclear magnetic moments are well known. This would be an interesting way to test our relativistic calculations.

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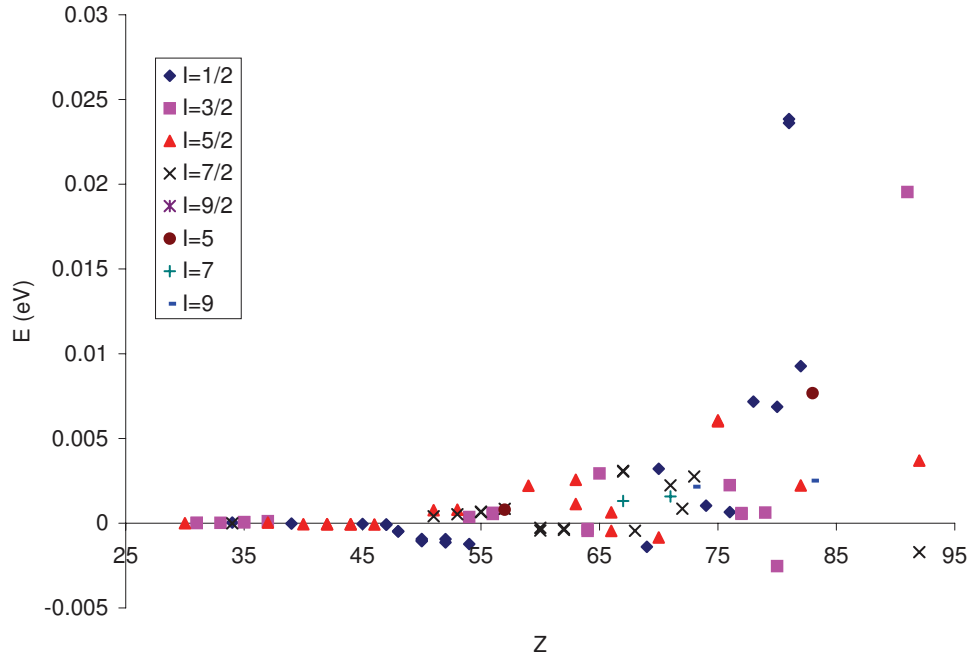
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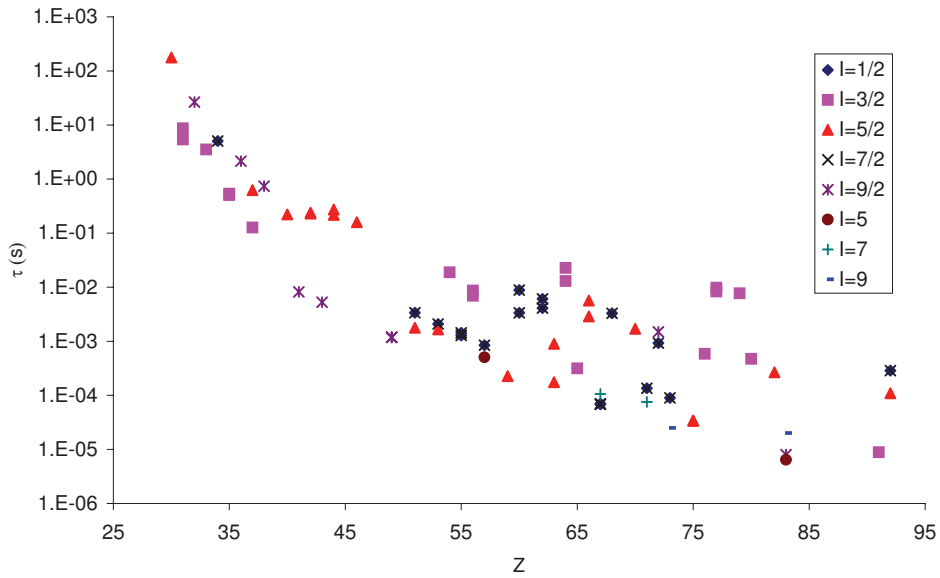
**Table 1.** Contribution to the energy of the  $4s4p\ ^3P_0, ^3P_1$  and  $^1P_1$  levels (in eV).

	$^3P_0$	$^3P_1$	$^1P_1$
		$Z = 36$	
Coulomb <sup>†</sup>	-75616.613	-75616.265	-75609.264
Magnetic <sup>†</sup>	42.785	42.779	42.765
Retardation (order $\omega^2$ ) <sup>†</sup>	-4.095	-4.095	-4.094
Retardation ( $> \omega^2$ )	-0.197	-0.197	-0.197
Self-energy (SE)	31.358	31.363	31.366
Self-energy screening	-2.735	-2.740	-2.743
VP [ $\alpha(Z\alpha)$ ]correction to e-e interaction	0.033	0.033	0.033
Vacuum Polarization $\alpha(Z\alpha)^3 + \alpha^2(Z\alpha)$	0.010	0.010	0.010
2nd order (SE-SE + SE-VP + S-VP-E) <sup>‡</sup>	-0.031	-0.031	-0.031
Recoil	-0.003	-0.003	-0.003
Relativistic Recoil <sup>‡</sup>	0.009	0.009	0.009
Total energy	-75549.478	-75549.137	-75542.148
		$Z = 54$	
Coulomb <sup>†</sup>	-195348.063	-195344.974	-195317.667
Magnetic <sup>†</sup>	171.169	171.142	170.943
Retardation (order $\omega^2$ ) <sup>†</sup>	-17.418	-17.418	-17.417
Retardation ( $> \omega^2$ )	-1.711	-1.713	-1.731
Self-energy (SE)	127.610	127.615	127.669
Self-energy screening	-9.166	-9.170	-9.205
VP [ $\alpha(Z\alpha)$ ]correction to e-el interaction	0.124	0.124	0.124
Vacuum Polarization $\alpha(Z\alpha)^3 + \alpha^2(Z\alpha)$	0.242	0.242	0.242
2nd order (SE-SE + SE-VP + S-VP-E) <sup>‡</sup>	-0.253	-0.253	-0.253
Recoil	-0.012	-0.012	-0.012
Relativistic Recoil <sup>‡</sup>	0.042	0.042	0.042
Total energy	-195077.437	-195074.376	-195047.265
		$Z = 82$	
Coulomb <sup>†</sup>	-513070.812	-513061.827	-512876.005
Magnetic <sup>†</sup>	710.586	710.511	708.897
Retardation (order $\omega^2$ ) <sup>†</sup>	-72.655	-72.654	-72.655
Retardation ( $> \omega^2$ )	-14.666	-14.668	-14.965
Self-energy (SE)	600.177	600.173	600.249
Self-energy screening	-38.198	-38.203	-38.200
VP [ $\alpha(Z\alpha)$ ]correction to e-e interaction	0.572	0.572	0.570
Vacuum Polarization $\alpha(Z\alpha)^3 + \alpha^2(Z\alpha)$	4.192	4.192	4.189
2nd order (SE-SE + SE-VP + S-VP-E) <sup>‡</sup>	-2.975	-2.975	-2.974
Recoil	-0.051	-0.051	-0.051
Relativistic Recoil <sup>‡</sup>	0.191	0.191	0.191
Total energy	-511883.638	-511874.738	-511690.755

<sup>†</sup> Contains the Uehling potential contribution to all order and all order Breit interaction.<sup>‡</sup> Calculated using the results of Ref. [33,34,35].<sup>‡</sup> The formulas and definitions used to evaluate this term are Appendix A of Ref. [36]



**Fig. 2.** Influence of the hyperfine interaction on the  $4s4p\ ^3P_1 - ^3P_0$  energy separation, as a function of the nuclear spin  $I$  and the atomic number  $Z$ . The quantity  $E = \Delta E_{\text{hf}} - \Delta E_0$  is the contribution of the hyperfine interaction to the fine structure splitting  $\Delta E_0$ . The symbols represent values for the different nuclear spins; some elements have several isotopes with identical spins but different  $\mu_I$  values.



**Fig. 3.** Influence of the hyperfine interaction on the lifetime of the  $4s4p\ ^3P_0$  level, as a function of the nuclear spin  $I$  and the atomic number  $Z$ . The unperturbed lifetime is, to a very good approximation, infinite.

**Table 2.** Hyperfine Matrix elements  $W_{i,j}$  in eV. The indexes 0, 1, and 2 stand for  $^3P_0$ ,  $^3P_1$ , and  $^1P_1$  respectively.  $I$  is the nuclear spin and  $\mu$  the nuclear magnetic moment in nuclear magneton units.

$I$	$Z$	$A$	$W_{1,1}$	$W_{2,2}$	$W_{0,1}$	$W_{0,2}$	$W_{1,2}$	$\mu_I$	
1/2	34	77	$-3.312 \times 10^{-05}$	$-3.245 \times 10^{-06}$	$-2.382 \times 10^{-05}$	$1.708 \times 10^{-05}$	$3.268 \times 10^{-05}$	0.535042	
	39	89	$2.697 \times 10^{-05}$	$4.139 \times 10^{-07}$	$1.670 \times 10^{-05}$	$-1.055 \times 10^{-05}$	$-2.199 \times 10^{-05}$	-0.137415	
	45	103	$4.648 \times 10^{-05}$	$-4.491 \times 10^{-06}$	$2.564 \times 10^{-05}$	$-1.321 \times 10^{-05}$	$-2.923 \times 10^{-05}$	-0.0884	
	47	107	$7.878 \times 10^{-05}$	$-1.013 \times 10^{-05}$	$4.218 \times 10^{-05}$	$-2.011 \times 10^{-05}$	$-4.507 \times 10^{-05}$	-0.11368	
	47	109	$9.057 \times 10^{-05}$	$-1.164 \times 10^{-05}$	$4.849 \times 10^{-05}$	$-2.311 \times 10^{-05}$	$-5.182 \times 10^{-05}$	-0.130691	
	48	111	$4.696 \times 10^{-04}$	$-6.718 \times 10^{-05}$	$2.480 \times 10^{-04}$	$-1.136 \times 10^{-04}$	$-2.560 \times 10^{-04}$	-0.594886	
	48	113	$4.912 \times 10^{-04}$	$-7.027 \times 10^{-05}$	$2.594 \times 10^{-04}$	$-1.188 \times 10^{-04}$	$-2.678 \times 10^{-04}$	-0.622301	
	50	115	$9.289 \times 10^{-04}$	$-1.573 \times 10^{-04}$	$4.788 \times 10^{-04}$	$-2.018 \times 10^{-04}$	$-4.591 \times 10^{-04}$	-0.91883	
	50	117	$1.012 \times 10^{-03}$	$-1.714 \times 10^{-04}$	$5.216 \times 10^{-04}$	$-2.199 \times 10^{-04}$	$-5.002 \times 10^{-04}$	-1.00104	
	50	119	$1.059 \times 10^{-03}$	$-1.793 \times 10^{-04}$	$5.457 \times 10^{-04}$	$-2.301 \times 10^{-04}$	$-5.233 \times 10^{-04}$	-1.04728	
	52	123	$9.385 \times 10^{-04}$	$-1.801 \times 10^{-04}$	$4.734 \times 10^{-04}$	$-1.834 \times 10^{-04}$	$-4.199 \times 10^{-04}$	-0.736948	
	52	125	$1.132 \times 10^{-03}$	$-2.172 \times 10^{-04}$	$5.708 \times 10^{-04}$	$-2.212 \times 10^{-04}$	$-5.063 \times 10^{-04}$	-0.888505	
	54	129	$1.232 \times 10^{-03}$	$-2.602 \times 10^{-04}$	$6.102 \times 10^{-04}$	$-2.170 \times 10^{-04}$	$-4.984 \times 10^{-04}$	-0.777976	
	69	169	$1.391 \times 10^{-03}$	$-3.983 \times 10^{-04}$	$6.364 \times 10^{-04}$	$-1.217 \times 10^{-04}$	$-2.710 \times 10^{-04}$	-0.231	
	70	171	$-3.210 \times 10^{-03}$	$9.272 \times 10^{-04}$	$-1.463 \times 10^{-03}$	$2.694 \times 10^{-04}$	$5.974 \times 10^{-04}$	0.49367	
	74	183	$-1.035 \times 10^{-03}$	$3.081 \times 10^{-04}$	$-4.659 \times 10^{-04}$	$7.391 \times 10^{-05}$	$1.610 \times 10^{-04}$	0.117785	
	76	187	$-6.581 \times 10^{-04}$	$1.982 \times 10^{-04}$	$-2.942 \times 10^{-04}$	$4.344 \times 10^{-05}$	$9.378 \times 10^{-05}$	0.064652	
	78	195	$-7.176 \times 10^{-03}$	$2.186 \times 10^{-03}$	$-3.189 \times 10^{-03}$	$4.387 \times 10^{-04}$	$9.383 \times 10^{-04}$	0.60952	
	80	199	$-6.867 \times 10^{-03}$	$2.112 \times 10^{-03}$	$-3.030 \times 10^{-03}$	$3.892 \times 10^{-04}$	$8.250 \times 10^{-04}$	0.505885	
	81	203	$-2.364 \times 10^{-02}$	$7.305 \times 10^{-03}$	$-1.040 \times 10^{-02}$	$1.290 \times 10^{-03}$	$2.724 \times 10^{-03}$	1.622258	
	81	205	$-2.387 \times 10^{-02}$	$7.377 \times 10^{-03}$	$-1.050 \times 10^{-02}$	$1.303 \times 10^{-03}$	$2.751 \times 10^{-03}$	1.638215	
	82	207	$-9.267 \times 10^{-03}$	$2.876 \times 10^{-03}$	$-4.060 \times 10^{-03}$	$4.873 \times 10^{-04}$	$1.024 \times 10^{-03}$	0.592583	
	3/2	31	69	$-1.363 \times 10^{-05}$	$-1.123 \times 10^{-06}$	$-2.652 \times 10^{-05}$	$2.024 \times 10^{-05}$	$1.535 \times 10^{-05}$	2.01659
		31	71	$-1.732 \times 10^{-05}$	$-1.427 \times 10^{-06}$	$-3.370 \times 10^{-05}$	$2.572 \times 10^{-05}$	$1.950 \times 10^{-05}$	2.56227
		33	75	$-2.179 \times 10^{-05}$	$-2.286 \times 10^{-06}$	$-3.670 \times 10^{-05}$	$2.687 \times 10^{-05}$	$2.235 \times 10^{-05}$	1.43948
		35	79	$-5.715 \times 10^{-05}$	$-4.904 \times 10^{-06}$	$-8.849 \times 10^{-05}$	$6.206 \times 10^{-05}$	$5.431 \times 10^{-05}$	2.1064
		35	81	$-6.161 \times 10^{-05}$	$-5.286 \times 10^{-06}$	$-9.538 \times 10^{-05}$	$6.690 \times 10^{-05}$	$5.855 \times 10^{-05}$	2.270562
		37	87	$-1.198 \times 10^{-04}$	$-6.370 \times 10^{-06}$	$-1.745 \times 10^{-04}$	$1.165 \times 10^{-04}$	$1.056 \times 10^{-04}$	2.75131
		54	131	$-3.649 \times 10^{-04}$	$7.708 \times 10^{-05}$	$-4.042 \times 10^{-04}$	$1.438 \times 10^{-04}$	$1.477 \times 10^{-04}$	0.6915
		56	135	$-5.429 \times 10^{-04}$	$1.235 \times 10^{-04}$	$-5.920 \times 10^{-04}$	$1.932 \times 10^{-04}$	$1.987 \times 10^{-04}$	0.837953
		56	137	$-6.073 \times 10^{-04}$	$1.382 \times 10^{-04}$	$-6.622 \times 10^{-04}$	$2.161 \times 10^{-04}$	$2.222 \times 10^{-04}$	0.937365
		64	155	$3.449 \times 10^{-04}$	$-9.328 \times 10^{-05}$	$3.595 \times 10^{-04}$	$-8.381 \times 10^{-05}$	$-8.504 \times 10^{-05}$	-0.2572
64		157	$4.556 \times 10^{-04}$	$-1.232 \times 10^{-04}$	$4.750 \times 10^{-04}$	$-1.107 \times 10^{-04}$	$-1.123 \times 10^{-04}$	-0.3398	
65		159	$-2.936 \times 10^{-03}$	$8.052 \times 10^{-04}$	$-3.049 \times 10^{-03}$	$6.825 \times 10^{-04}$	$6.901 \times 10^{-04}$	2.014	
76		189	$-2.239 \times 10^{-03}$	$6.745 \times 10^{-04}$	$-2.238 \times 10^{-03}$	$3.305 \times 10^{-04}$	$3.191 \times 10^{-04}$	0.659933	
77		191	$-5.500 \times 10^{-04}$	$1.666 \times 10^{-04}$	$-5.481 \times 10^{-04}$	$7.811 \times 10^{-05}$	$7.506 \times 10^{-05}$	0.1507	
77		193	$-5.974 \times 10^{-04}$	$1.810 \times 10^{-04}$	$-5.954 \times 10^{-04}$	$8.484 \times 10^{-05}$	$8.154 \times 10^{-05}$	0.1637	
79		197	$-6.243 \times 10^{-04}$	$1.911 \times 10^{-04}$	$-6.182 \times 10^{-04}$	$8.215 \times 10^{-05}$	$7.824 \times 10^{-05}$	0.148158	
80		201	$2.535 \times 10^{-03}$	$-7.798 \times 10^{-04}$	$2.501 \times 10^{-03}$	$-3.212 \times 10^{-04}$	$-3.045 \times 10^{-04}$	-0.560226	
91		231	$-1.960 \times 10^{-02}$	$6.265 \times 10^{-03}$	$-1.845 \times 10^{-02}$	$1.656 \times 10^{-03}$	$1.506 \times 10^{-03}$	2.01	
5/2		30	67	$-1.975 \times 10^{-06}$	$-3.436 \times 10^{-08}$	$-6.734 \times 10^{-06}$	$5.115 \times 10^{-06}$	$2.303 \times 10^{-06}$	0.875479
		37	85	$-3.537 \times 10^{-05}$	$-1.880 \times 10^{-06}$	$-7.865 \times 10^{-05}$	$5.252 \times 10^{-05}$	$3.118 \times 10^{-05}$	1.353352
	40	91	$6.158 \times 10^{-05}$	$-2.554 \times 10^{-07}$	$1.273 \times 10^{-04}$	$-7.804 \times 10^{-05}$	$-4.821 \times 10^{-05}$	-1.30362	
	42	95	$6.081 \times 10^{-05}$	$-2.597 \times 10^{-06}$	$1.207 \times 10^{-04}$	$-6.931 \times 10^{-05}$	$-4.375 \times 10^{-05}$	-0.9142	
	42	97	$6.209 \times 10^{-05}$	$-2.652 \times 10^{-06}$	$1.233 \times 10^{-04}$	$-7.078 \times 10^{-05}$	$-4.467 \times 10^{-05}$	-0.9335	
	44	99	$5.826 \times 10^{-05}$	$-4.623 \times 10^{-06}$	$1.116 \times 10^{-04}$	$-5.968 \times 10^{-05}$	$-3.835 \times 10^{-05}$	-0.6413	
	44	101	$6.530 \times 10^{-05}$	$-5.182 \times 10^{-06}$	$1.251 \times 10^{-04}$	$-6.689 \times 10^{-05}$	$-4.298 \times 10^{-05}$	-0.7188	
	46	105	$7.773 \times 10^{-05}$	$-8.788 \times 10^{-06}$	$1.442 \times 10^{-04}$	$-7.151 \times 10^{-05}$	$-4.664 \times 10^{-05}$	-0.642	
	51	121	$-7.648 \times 10^{-04}$	$1.385 \times 10^{-04}$	$-1.332 \times 10^{-03}$	$5.382 \times 10^{-04}$	$3.597 \times 10^{-04}$	3.3634	
	53	127	$-8.004 \times 10^{-04}$	$1.617 \times 10^{-04}$	$-1.366 \times 10^{-03}$	$5.072 \times 10^{-04}$	$3.406 \times 10^{-04}$	2.813273	
	59	141	$-2.218 \times 10^{-03}$	$5.482 \times 10^{-04}$	$-3.622 \times 10^{-03}$	$1.040 \times 10^{-03}$	$6.985 \times 10^{-04}$	4.2754	
	63	151	$-2.567 \times 10^{-03}$	$6.842 \times 10^{-04}$	$-4.106 \times 10^{-03}$	$9.970 \times 10^{-04}$	$6.642 \times 10^{-04}$	3.4717	
	63	153	$-1.133 \times 10^{-03}$	$3.020 \times 10^{-04}$	$-1.812 \times 10^{-03}$	$4.401 \times 10^{-04}$	$2.932 \times 10^{-04}$	1.5324	
	66	161	$4.556 \times 10^{-04}$	$-1.264 \times 10^{-04}$	$7.196 \times 10^{-04}$	$-1.548 \times 10^{-04}$	$-1.021 \times 10^{-04}$	-0.4803	
	66	163	$-6.384 \times 10^{-04}$	$1.771 \times 10^{-04}$	$-1.008 \times 10^{-03}$	$2.169 \times 10^{-04}$	$1.431 \times 10^{-04}$	0.673	
	70	173	$8.428 \times 10^{-04}$	$-2.434 \times 10^{-04}$	$1.312 \times 10^{-03}$	$-2.416 \times 10^{-04}$	$-1.568 \times 10^{-04}$	-0.648	
	75	185	$-6.034 \times 10^{-03}$	$1.807 \times 10^{-03}$	$-9.246 \times 10^{-03}$	$1.415 \times 10^{-03}$	$8.981 \times 10^{-04}$	3.1871	
	75	187	$-6.096 \times 10^{-03}$	$1.826 \times 10^{-03}$	$-9.341 \times 10^{-03}$	$1.429 \times 10^{-03}$	$9.073 \times 10^{-04}$	3.2197	
82	205	$-2.226 \times 10^{-03}$	$6.908 \times 10^{-04}$	$-3.331 \times 10^{-03}$	$3.998 \times 10^{-04}$	$2.460 \times 10^{-04}$	0.7117		
92	233	$1.702 \times 10^{-03}$	$-5.451 \times 10^{-04}$	$3.263 \times 10^{-03}$	$-2.839 \times 10^{-04}$	$-1.256 \times 10^{-04}$	0.59		

Table 2. *Continued*

$I$	$Z$	$A$	$W_{1,1}$	$W_{2,2}$	$W_{0,1}$	$W_{0,2}$	$W_{1,2}$	$\mu_I$	
7/2	34	79	$9.003 \times 10^{-06}$	$8.821 \times 10^{-07}$	$2.967 \times 10^{-05}$	$-2.128 \times 10^{-05}$	$-8.883 \times 10^{-06}$	-1.018	
	51	123	$-4.141 \times 10^{-04}$	$7.500 \times 10^{-05}$	$-9.673 \times 10^{-04}$	$3.910 \times 10^{-04}$	$1.948 \times 10^{-04}$	2.5498	
	53	129	$-5.326 \times 10^{-04}$	$1.076 \times 10^{-04}$	$-1.220 \times 10^{-03}$	$4.528 \times 10^{-04}$	$2.267 \times 10^{-04}$	2.621	
	55	133	$-6.480 \times 10^{-04}$	$1.424 \times 10^{-04}$	$-1.459 \times 10^{-03}$	$4.970 \times 10^{-04}$	$2.494 \times 10^{-04}$	2.582025	
	55	135	$-6.857 \times 10^{-04}$	$1.507 \times 10^{-04}$	$-1.544 \times 10^{-03}$	$5.260 \times 10^{-04}$	$2.639 \times 10^{-04}$	2.7324	
	57	139	$-8.527 \times 10^{-04}$	$2.001 \times 10^{-04}$	$-1.892 \times 10^{-03}$	$5.915 \times 10^{-04}$	$2.968 \times 10^{-04}$	2.783046	
	60	143	$4.324 \times 10^{-04}$	$-1.093 \times 10^{-04}$	$9.424 \times 10^{-04}$	$-2.593 \times 10^{-04}$	$-1.296 \times 10^{-04}$	-1.065	
	60	145	$2.664 \times 10^{-04}$	$-6.731 \times 10^{-05}$	$5.805 \times 10^{-04}$	$-1.597 \times 10^{-04}$	$-7.984 \times 10^{-05}$	-0.656	
	62	147	$3.935 \times 10^{-04}$	$-1.032 \times 10^{-04}$	$8.483 \times 10^{-04}$	$-2.147 \times 10^{-04}$	$-1.069 \times 10^{-04}$	-0.812	
	62	149	$3.235 \times 10^{-04}$	$-8.486 \times 10^{-05}$	$6.975 \times 10^{-04}$	$-1.765 \times 10^{-04}$	$-8.790 \times 10^{-05}$	-0.6677	
	67	163	$-3.107 \times 10^{-03}$	$8.721 \times 10^{-04}$	$-6.561 \times 10^{-03}$	$1.356 \times 10^{-03}$	$6.644 \times 10^{-04}$	4.23	
	67	165	$-3.063 \times 10^{-03}$	$8.598 \times 10^{-04}$	$-6.467 \times 10^{-03}$	$1.337 \times 10^{-03}$	$6.550 \times 10^{-04}$	4.17	
	68	167	$4.484 \times 10^{-04}$	$-1.271 \times 10^{-04}$	$9.431 \times 10^{-04}$	$-1.875 \times 10^{-04}$	$-9.150 \times 10^{-05}$	-0.56385	
	71	175	$-2.238 \times 10^{-03}$	$6.514 \times 10^{-04}$	$-4.657 \times 10^{-03}$	$8.257 \times 10^{-04}$	$3.979 \times 10^{-04}$	2.2323	
	72	177	$-8.583 \times 10^{-04}$	$2.519 \times 10^{-04}$	$-1.781 \times 10^{-03}$	$3.041 \times 10^{-04}$	$1.459 \times 10^{-04}$	0.7935	
	73	181	$-2.765 \times 10^{-03}$	$8.176 \times 10^{-04}$	$-5.722 \times 10^{-03}$	$9.414 \times 10^{-04}$	$4.495 \times 10^{-04}$	2.3705	
	92	235	$-3.699 \times 10^{-03}$	$1.185 \times 10^{-03}$	$-5.287 \times 10^{-03}$	$4.599 \times 10^{-04}$	$2.731 \times 10^{-04}$	-0.38	
	9/2	32	73	$3.089 \times 10^{-06}$	$3.176 \times 10^{-07}$	$1.419 \times 10^{-05}$	$-1.060 \times 10^{-05}$	$-3.306 \times 10^{-06}$	-0.879468
		36	83	$1.124 \times 10^{-05}$	$7.920 \times 10^{-07}$	$4.326 \times 10^{-05}$	$-2.963 \times 10^{-05}$	$-1.029 \times 10^{-05}$	-0.970669
		38	87	$1.959 \times 10^{-05}$	$6.773 \times 10^{-07}$	$7.138 \times 10^{-05}$	$-4.640 \times 10^{-05}$	$-1.662 \times 10^{-05}$	-1.093603
41		93	$-1.930 \times 10^{-04}$	$4.552 \times 10^{-06}$	$-6.573 \times 10^{-04}$	$3.901 \times 10^{-04}$	$1.449 \times 10^{-04}$	6.1705	
43		99	$-2.464 \times 10^{-04}$	$1.512 \times 10^{-05}$	$-8.076 \times 10^{-04}$	$4.478 \times 10^{-04}$	$1.696 \times 10^{-04}$	5.6847	
49		113	$-5.500 \times 10^{-04}$	$8.617 \times 10^{-05}$	$-1.648 \times 10^{-03}$	$7.241 \times 10^{-04}$	$2.856 \times 10^{-04}$	5.5289	
49		115	$-5.512 \times 10^{-04}$	$8.636 \times 10^{-05}$	$-1.651 \times 10^{-03}$	$7.257 \times 10^{-04}$	$2.862 \times 10^{-04}$	5.5408	
72		179	$5.392 \times 10^{-04}$	$-1.582 \times 10^{-04}$	$1.403 \times 10^{-03}$	$-2.395 \times 10^{-04}$	$-9.166 \times 10^{-05}$	-0.6409	
83		209	$-7.663 \times 10^{-03}$	$2.388 \times 10^{-03}$	$-1.922 \times 10^{-02}$	$2.230 \times 10^{-03}$	$8.126 \times 10^{-04}$	4.1103	
5		57	138	$-7.965 \times 10^{-04}$	$1.869 \times 10^{-04}$	$-2.439 \times 10^{-03}$	$7.625 \times 10^{-04}$	$2.772 \times 10^{-04}$	3.713646
83	208	$-7.773 \times 10^{-03}$	$2.422 \times 10^{-03}$	$-2.146 \times 10^{-02}$	$2.491 \times 10^{-03}$	$8.243 \times 10^{-04}$	4.633		
	67	166	$-1.322 \times 10^{-03}$	$3.711 \times 10^{-04}$	$-5.264 \times 10^{-03}$	$1.088 \times 10^{-03}$	$2.827 \times 10^{-04}$	3.6	
7	71	176	$-1.588 \times 10^{-03}$	$4.624 \times 10^{-04}$	$-6.233 \times 10^{-03}$	$1.105 \times 10^{-03}$	$2.824 \times 10^{-04}$	3.169	
	9	73	180	$-2.189 \times 10^{-03}$	$6.472 \times 10^{-04}$	$-1.083 \times 10^{-02}$	$1.781 \times 10^{-03}$	4.825	
83	210	$-2.545 \times 10^{-03}$	$7.929 \times 10^{-04}$	$-1.217 \times 10^{-02}$	$1.412 \times 10^{-03}$	$2.699 \times 10^{-04}$	2.73		

**Table 3.** Influence of the hyperfine interaction on the  $4s4p\ ^3P_1 - 4s4p\ ^3P_0$  energy separation and on the lifetime of the  $^3P_0$  level, as a function of the nuclear spin  $I$  and the atomic number  $Z$ .  $\Delta E_0$  is the unperturbed energy separation (in eV), and  $\Delta E_{hf}$  is the perturbed energy (in eV) when the hyperfine interaction is taken into account (the 5 digits do not necessarily represent the accuracy of the calculation - they are intended to show the effect at low  $Z$ ).  $\tau_0$ ,  $\tau_1$  and  $\tau_2$  represent the perturbed lifetimes (in s) of  $4s4p\ ^3P_0$ ,  $^3P_1$  and  $^1P_1$  levels respectively.

I	Z	A	$\Delta E_0$	$\Delta E_{hf}$	$\tau_1$	$\tau_2$	$\tau_0$	
1/2	34	77	0.20461	0.20458	$2.139 \times 10^{-07}$	$1.340 \times 10^{-10}$	$7.847 \times 10^{+00}$	
	39	89	0.61806	0.61809	$1.393 \times 10^{-08}$	$5.025 \times 10^{-11}$	$1.317 \times 10^{+01}$	
	45	103	1.41748	1.41753	$1.976 \times 10^{-09}$	$2.375 \times 10^{-11}$	$5.111 \times 10^{+00}$	
	47	107	1.74706	1.74714	$1.221 \times 10^{-09}$	$1.923 \times 10^{-11}$	$1.847 \times 10^{+00}$	
	47	109	1.74706	1.74715	$1.221 \times 10^{-09}$	$1.923 \times 10^{-11}$	$1.397 \times 10^{+00}$	
	48	111	1.92132	1.92179	$9.804 \times 10^{-10}$	$1.739 \times 10^{-11}$	$5.286 \times 10^{-02}$	
	48	113	1.92132	1.92181	$9.804 \times 10^{-10}$	$1.739 \times 10^{-11}$	$4.831 \times 10^{-02}$	
	50	115	2.28581	2.28674	$6.623 \times 10^{-10}$	$1.433 \times 10^{-11}$	$1.391 \times 10^{-02}$	
	50	117	2.28581	2.28682	$6.623 \times 10^{-10}$	$1.433 \times 10^{-11}$	$1.172 \times 10^{-02}$	
	50	119	2.28581	2.28687	$6.623 \times 10^{-10}$	$1.433 \times 10^{-11}$	$1.071 \times 10^{-02}$	
	52	123	2.66778	2.66872	$4.673 \times 10^{-10}$	$1.185 \times 10^{-11}$	$1.397 \times 10^{-02}$	
	52	125	2.66778	2.66891	$4.673 \times 10^{-10}$	$1.185 \times 10^{-11}$	$9.609 \times 10^{-03}$	
	54	129	3.06295	3.06418	$3.425 \times 10^{-10}$	$9.804 \times 10^{-12}$	$8.274 \times 10^{-03}$	
	69	169	6.17717	6.17856	$7.752 \times 10^{-11}$	$2.398 \times 10^{-12}$	$7.234 \times 10^{-03}$	
	70	171	6.38647	6.38326	$7.246 \times 10^{-11}$	$2.174 \times 10^{-12}$	$1.367 \times 10^{-03}$	
	74	183	7.22323	7.22219	$5.650 \times 10^{-11}$	$1.464 \times 10^{-12}$	$1.353 \times 10^{-02}$	
	76	187	7.64172	7.64106	$5.051 \times 10^{-11}$	$1.196 \times 10^{-12}$	$3.400 \times 10^{-02}$	
	78	195	8.06074	8.05357	$4.545 \times 10^{-11}$	$9.804 \times 10^{-13}$	$2.895 \times 10^{-04}$	
	80	199	8.48051	8.47364	$4.115 \times 10^{-11}$	$7.937 \times 10^{-13}$	$3.215 \times 10^{-04}$	
	81	203	8.69084	8.66722	$3.922 \times 10^{-11}$	$7.194 \times 10^{-13}$	$2.725 \times 10^{-05}$	
	81	205	8.69084	8.66699	$3.922 \times 10^{-11}$	$7.194 \times 10^{-13}$	$2.672 \times 10^{-05}$	
	82	207	8.90152	8.89226	$3.745 \times 10^{-11}$	$6.494 \times 10^{-13}$	$1.794 \times 10^{-04}$	
	3/2	31	69	0.06865	0.06864	$4.446 \times 10^{-06}$	$4.528 \times 10^{-10}$	$8.711 \times 10^{+00}$
		31	71	0.06865	0.06863	$4.446 \times 10^{-06}$	$4.528 \times 10^{-10}$	$5.395 \times 10^{+00}$
		33	75	0.14992	0.14990	$4.831 \times 10^{-07}$	$1.808 \times 10^{-10}$	$3.524 \times 10^{+00}$
		35	79	0.26849	0.26843	$1.071 \times 10^{-07}$	$1.044 \times 10^{-10}$	$5.357 \times 10^{-01}$
		35	81	0.26849	0.26843	$1.071 \times 10^{-07}$	$1.044 \times 10^{-10}$	$4.610 \times 10^{-01}$
		37	87	0.42429	0.42417	$3.448 \times 10^{-08}$	$6.944 \times 10^{-11}$	$1.270 \times 10^{-01}$
54		131	3.06295	3.06259	$3.425 \times 10^{-10}$	$9.804 \times 10^{-12}$	$1.883 \times 10^{-02}$	
56		135	3.46760	3.46706	$2.611 \times 10^{-10}$	$8.197 \times 10^{-12}$	$8.660 \times 10^{-03}$	
56		137	3.46760	3.46699	$2.611 \times 10^{-10}$	$8.197 \times 10^{-12}$	$6.920 \times 10^{-03}$	
64		155	5.12988	5.13022	$1.130 \times 10^{-10}$	$3.876 \times 10^{-12}$	$2.277 \times 10^{-02}$	
64		157	5.12988	5.13034	$1.130 \times 10^{-10}$	$3.876 \times 10^{-12}$	$1.304 \times 10^{-02}$	
65		159	5.33938	5.33645	$1.041 \times 10^{-10}$	$3.534 \times 10^{-12}$	$3.155 \times 10^{-04}$	
76		189	7.64172	7.63948	$5.051 \times 10^{-11}$	$1.196 \times 10^{-12}$	$5.872 \times 10^{-04}$	
77		191	7.85115	7.85060	$4.785 \times 10^{-11}$	$1.081 \times 10^{-12}$	$9.806 \times 10^{-03}$	
77		193	7.85115	7.85055	$4.785 \times 10^{-11}$	$1.081 \times 10^{-12}$	$8.310 \times 10^{-03}$	
79		197	8.27049	8.26987	$4.329 \times 10^{-11}$	$8.850 \times 10^{-13}$	$7.729 \times 10^{-03}$	
80		201	8.48051	8.48305	$4.115 \times 10^{-11}$	$7.937 \times 10^{-13}$	$4.730 \times 10^{-04}$	
91		231	10.82070	10.80116	$2.591 \times 10^{-11}$	$2.538 \times 10^{-13}$	$8.872 \times 10^{-06}$	
5/2		30	67	0.04546	0.04546	$1.834 \times 10^{-05}$	$1.371 \times 10^{-09}$	$1.771 \times 10^{+02}$
		37	85	0.42429	0.42425	$3.448 \times 10^{-08}$	$6.944 \times 10^{-11}$	$6.251 \times 10^{-01}$
	40	91	0.72914	0.72920	$9.434 \times 10^{-09}$	$4.348 \times 10^{-11}$	$2.221 \times 10^{-01}$	
	42	95	0.97889	0.97895	$4.695 \times 10^{-09}$	$3.344 \times 10^{-11}$	$2.395 \times 10^{-01}$	
	42	97	0.97889	0.97895	$4.695 \times 10^{-09}$	$3.344 \times 10^{-11}$	$2.297 \times 10^{-01}$	
	44	99	1.26339	1.26345	$2.584 \times 10^{-09}$	$2.646 \times 10^{-11}$	$2.732 \times 10^{-01}$	
	44	101	1.26339	1.26346	$2.584 \times 10^{-09}$	$2.646 \times 10^{-11}$	$2.175 \times 10^{-01}$	
	46	105	1.57892	1.57900	$1.541 \times 10^{-09}$	$2.132 \times 10^{-11}$	$1.597 \times 10^{-01}$	
	51	121	2.47489	2.47413	$5.525 \times 10^{-10}$	$1.302 \times 10^{-11}$	$1.779 \times 10^{-03}$	
	53	127	2.86396	2.86316	$3.984 \times 10^{-10}$	$1.079 \times 10^{-11}$	$1.661 \times 10^{-03}$	
	59	141	4.08585	4.08364	$1.828 \times 10^{-10}$	$6.211 \times 10^{-12}$	$2.274 \times 10^{-04}$	
	63	151	4.92049	4.91793	$1.233 \times 10^{-10}$	$4.274 \times 10^{-12}$	$1.747 \times 10^{-04}$	
	63	153	4.92049	4.91936	$1.233 \times 10^{-10}$	$4.274 \times 10^{-12}$	$8.977 \times 10^{-04}$	
	66	161	5.54887	5.54933	$9.615 \times 10^{-11}$	$3.205 \times 10^{-12}$	$5.667 \times 10^{-03}$	
	66	163	5.54887	5.54823	$9.615 \times 10^{-11}$	$3.205 \times 10^{-12}$	$2.885 \times 10^{-03}$	
	70	173	6.38647	6.38731	$7.246 \times 10^{-11}$	$2.174 \times 10^{-12}$	$1.702 \times 10^{-03}$	

Table 3. *Continued*

I	Z	A	$\Delta E_0$	$\Delta E_{hf}$	$\tau_1$	$\tau_2$	$\tau_0$
5/2	75	185	7.43246	7.42645	$5.348 \times 10^{-11}$	$1.325 \times 10^{-12}$	$3.433 \times 10^{-05}$
	75	187	7.43246	7.42639	$5.348 \times 10^{-11}$	$1.325 \times 10^{-12}$	$3.364 \times 10^{-05}$
	82	205	8.90152	8.89930	$3.745 \times 10^{-11}$	$6.494 \times 10^{-13}$	$2.670 \times 10^{-04}$
	92	233	11.03725	11.03356	$2.506 \times 10^{-11}$	$2.288 \times 10^{-13}$	$1.090 \times 10^{-04}$
7/2	34	79	0.20461	0.20462	$2.139 \times 10^{-07}$	$1.340 \times 10^{-10}$	$5.060 \times 10^{+00}$
	51	123	2.47489	2.47448	$5.525 \times 10^{-10}$	$1.302 \times 10^{-11}$	$3.371 \times 10^{-03}$
	53	129	2.86396	2.86343	$3.984 \times 10^{-10}$	$1.079 \times 10^{-11}$	$2.084 \times 10^{-03}$
	55	133	3.26430	3.26365	$2.976 \times 10^{-10}$	$8.929 \times 10^{-12}$	$1.435 \times 10^{-03}$
	55	135	3.26430	3.26362	$2.976 \times 10^{-10}$	$8.929 \times 10^{-12}$	$1.281 \times 10^{-03}$
	57	139	3.67249	3.67164	$2.304 \times 10^{-10}$	$7.463 \times 10^{-12}$	$8.424 \times 10^{-04}$
	60	143	4.29381	4.29424	$1.645 \times 10^{-10}$	$5.650 \times 10^{-12}$	$3.350 \times 10^{-03}$
	60	145	4.29381	4.29408	$1.645 \times 10^{-10}$	$5.650 \times 10^{-12}$	$8.830 \times 10^{-03}$
	62	147	4.71127	4.71166	$1.351 \times 10^{-10}$	$4.695 \times 10^{-12}$	$4.108 \times 10^{-03}$
	62	149	4.71127	4.71159	$1.351 \times 10^{-10}$	$4.695 \times 10^{-12}$	$6.075 \times 10^{-03}$
	67	163	5.75836	5.75527	$8.929 \times 10^{-11}$	$2.915 \times 10^{-12}$	$6.802 \times 10^{-05}$
	67	165	5.75836	5.75531	$8.929 \times 10^{-11}$	$2.915 \times 10^{-12}$	$6.999 \times 10^{-05}$
	68	167	5.96779	5.96824	$8.264 \times 10^{-11}$	$2.646 \times 10^{-12}$	$3.294 \times 10^{-03}$
	71	175	6.59569	6.59346	$6.757 \times 10^{-11}$	$1.972 \times 10^{-12}$	$1.350 \times 10^{-04}$
	72	177	6.80490	6.80404	$6.369 \times 10^{-11}$	$1.786 \times 10^{-12}$	$9.241 \times 10^{-04}$
	73	181	7.01409	7.01133	$5.988 \times 10^{-11}$	$1.618 \times 10^{-12}$	$8.953 \times 10^{-05}$
	92	235	11.03725	11.03895	$2.506 \times 10^{-11}$	$2.288 \times 10^{-13}$	$2.864 \times 10^{-04}$
9/2	32	73	0.10442	0.10442	$1.298 \times 10^{-06}$	$2.644 \times 10^{-10}$	$2.655 \times 10^{+01}$
	36	83	0.34167	0.34168	$5.882 \times 10^{-08}$	$8.403 \times 10^{-11}$	$2.142 \times 10^{+00}$
	38	87	0.51641	0.51643	$2.146 \times 10^{-08}$	$5.882 \times 10^{-11}$	$7.376 \times 10^{-01}$
	41	93	0.84950	0.84931	$6.536 \times 10^{-09}$	$3.802 \times 10^{-11}$	$8.194 \times 10^{-03}$
	43	99	1.11697	1.11672	$3.448 \times 10^{-09}$	$2.967 \times 10^{-11}$	$5.277 \times 10^{-03}$
	49	113	2.10110	2.10055	$8.000 \times 10^{-10}$	$1.577 \times 10^{-11}$	$1.184 \times 10^{-03}$
	49	115	2.10110	2.10055	$8.000 \times 10^{-10}$	$1.577 \times 10^{-11}$	$1.179 \times 10^{-03}$
	72	179	6.80490	6.80544	$6.369 \times 10^{-11}$	$1.786 \times 10^{-12}$	$1.491 \times 10^{-03}$
	83	209	9.11260	9.10502	$3.584 \times 10^{-11}$	$5.848 \times 10^{-13}$	$8.025 \times 10^{-06}$
	5	57	138	3.67249	3.67170	$2.304 \times 10^{-10}$	$7.463 \times 10^{-12}$
	83	208	9.11260	9.10493	$3.584 \times 10^{-11}$	$5.848 \times 10^{-13}$	$6.433 \times 10^{-06}$
7	67	166	5.75836	5.75705	$8.929 \times 10^{-11}$	$2.915 \times 10^{-12}$	$1.057 \times 10^{-04}$
	71	176	6.59569	6.59411	$6.757 \times 10^{-11}$	$1.972 \times 10^{-12}$	$7.540 \times 10^{-05}$
9	73	180	7.01409	7.01193	$5.988 \times 10^{-11}$	$1.618 \times 10^{-12}$	$2.501 \times 10^{-05}$
	83	210	9.11260	9.11009	$3.584 \times 10^{-11}$	$5.848 \times 10^{-13}$	$2.003 \times 10^{-05}$