

# Modeling of a Non-Linear Conductive Magnetic Circuit

## Part 1 : Definition and Experimental Validation of an Equivalent Problem

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**Abstract**—Dynamic representation model of a magnetic circuit including effects of hysteresis and transients is described. This dynamic modelling is worthwhile regarding time spent on different calculations and it requires only two parameters for the entire simulation. An experimental validation is presented on industrial cases.

### I. INTRODUCTION

The behavior of an electrical circuit involving wound components is mainly modified by the properties of the magnetic materials used for its construction. In addition to the circuit's equations, magnetic formulations have to be performed.

The classical self-inductance and mutual inductance quantities, using linearization techniques, lead to problems in high level signal operations, or in fast transient conditions. We propose circuit representation, an alternative formulation for simple shaped magnetic describing the instantaneous evolution of the mean flux in a cross-section coupled with electrical equations.

The magnetic behavior of a conducting circuit is time dependent. For fast solicitations (power electronics), this effect can be very important.

### II. MODELING TECHNIQUE

An insulating magnetic circuit is fully described by the quasi-static characteristic, with no time dependence thus excitation inputs. For a conductive circuit, we assume that local eddy-currents or dynamical properties can be represented by a lumped fictitious winding of "n" turns, shorted with a resistor "r" on an insulating magnetic circuit with the same magnetic characteristics (Fig. 1). The addition of applied and fictitious ampere-turns verify the quasi-static characteristic.

With these considerations we obtain the instantaneous value of the flux  $\Phi(t)$  as a function of the input ampere-turns  $N \cdot I_p$ .

$$\frac{d\Phi}{dt} = \frac{r}{n^2} \cdot (N \cdot I_p - F^{-1}(\Phi)) \quad (1)$$

The " $n^2/r$ " quantity defined in the equivalent problem is theoretically independent of the inputs.

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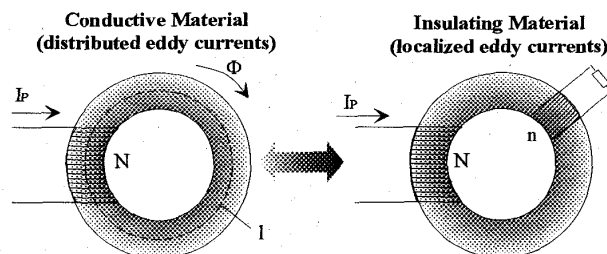


Fig. 1. Model of equivalent problem.

$\Phi = F(N \cdot I)$  or  $N \cdot I = F^{-1}(\Phi)$  represent the quasi-static characteristic.

For this case the magnetic characteristic of the equivalent circuit material is represented by a static curve. If hysteresis effects are not taken into account, this quasi-static characteristic is reduced to the first magnetization curve. To take into account the hysteresis phenomena we substitute this static characteristic by static generator which allows to describe any hysteresis loops (major and minor loops). We use the Preisach-Neel model for its ease of its description of non-symmetrical loops [1], [2]. We use the discrete form of this model, proposed by Biorci and Pescetti [3].

The coupling with the electrical circuit equations can be performed by the use of the quantity  $d\Phi/dt$  available in the simulation.

### III. RESULTS AND DISCUSSION

All these experimentations have been carried out through accurate measures involving the primary current and the integral of the secondary induced voltage. The only data used to elaborate the Preisach - Neel model is the first static magnetization linked to the descending saturation cycle.

The  $n^2/r$  factor can be determined by the first dynamic magnetization cycle of any transient. With these two static and dynamic experiments, the model is fully described for a particular circuit, and does not need an adjustment on any parameter in relation with the inputs.

With this representation, we obtain accurate results for the magnetic behavior  $B(H)$  of a magnetic material, and for the electrical behavior  $V_2(t)$  of the secondary circuit.

Many experiments on materials and different samples have pointed out that the equivalence principle gives good results in harmonic excitations for frequencies up to ten times the nominal frequency.

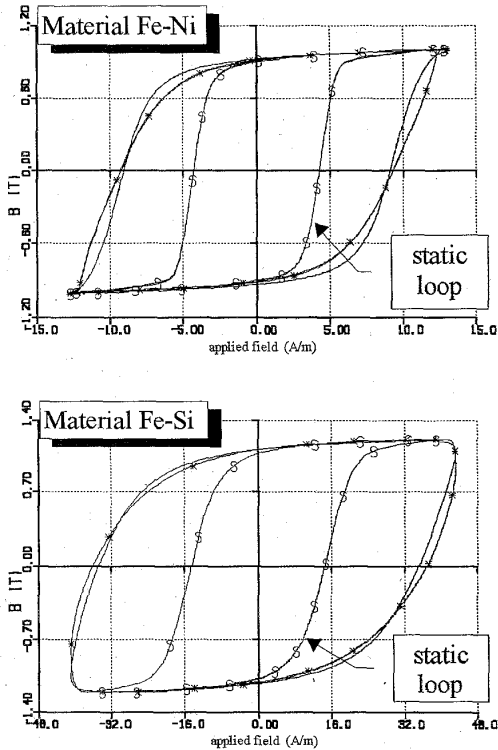


Fig. 2. Dynamical validation for two materials (Fe-Ni and Fe-Si) : induction B vs. applied field; Comparison between measurement (—) and numerical simulation (-\*) for a 50Hz sine current excitation ; the calculated static loop (-s-) is plot as a reference.

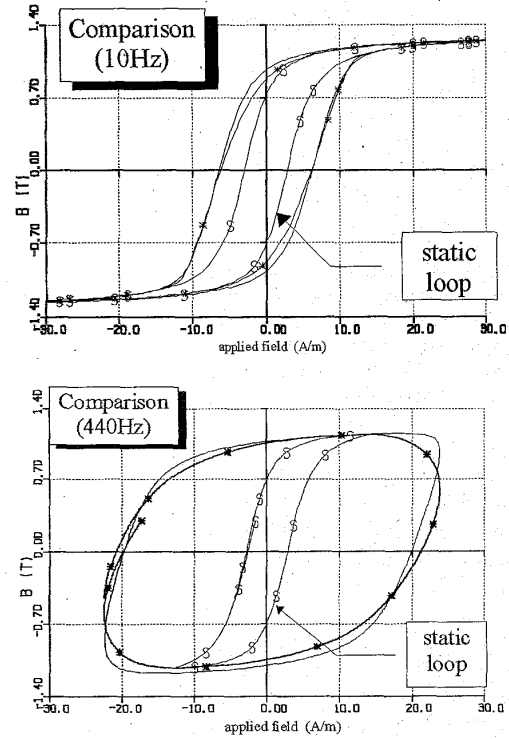


Fig. 3. Dynamical validation for two frequencies (10Hz and 440Hz) : induction B vs. applied field; Comparison between measurement (—) and numerical simulation (-\*) for a sine current excitation; the calculated static loop (-s-) is plot as a reference.

In order to illustrate the validity of the method, we present a comparison between experimental and simulated values of magnetic and electrical variables in several cases :

- for two materials (fig. 2),
- for two sine current excitation frequencies (fig. 3),
- for a transient under a full rectified sine voltage excitation , with a previously demagnetized material (fig. 4).
- The figures 5a, 5b show a coupling of the magnetic model with electrical circuit equations in an industrial application : a current sensor feeds the passive circuit of a fault detector.

All these simulations are performed without parameter adjustment, with fixed values of the  $r/n^2$  coefficient previously identified on a single test for each magnetic circuit.

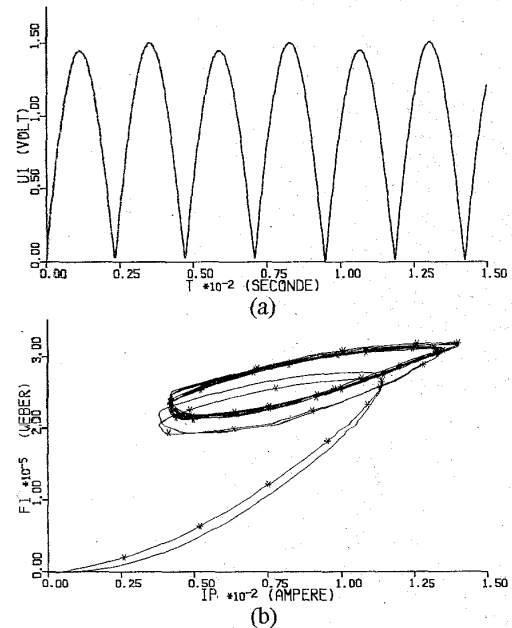


Fig. 4. Transient working : (a) - sine voltage excitation U1 vs. time ; (b) - flux-current comparison between measurement (—) and numerical simulation (-\*)

## IV. CONCLUSION

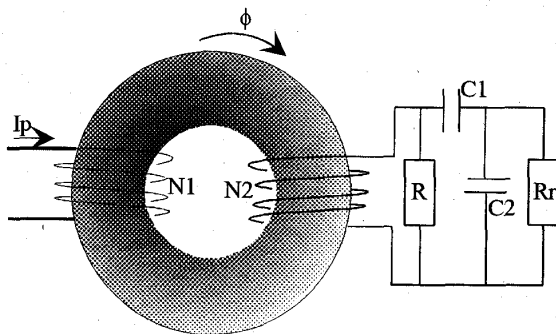


Fig. 5a. Example of magnetic and electric coupling.

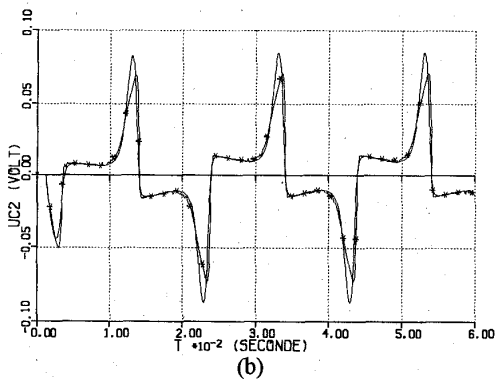
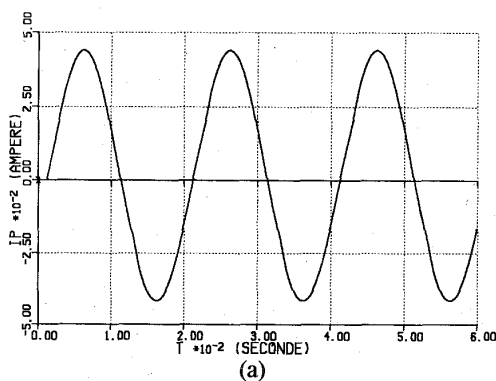


Fig. 5b. (a) - sine current excitation vs. time ; (b) - voltage simulated (---) and mesured (—) across the capacitor C2 (UC2).

The use of a resistor in order to represent eddy currents is classical [5]-[7]. Generally authors characterize the magnetic core without loss, then add a resistor on an equivalent electrical circuit and place it in parallel with the exciting winding. The resistance is determined by using the equivalent mean iron losses of energy. The experiment shows that this value of resistance varies under exciting conditions.

In the model we propose, the current flowing in the fictitious secondary circuit represents iron losses and changes in the stored magnetic energy.

The evaluation of this contribution is performed starting from data which is uncorrelated with the inputs (fictitious circuit components and quasi-static characteristics) in the area where the dynamic local effects can be globalized. The limits of this area have been achieved by experimental evaluations. The experimental results support these theoretical hypothesis and show that with constant parameters it is possible to give a quite precise representation of the dynamic behavior of the magnetic circuit, without any knowledge of the exciting conditions. This last point is particularly important for the description of transient working conditions. The corresponding non linear differential equation has been coupled with the circuit electrical equations in order to take into account easily coupling effects with other physical energies (electrical, mechanical, ...) a bond-graph formulation has been performed. This work is presented a second paper entitled "Modeling of a Non-Linear Conductive Magnetic Circuit Part 2 : Bond Graph formulation".

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