# A Non Linear Model for Surface Conduction

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*Abstract* — **A non linear conducting layer, at the surface of an insulating material, is modeled by a surface with a conductivity which depends on the tangential field at the interface. The model, formulated for 2D and axial symmetric problems, was embedded in a field computation software based on boundary elements method. The non linear equations on the conducting surface are discretised with one dimensional finite elements. The results agree with the solution of the differential equation which governs the potential in a simplified configuration.**

*Index terms* — **Boundary Integral Equation Method (BIEM), electrostatics, surface conduction, finite elements method (FEM), non linear, thin layer.**

## I. INTRODUCTION

In some electrical apparatuses, the presence of a conducting zone modifies the potential distribution. This situation is met, for example, in the pollution of insulators or in electrical bushings [1, 2]. In these cases, the conducting zone has a thin thickness and its discretization with standard methods may lead to numerical problems [3]. In order to avoid these difficulties, surface models are formulated [4-6]. In these studies, the conducting layer is characterized by a constant conductivity and, therefore, has a linear behavior. In this paper, we propose an extension of this model to non linear conducting zone for 2D and axisymmetric problems.

## II. SURFACE MODEL

In the conductive zone, with current density $\mathbf{J}_v$, the conservation of current is expressed as :

$$\mathrm{div}(\mathbf{J}_V + \partial \mathbf{D} / \partial t) = 0 \qquad (1)$$

$\mathbf{D}$ : electric induction [C.m$^{-2}$]
t : time [s]

As the layer has a thin thickness $h$, we introduce the surface current density $\mathbf{J}_s$ as :

$$\mathbf{J}_s = \mathbf{J}_v . h \qquad [\mathrm{A/m}] \qquad (2)$$

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$D_{n1}$ and $D_{n2}$ are the electric normal induction at each side of the conducting interface. We define the surface charge density :

$$\rho_s = D_{n1} + D_{n2} \qquad [\mathrm{C/m^2}] \qquad (3)$$

This allows to rewrite (1) as :

$$\mathrm{div}_s \mathbf{J}_s + \partial \rho_s / \partial t = 0 \qquad (4)$$

The surface current is related to the potential by :

$$\mathbf{J}_s = -\sigma_s . \mathbf{grad} V \qquad [\mathrm{A/m}] \qquad (5)$$

In the case of anti-corona coating (electrical bushings), we have experimentally characterized the conductivity of the layer : it has been found to vary as an exponential function of the modulus of the tangential field ([1], Fig. 1) :

$$\sigma_s = \sigma_0 . \exp(\alpha . E_t) \qquad [\mathrm{S}] \qquad (6)$$

where $\sigma_0$ and $\alpha$ are positive constants. It must be emphasized that it is a very strong non linearity.

| Surf. conductivity | $\sigma_0$[S] | $\alpha$ [m/V] |
|---|---|---|
| C1 - - - - - - | $10^{-9}$ | 0 |
| C2 ........... | $10^{-5}$ | 0 |
| C3 ——— | $10^{-9}$ | $4{,}46.10^{-5}$ |

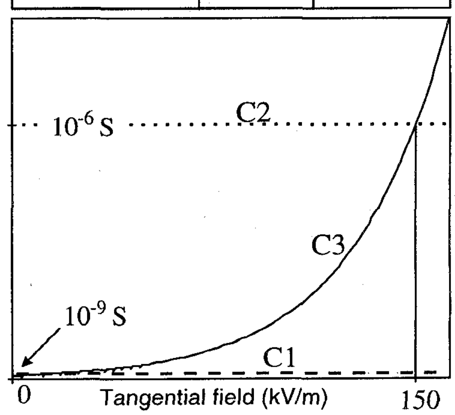


Fig. 1 : Example of linear and non-linear surface conductivities

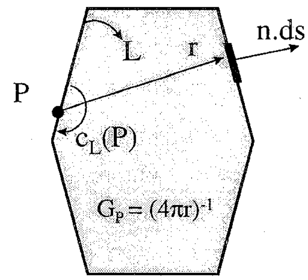


Fig. 2 : BIE formulation for a dielectric area of contour L.

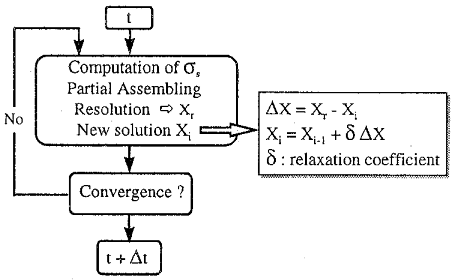


Fig. 3 : Flow chart for the non linear resolution

### III. Resolution

*A. Dielectric domains.* Apart from the conducting zone, the distribution of potential obeys Laplace's equation ; we write the solution in the dielectric areas and in the air under its integral form, using the classical boundary integral equations method (BIEM [3]) :

$$c_L(P).V(P) = \oint_L \left( V \frac{\partial G_P}{\partial n} - G_P \frac{\partial V}{\partial n} \right).dl \qquad (7)$$

where $G_P$ is the Green function and $c_L(P)$ is proportional to the solid angle from P to L (Fig. 2). The interface condition between to dielectric regions is given by (3), without surface charge density :

$$\varepsilon_{r1}.\frac{\partial V}{\partial n_1}\bigg|_{region\ 1} = -\ \varepsilon_{r2}.\frac{\partial V}{\partial n_2}\bigg|_{region\ 2} \qquad (8)$$

The unknowns taken into account with this formulation are the potential and its two normal derivatives on the boundaries. The corresponding equations are 2 boundary integral equations (7) - one for each side of the interface - and the interface equation (8).

*B. Conducting interface.* In the case of a conducting interface, we keep the previous equations and unknowns, but the surface charge density (3) intervenes as an additional unknown. The corresponding equation is the conservation of the surface current (4) :

$$\mathrm{div}_s(\sigma_s \mathbf{grad} V) = \partial\rho_s/\partial t \qquad (9)$$

which is a 1D differential equation if the geometry is 2D or axisymmetric. The finite element method is well suited to solve it (with the boundary mesh used for the BIE), also if $\sigma_s$ depends on the local field (non linear problem) :

- *2D problems :*

$$\oint_L \frac{\partial \rho_s}{\partial t} w.dl + \int_L \sigma_s \nabla V.\nabla w.dl = 0 \qquad (10)$$

- *axisymmetric problems :*

$$\oint_L \frac{\partial \rho_s}{\partial t} w.rdl + \int_L \sigma_s \nabla V.\nabla w.rdl = 0 \qquad (11)$$

where w is the weighting function.

The time derivative is replaced according to the Euler implicit method [1] :

$$\partial\rho_s/\partial t = [\rho_s(t+\Delta t) - \rho_s(t)]/\Delta t \qquad (12)$$

and the time behavior is described with a step by step method.

Equations (7) to (11) are discretized with quadratic finite elements and we get a linear system (time t):

$$A_t.X_t = B_t \qquad (13)$$

where $X_t$ is the unknown vector constituted by the potential V, the normal derivative $\partial V/\partial n$ and the charge density at the next time step $\rho_s(t+\Delta t)$.

Some coefficients of the matrix A depend on the conductivity. Then, in the non linear case, A depends on the solution : so the problem (at a given time step) is solved using a substitution method, with relaxation (Flow chart on Fig. 3).

### IV. Test problem

The test problem is defined in Fig. 4. Given that the thickness a of the dielectric is much less than the length L of the device, the problem is in reality a 1-D problem, and the potential is the solution of [8] :

$$\frac{\varepsilon}{a}\frac{\partial V}{\partial t} - \frac{\partial}{\partial x}(\sigma_s.\frac{\partial V}{\partial x}) = 0 \quad \textit{(2D problem)} \qquad (14)$$

$$\frac{\varepsilon}{a} r \frac{\partial V}{\partial t} - \frac{\partial}{\partial r}(\sigma_s.r\frac{\partial V}{\partial r}) = 0 \quad \textit{(axisym. problem)} \qquad (15)$$

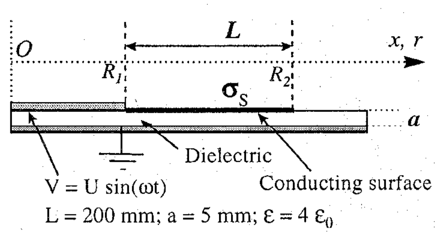


Fig. 4 : Definition of the test problem

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[1] If the conductivity is constant, and if the source potential is harmonic, a simple complex representation is used.

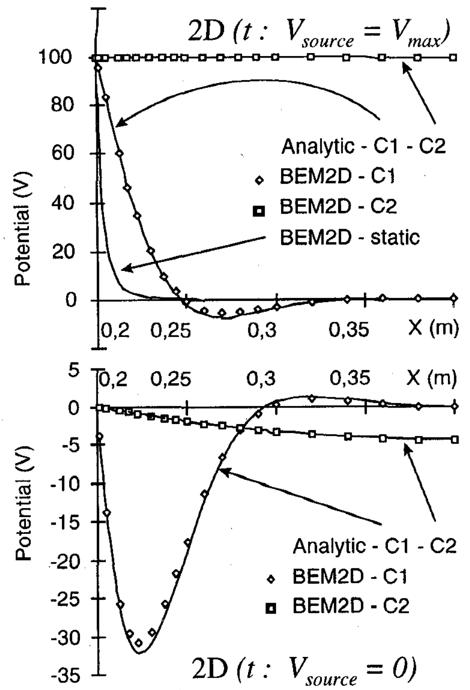


Fig. 5 (a) : Comparison analytic/numeric in the 2D case.

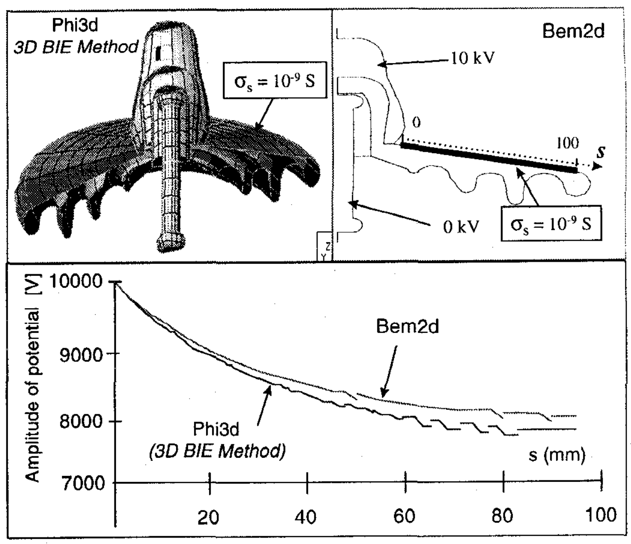


Fig. 6 : Comparison with 3D numerical results in the axisymmetric case.

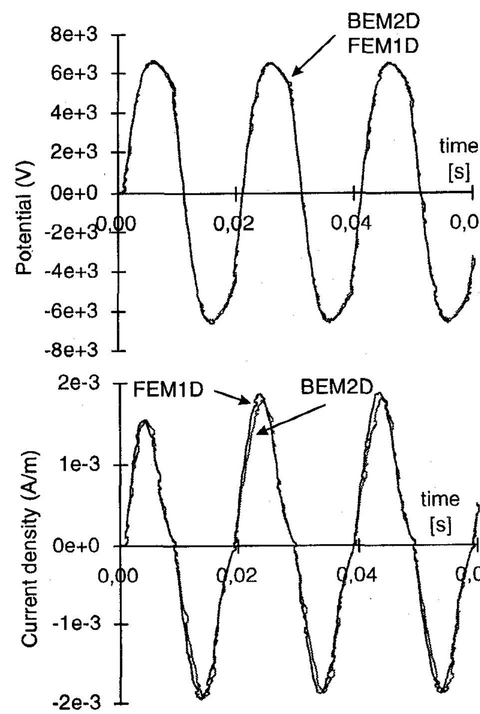


Fig. 7 : Non linear case : comparison 1D/2D.
Example of evolutions in respect of time (non linear 2D case, x=270mm).
*2 or 3 iterations/step; 500 steps/period; > 10 hours CPU (HP 720)*

## V. VALIDATION

To test the 2D and axisymmetric BIEM/FEM step by step solutions, we considered successively the linear case, then the non-linear case.

*A. Linear case.* In the linear case, the harmonic permanent responses for the test problem of Fig. 4 can be analytically expressed :

$$V(x) = U.\cosh[k(1+j)(L-x)]/\cosh[k(1+j)L]$$
$$\text{with : } k^2 = \omega\varepsilon/2a\sigma \qquad \textit{in 2D} \qquad (16)$$
$$V(r) = A_1J_0(kr) + A_2Y_0(kr)$$
$$\text{with } k^2 = j\omega\varepsilon/a\sigma \qquad \textit{in axisym.} \qquad (17)$$

and we can compare them with the numerical results of BEM2D, for the conductivities C1 and C2. The analytic and numerical results cannot be distinguished (Fig. 5).

The axisymmetric model was also tested in comparison with the 3D results obtained with the software Phi3d [4]. Figure 6 presents for example the variation of the electric potential (modulus) along the pollution layer on the surface of an insulator : the results are in good agreement.

*B. Non linear case.* In the non linear case, there is no analytic solution available for the test problem: we have to compare with the numerical solution of (14) or (15) obtained using a 1D FEM step by step method (« FEM1D »). An example of time evolutions (potential, current surface density) is shown on Fig. 7.

The FEM1D and BEM2D solutions are in agreement. A distortion of potential and current shapes appears, due to the non linearity.

## VI. Utility of conducting coating

Conducting layers are intentionally put on the insulation of Cu-bars, on the stator end region of some HV rotating machines, to improve the insulation behavior by preventing corona effects. We can easily understand the influence of the layer with the help of Fig. 8 : the sketch shows how the layer pushes the equipotential lines back, and thus reduces the electric field. We have also computed and plotted the values of the tangential field at the interface (configuration of Fig. 4) without and with the layer : the interest of the conducting coating clearly appears.

Figure 9 compares the behaviors of linear (constant conductivity C1) and non linear (conductivity C3) layers, for low and high voltages, respectively. For the non linear coating, the higher the field gets, the higher the conductivity becomes: the tangential field does not increase proportionally to the source voltage. At 10kV, the non linearity reduces the maximum field by a factor 4.

## Conclusion

We have presented a method, which allows to compute the electric potential and field in 2D insulating structures, in presence of thin layers with a field dependent conductivity. The numerical validation was successfully carried out. However, the cost of the step by step procedure used to solve this time dependent and badly non linear problem is very high : the planned transition to 3D problems will impose to improve it.

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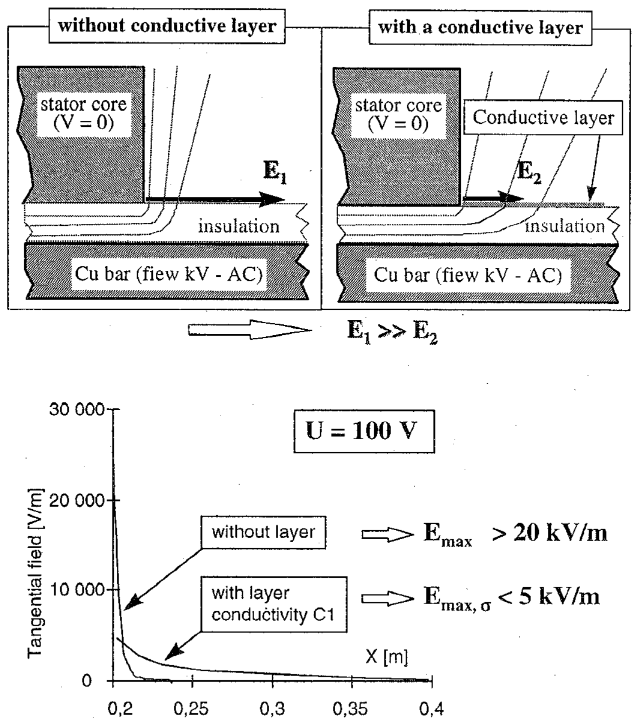


Fig. 8 : Interest of the conductive layer :
Reduction of the maximum electrical stress.

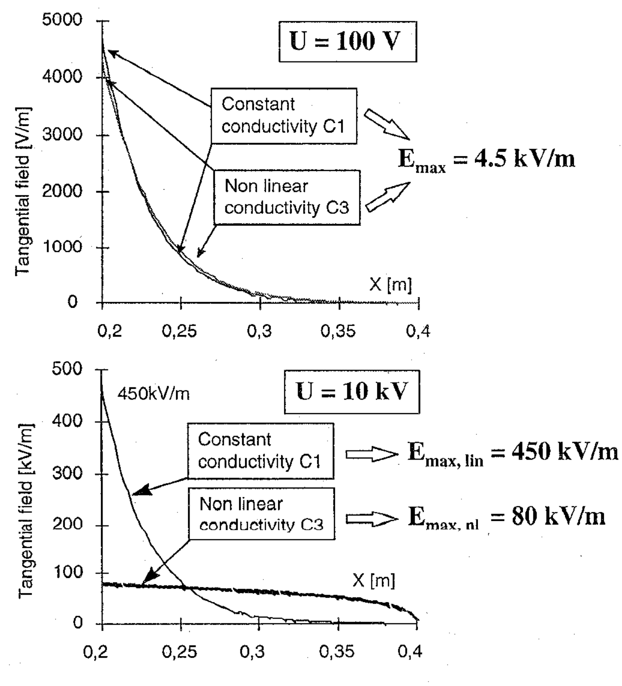


Fig. 9 : Non linear conductive layer :
Homogenization of the electric stress for High Voltage excitation.