

Niching Genetic Algorithms for Optimization in Electromagnetics

I. Fundamentals

B. Sareni, L. Krähenbühl, A. Nicolas
 CEGELY - UPRESA CNRS 5005 - École Centrale de Lyon
 BP 163 - 69131 Ecully Cedex - France.

Abstract—Niching methods extend genetic algorithms and permit the investigation of multiple optimal solutions in the search space. In this paper, we review and discuss various strategies of niching for optimization in electromagnetics. Traditional mathematical problems and an electromagnetic benchmark are solved using niching genetic algorithms to show their interest in real world optimization.

Index terms—Genetic algorithms, niching methods, sharing, crowding, clearing, shape optimization, magnetizer.

I. INTRODUCTION

Real world optimization problems often present multiple optima in the feasible domain. In particular, it has been shown that electromagnetic problems are frequently multimodal [1]-[3]. In this context, global optimization techniques such as evolution strategies [1], genetic algorithms [2][4] or simulated annealing [1][3] have been successfully applied to find a global optimum. However, in case of shape or structural optimization, it could be advantageous to identify multiple optimal profiles by locating global as well as local optima. For that purpose, niching methods extend genetic algorithms by promoting the formation of stable subpopulations in the neighborhood of optimal solutions [5].

Section II presents an overview of niching methods and discusses advantages and drawbacks of each of them when they are applied to real optimization problems. Section III investigates the role of the distance criterion which allows the formation of niches. Section IV presents a comparison of the efficiency of the studied multimodal GAs for standard mathematical test functions. Section V proposes the application of the niching GAs on an electromagnetic benchmark similar to that reported in [4].

II. NICHING GENETIC ALGORITHMS

Genetic Algorithms (GAs) are stochastic optimization methods based on the mechanics of natural evolution and natural genetics [2][4]-[8]. They work with a population of individuals, each representing a feasible solution in the search space. A fitness score (namely the objective function) measures the adaptation of individuals in their environment. For each individual, the set of parameters are coded into a finite-length character string (chromosome). The convergence of the population to a global optimum of the space is obtained by applying sequentially three genetic operators: selection,

crossover and mutation. However, for simple genetic algorithms, all the individuals in the population converge to a single solution representing the global solution of the problem. Niching methods have been developed to minimize the effect of genetic drift resulting from the selection operator in the traditional GA in order to allow the parallel investigation of many solutions in the population. In natural ecosystems, animals compete and survive in many ways (by grazing and hunting for example) and different species evolve to fill each role. A niche can be viewed as an organism task which permits species to survive in their environment. Species are defined as a collection of similar organisms with similar features. For each niche, the physical resources are finite and must be shared among the population of that niche. By analogy, in multimodal GAs, niching methods tend to achieve a natural emergence of niches and species in the search space. A niche is commonly referred to as an optimum of the domain, the fitness representing the resources of that niche. Species can be defined as similar individuals in terms of similarity metrics. An important number of niching GAs have been reported in the literature for example [5]-[9].

A. Fitness Sharing

Fitness sharing modifies the search landscape by reducing the payoff in densely-populated regions. It derates each population element's fitness by an amount nearly equal to the number of similar individuals in the population. Typically, the shared fitness f_i of an individual i with fitness f_i is simply:

$$f_i' = f_i / \sum_{j=1}^N sh(d_{ij}) \quad (1)$$

$$\text{with } sh(d_{ij}) = \begin{cases} 1 - (d_{ij} / \sigma_s)^\alpha & \text{if } d_{ij} < \sigma_s \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where N denotes the population size and d_{ij} represents the distance between the individual i and the individual j . The sharing function (sh) measures the similarity level between two population elements according to a threshold of dissimilarity σ_s (also the distance cutoff or the niche radius). α is a constant parameter which regulates the shape of the sharing function (typically $\alpha=1$). The effect of this scheme is to encourage search in unexplored regions.

B. Crowding Methods

Crowding techniques insert new elements into the population by replacing similar elements. We report two interesting crowding schemes.

Manuscript received November 3, 1997.
 bruno.sareni@ec-lyon.fr, laurent.krähenbühl@ec-lyon.fr,
 http://www.cegely.ec-lyon.fr

1) *Deterministic crowding (DC)*: Mahfoud improved standard crowding of De Jong by introducing competition between children and parents of identical niches [5]. After crossover and eventually mutation, each child replaces the nearest parent if he has an higher fitness. Thus, DC results in two sets of tournaments: (parent 1 against child 1 and parent 2 against child 2) or (parent 1 against child 2 and parent 2 against child 1). The set of tournaments that yields the closest competitions is held.

2) *Restricted Tournament Selection (RTS)*: RTS adapts tournament selection for multimodal optimization [6]. RTS initially selects two elements from the population to undergo crossover and mutation. After recombination, a random sample of w individuals is taken from the population. Following this way, each offspring competes with the closest sample element. The winners are inserted into the population. This procedure is $N/2$ times repeated per generation.

C. Clearing

Clearing is a recent promising multimodal method which has been successfully applied to difficult mathematical problems [7]. It is very similar to sharing but uses the concept of limited resources in the environment. Instead of sharing resources between all individuals of a same niche as in the fitness sharing scheme, clearing attributes them only to the best members of the niche. In practice, the capacity k of a niche specifies the maximum number of elements that this niche can accept. Thus, clearing preserves the fitness of the k best individuals (dominant individuals) of the niche and resets the fitness of the others that belong to the same subpopulation (dominated individuals). As in the sharing method, individuals belong to the same niche (or subpopulation) if their distance in the search space is less than a dissimilarity threshold σ_s (clearing radius). Clearing can be coupled with elitism strategies to preserve the best elements of the niches during the generations.

D. Other niching methods

To give a complete description of niching GAs, we also mention Sequential Niching, Ecological GAs reported in [5] and Immune Systems which have been already applied to solve electromagnetic optimization problems [9].

III. DISTANCE CRITERION

All niching GAs must differentiate similar individuals from dissimilar ones. The similarity metric can be based on either genotype or phenotype similarity. Genotypic similarity is directly linked to bit string representation (binary GAs) and is commonly referred to as the Hamming distance. Phenotypic similarity is related to real parameters of the search space. This can be the Metropolis or Euclidean distances for mathematical problems since all parameters have the same dimension. For real problems, we must use a normalized distance because parameters generally have various physics dimensions. We propose the following distance to

characterize the similarity level between an individual x_1 and an individual x_2 in the domain,

$$d(x_1, x_2) = \max_{i=1..n} \left| \frac{x_{1i} - x_{2i}}{x_{i\max} - x_{i\min}} \right| \quad (3)$$

where n is the number of parameters (also the space dimension), x_{1i} and x_{2i} denote the i th parameter of the individual x_1 and x_2 respectively, $x_{i\min}$ and $x_{i\max}$ being the extreme values of the i th parameter. As it can be seen in (3), this distance represents the maximum deviation of normalized parameters taken in all directions of the space.

Niching GAs can be classed into two different groups: - The first one involves GAs characterized by an explicit neighborhood since they need an explicit distance cutoff to set the dissimilarity threshold (clearing and sharing for example). In that case, we need *a priori* to know how far the optima are. However, for real optimization problems, we have generally no information about the search space and the distribution of the optima until we begin the search. This can be an important drawback and cause these methods to fail if the minimum distance between two optima is not correctly estimated.

- The second one consists of techniques for which the neighborhood is implicit (crowding schemes). In that case, the algorithm requires no information about the search space and can be easily applied to various problems without the previous restrictions.

IV. MATHEMATICAL TESTS

A. Test functions

We consider five multimodal functions of different difficulty [5] displayed in Fig. 1.

B. Performance criteria for niching GAs tests

1) *The Maximum peak ratio*: It is the sum of the fitness of

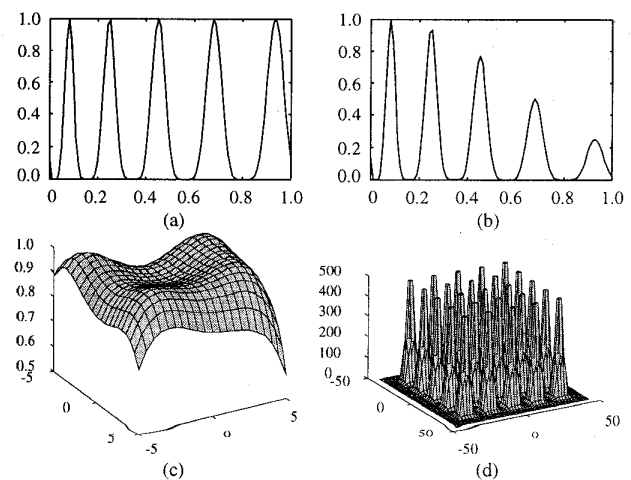


Fig. 1. Test functions (a) $M3$ uniform sine (b) $M4$ nonuniform sine (c) $M5$ Modified Himmelbau's function (d) $M6$ Modified Foxholes function

the local optima identified by the niching technique divided by the sum of the fitness of the actual optima in the search space [9]. An optimum is considered to be detected if it is within a niche radius of the real optimum and if its fitness value is at least 80 % of the real optimum. When an optimum is not identified, the local optimum value is set to zero. Thence, the maximum value for the maximum peak ratio is 1 corresponding to a perfect detection of all optima.

2) *The effective number of maintained peaks:* We also consider the effective number of optima maintained at the end of the search according to the previous definitions.

3) *The chi-square like performance statistic:* It measures the deviation between the population distribution and an ideal proportionally populated distribution (see [9] for more details). It characterizes the ability of the niching technique to proportionally populate the niches of the search space. The smaller the measure, the better the method. An ideal distribution is of value 0.

4) *The number of fitness function evaluations:* In many real applications such as electromagnetic design, the computational cost of objective functions can be high. Therefore, we are interested in assessing the efficiency of niching methods at limited number of function evaluations. Experiments are investigated for 900 fitness function evaluations (30 individuals, 30 generations).

C. Test results

We first examine the efficiency of the niching GAs on functions M3 and M4. Table I shows statistics relative to these problems on an average of ten runs.

TABLE I
NICHING GAS COMPARED - STATISTICS FOR THE FUNCTIONS M3-M4

Niching GA	Nb Peaks maintained		Maximum Peak Ratio		Mean Chi-Square		End Chi-Square	
	F3	F4	F3	F4	F3	F4	F3	F4
Sharing $\sigma_s=0.1$	4.8	4.8	0.935	0.941	2.957	3.461	1.789	2.281
Clearing $\sigma_s=0.1$	5.0	4.8	0.990	0.933	1.045	2.397	0.129	1.477
DC	5.0	4	0.999	0.768	3.537	5.263	3.155	5.264
RTS	4.8	5	0.958	0.998	2.743	3.698	3.763	2.924

Clearing surpassed all other niching GAs by combining a very low chi-square-like deviation with a good detection of the peaks. Crowding schemes were unable to maintain low chi-square-like deviations during the generations but preserved peaks thanks to elitism inherent to these techniques. Sharing worked well on these easy problems but presented difficulty to perfectly detect the optima.

Table II displays statistics relative to the functions M5-M6. All niching GAs performed well on the modified Himmelbau's function but were unable to maintain all optima of the modified Scheckel's foxholes function because of the small population size. Results indicate that it is necessary to have more than one individual per optimum to maintain all optima. Niching GAs with explicit neighborhood surpassed DC and RTS with small window size w . However, it should be noted that the best result for this problem was obtained

with RTS with the maximum window size ($w=N$). The maximum peak ratio and the effective number of maintained peaks were respectively 0.874 and 21.9.

TABLE II
NICHING GAS COMPARED - STATISTICS FOR THE FUNCTIONS M5-M6

Niching GA	M5		M6	
	Nb Peaks maintained	Maximum Peak Ratio	Nb Peaks maintained	Maximum Peak Ratio
Sharing $\sigma_s=0.05$	4	0.986	15.5	0.607
Clearing $\sigma_s=0.05$	4	0.995	16.4	0.655
DC	4	0.992	8.0	0.322
RTS ($w=30\%N$)	4	1.000	7.9	0.318

V. ÜLER'S BENCHMARK

Niching experiments were also carried out on an electromagnetic benchmark similar to that reported in [4]. The magnetizer geometry has been modified to depend on three parameters only. The pole shape consists of a circular arc centered at the point O of coordinates $(0, x_1)$ and joining the point P of coordinates (x_2, x_3) (see Fig. 2).

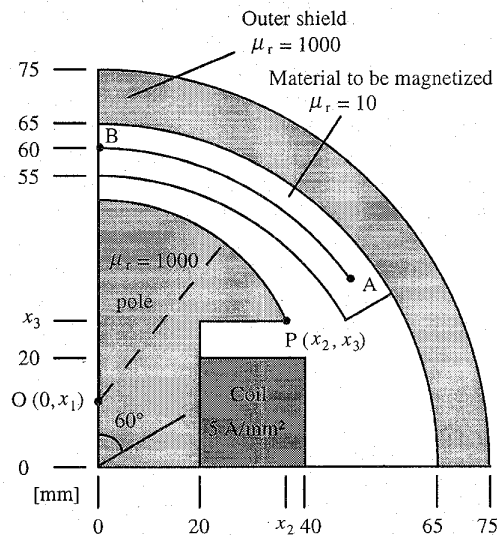


Fig. 2 Geometry of the magnetizer

Table III shows the range of the design variables used in this magnetizer problem. An additional constraint imposes a minimum distance of 55 mm between the point P and the point O to prevent the interception of the pole with the material to be magnetized.

TABLE III
DESIGN VARIABLES RANGES

Design variable	Minimum value	Maximum value
x_1	-5 mm	25 mm
x_2	25 mm	45.9 mm
x_3	25 mm	26.5 mm

Unlike the optimization objective in [4], we want to obtain an uniform magnetic flux density on the chord AB. Moreover, we specify no normative value for the magnetic flux density. In this resulting inverse problem, specifically designed to multimodal optimization, the value of the induction is voluntarily not prescribed so as to allow multiple optimal solutions. The objective function to be minimized can be expressed by (9)

$$f_{obj} = \sqrt{\sum_{i=1}^n B_i^2 - \left(\sum_{i=1}^n B_i\right)^2} / \sum_{i=1}^n B_i \quad (9)$$

where B_i is the magnetic flux density at the point i , $n=50$ being the number of discretized points on the chord AB.

The optimization problem is coupled with a FEM code and solved using the niching GAs presented in the previous sections. We apply the death penalty method when the geometric constraint is violated [10] by rejecting the unfeasible individuals from the population.

Table IV shows a comparison of the niching GAs on the modified Üler's benchmark. A simple GA converged to a single configuration giving an uniform induction level on the chord AB of value 0.208 T. The investigated niching GAs were able to maintain more than one solution. Best results were obtained with crowding schemes. Sharing performed poorly reflecting its difficulty to stabilize its population around the optimal solutions. In fact, sharing detected an important number of quasi uniform induction levels but these solutions were worse than those found by the other GAs and below the quality criteria ($f_{obj} < 0.03$). Clearing was obviously better by identifying multiple close optimal solutions with a small niching radius and a few distinct ones with higher niche radii.

TABLE IV
EFFICIENCY OF THE NICHING GA ON THE ÜLER'S BENCHMARK

Optimization process	Nb of optimal solutions ($f_{obj} < 0.03$)	Induction levels detected (range)	Average objective function
simple GA	1	0.208 T	0.002
Clearing ($\sigma=0.1$ $k=1$)	14	[0.098 T-0.427 T]	0.014
Clearing ($\sigma=0.4$ $k=1$)	4	[0.125 T-0.442 T]	0.009
Sharing ($\sigma=0.1$)	4	[0.103 T-0.150 T]	0.024
DC	20	[0.117 T-0.563T]	0.011
RTS	20	[0.122 T-0.596T]	0.008

Fig. 3 shows examples of uniform induction levels detected with a run of RTS compared to that obtained with a simple GA and that corresponding to a non-optimized configuration.

VI. CONCLUSIONS

Niching methods are robust optimization techniques which allow multiple solutions in multimodal domains to be found. They can be easily coupled with GAs with only a small increase of the computational time resulting from the

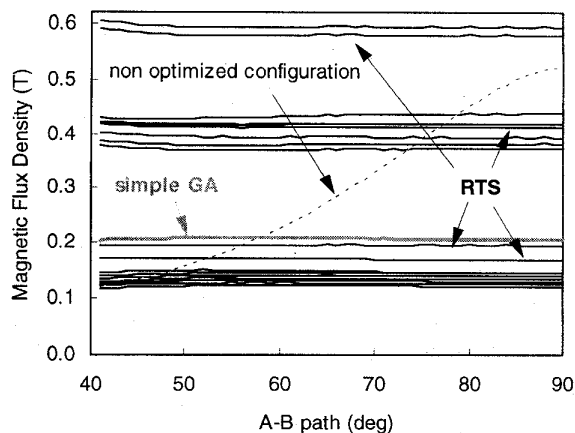


Fig. 3. Examples of uniform induction levels detected by a run of Restricted Tournament Selection (RTS). Optimal solutions are compared with that obtained with a simple GA and a non-optimized configuration

computation of the distances between individuals. Nevertheless, this drawback is minor in relation to the advantages of these methods. The benefit of the detection of distinct optimal solutions is particularly interesting for shape optimization problems and inverse problems for which the uniqueness of the solution is not fulfilled [11].

REFERENCES

- [1] P. Aloto, A.V. Kuntsevitch, Ch. Magele, G. Molinari, (...), "Multiobjective Optimization in Magnetostatics: A Proposal for Benchmark Problems," *IEEE T-Mag.* 32 n°3, pp. 1238-1241, 1996.
- [2] J.A. Vasconcelos, R. R. Saldanha, L. Krähenbühl, A. Nicolas, "Genetic Algorithm Coupled with a Deterministic Method for Optimization in Electromagnetics," *IEEE T-Mag.* 33 n°2, pp. 1860-1863, 1997.
- [3] J.A. Vasconcelos, L. Krähenbühl, A. Nicolas, "Simulated Annealing coupled with the Tabu Search Method for Continuum Optimization in Electromagnetics," *IEEE T-Mag.* 32 n°3, pp. 1206-1209, 1996.
- [4] G.F. Üler, O.A. Mohammed, C. Koh, "Design Optimization of Electrical Machines using Genetic Algorithms", *IEEE T-Mag.* 31 n°3, 1995.
- [5] S.W. Mahfoud, *Niching Methods for Genetic Algorithms*. IlliGAL Report 95001, Univ. of Illinois at Urbana Champaign, I.G.A. Lab., 1995. Available by [ftp://gal4.ge.uiuc.edu/pub/papers/](http://gal4.ge.uiuc.edu/pub/papers/)
- [6] G. Harrik, "Finding multimodal solutions using restricted tournament selection," in L.J. Eshelman (Ed.), *Proceedings of the 6th International Conference on Genetic Algorithms (ICGA6)*, San Francisco, CA: Morgan Kaufmann, pp. 24-31, 1995.
- [7] A. Petrowski, "A Clearing Procedure as a Niching Method for Genetic Algorithms", *IEEE 3rd International Conference on Evolutionary Computation (ICEC'96)*, Nagoya, 1996.
- [8] J.S. Chun, M.K. Kim, H.K. Jung, S.K. Hong, "Shape Optimization of Electromagnetic Devices Using Immune Algorithm," *IEEE T-Mag.* 33 n°2, pp. 1876-1879, 1997.
- [9] B.L. Miller, M.J. Shaw, "Genetic Algorithms with Dynamic Niche Sharing for Multimodal Function Optimization," *IEEE International Conference on Evolutionary Computation*, pp. 786-791, 1996.
- [10] Z. Michalewicz, D. Dasgupta, R. Leriche, M. Schoenauer, "Evolutionary Algorithms for Constrained Engineering Problems," *Computer and Industrial Engineering Journal*, vol. 30, No 2, 1996.
- [11] P. Di Barba, F. Dughiero, F. Trevisan, "Optimization of the Loney's Solenoid through Quasi-Analytical Strategies: a Benchmark Problem Reconsidered," *IEEE T-Mag* 33 n°2, pp. 1864-1867, 1997.