# WAVELET ESTIMATION IN HOMOMORPHIC DOMAIN BY SPECTRAL AVERAGING, FOR DECONVOLUTION OF SEISMIC DATA

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## ABSTRACT

In geophysics, a homomorphic system is used to modelize the convolution of an emitted wavelet (source) with the impulse response of the earth into the sum of the log spectra of the wavelet and the earth's response. If the source function is supposed to be stationary and the earth's response spatially nonstationary, by averaging the log spectra of several random reflection records, the log spectrum of the wavelet will be enhanced and the log spectrum of the earth's response will average out. In this paper, we take an interest in the application of the above method on synthetic seismic data, for estimating the theoretical wavelet spectrum. Then, the wavelet estimate is used to deconvolve the data for obtaining the earth's response.

## 1. INTRODUCTION

The seismic trace $d(t)$ is usually assumed to be the convolution of an emitted wavelet $w(t)$ with the impulse response of the earth $r(t)$. Homomorphic deconvolution was introduced by Oppenheim and Schafer [1], and has been used to deconvolve seismic records [2]: the method consists in separating the wavelet and earth's reflectivity components in homomorphic domain by filtering method.

Another technique was proposed by Otis & Smith [3], for estimating the actual source wavelet and then deconvolving the seismic data. It consists in averaging the log spectra of several reflection records, the log spectrum of the emitted wavelet (supposed stationary) will be enhanced and the log spectrum of the earth's response (nonstationary) will average out. Log spectral averaging has two important advantages: (1) the estimate of the source wavelet will contain phase information concerning the actual source; (2) if the estimate of the source is used to construct a Wiener inverse filter for deconvolving the seismogram, the assumption that the earth's response is a white series is eliminated.

## 2. HOMOMORPHIC SYSTEM AND WAVELET ESTIMATION

A commonly used homomorphic system $T$ is the cascade of (a) the Fourier transform of a time signal $x(t)$,

$$X(f) = F\left[x(t)\right]$$

and (b) the complex logarithm of the spectrum $X(f)$,

$$\log[X(f)] = \log[|X(f)|] + j\arg[X(f)], \qquad (1)$$

If we assume that the seismic reflection $d(t)$ (noiseless) is the convolution of a source $w(t)$ with the impulse response of the earth $r(t)$, the application of the previous steps (a) and (b) gives:

$$\log[D(f)] = \log[W(f)] + \log[R(f)] \qquad (2)$$

The real ($\mathcal{R}_e$) and imaginary ($\mathcal{I}_m$) parts of the resultant relation (2) are given by:

$$\begin{aligned} \mathcal{R}_e &: \quad \log|D(f)| = \log|W(f)| + \log|R(f)| && (3) \\ \mathcal{I}_m &: \quad \arg[D(f)] = \arg[W(f)] + \arg[R(f)] && (4) \end{aligned}$$

Now, let us consider $N_s$ sensors recording different reflections $\{d_s(t)\}_{s=1..N_S}$ to provide the seismic profile (see figure 1). If the source wavelet can be considered stationary and the geological structure considered spatially variable, then the log spectra of each trace (see eqs. (2), (3) and (4)) will be the sum of a constant function corresponding to the source wavelet and a variable function corresponding to the earth's response.

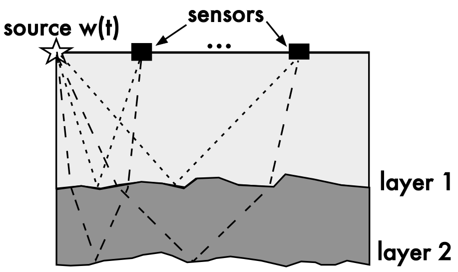


**Fig. 1**. Seismic reflections recorded by different sensors.

Next, we present the Log Spectral Averaging (LSA) method, whose objective is to enhance the unchanging function (source wavelet) and suppressing the changing signals (earth's responses).

### 2.1. Wavelet Estimation by Log Spectral Averaging

With source wavelet stationarity assumption, averaging the real and imaginary parts of log spectra (see eqs. (3) and (4)) gives:

$$\frac{1}{N_s}\sum_{s=1}^{N_s}\log\left(|D_s(f)|\right) = \log(|W(f)|) + \frac{1}{N_s}\sum_{s=1}^{N_s}\log(|R_s(f)|) \quad (5)$$

$$\frac{1}{N_s}\sum_{s=1}^{N_s}\arg[D_s(f)] = \arg[W(f)] + \frac{1}{N_s}\sum_{s=1}^{N_s}\arg[R_s(f)] \quad (6)$$

If for each $f$ the values $\{\log|R_s(f)|\}_{s=1..N_s}$ are random variables that have identical probability distributions with the same mean $K$ and variance $\sigma$, and $\{\arg[R_s(f)]\}_{s=1..N_s}$ are random variables distributed uniformly within a certain interval $[-\phi_1\ ,\ \phi_1]$, then with respect to the central limit theorem, we have:

$$\frac{1}{N_s}\sum_{s=1}^{N_s}\log|R_s(f)| \overset{N_s\to+\infty}{\longrightarrow} K$$

$$\frac{1}{N_s}\sum_{s=1}^{N_s}\arg[R_s(f)] \overset{N_s\to+\infty}{\longrightarrow} 0$$

In other words, the LSA will produce an estimate proportional to the actual wavelet magnitude $|W(f)|$ (see eq. (5)) and an estimate of the actual wavelet phase $\arg[W(f)]$ (see eq. (6)).

While the above estimation approach can be straighforward applied to recovering the amplitude $|W(f)|$, the situation is more complicated when the phases are considered. Indeed, the original data phase $\arg[D(f)]$ is always given as "modulus $2\pi$: $\text{ARG}[D(f)] = (\arg[D(f)])_{2\pi}$ (wrapped phase), i.e. the phase can never be retrieved as a continuous fonction of the frequency $f$, even if it is originally a continuous function [4]. Clearly, the phase wrapping makes the execution of phase averaging of no use. A possible solution to this deficiency is to reconstruct the original phase values via phase unwrapping methods before averaging is applied. Next, two phase unwrapping methods are described and used in our application, for recovering the original data phase and then the wavelet phase.

### 2.2. Phase unwrapping for wavelet phase recovering

#### *2.2.1. Oppenheim's phase unwrapping*

One solution was proposed by Oppenheim [4]: the unwrapped phase is recovered by computing the phase derivative and then integrating it. From eq. (1), and replacing $X(f)$ by $D(f)$, we denote $\hat{D}(f) = \log[D(f)]$ and

$$\begin{aligned}\hat{D}_R(f) &= \log[|D(f)|]\\ \hat{D}_I(f) &= \arg[D(f)]\end{aligned}$$

the real and imaginary parts of $\hat{D}(f)$ respectively. Referring to Oppenheim [4], we have:

$$\frac{d}{df}\arg[D(f)] = \frac{d}{df}\hat{D}_I(f) = \frac{D_I'(f)D_R(f) - D_I(f)D_R'(f)}{D_R(f)^2 + D_I(f)^2} \quad (7)$$

where $D_R, D_I$ are the real and imaginary parts of the data $D(f)$ and $D_R', D_I'$ their corresponding derivatives with respect to frequency. Once, the phase derivative is obtained, one can recover the unwrapped phase by integrating it:

$$\arg[D(f)] = \int_0^f \arg'[D(\xi)]d\xi \quad (8)$$

assuming $\arg[D(f)]\Big|_{f=0} = \arg\left[\sum_{n=-\infty}^{+\infty} d(n)\right] = 0.$

#### *2.2.2. Phase unwrapping by Matlab routine*

The Matlab phase unwrapping consists in correcting the radian phase angles in the wrapped phase $\text{ARG}[D(f)]$ by adding multiples of $2\pi$ when absolute jumps between consecutive elements of $\text{ARG}[D(f)]$ are greater than the jump tolerance of $\pi$ radians.

#### *2.2.3. Deramping the unwrapped phase*

In this paragraph, we show that usual conditions imposed to the unwrapped data phase $\arg[D(f)]$ lead us to suppress the linear component of the phase [5]. To do so, let us write the discrete Fourier transform as:

$$D(\omega) = \sum_{n=-\infty}^{+\infty} d(n)\exp\left[-j\omega n\right] \quad (9)$$

Due to the fact that $D^*(-\omega) = D(\omega)$, the phase must be an odd function:

$$-\arg[D(-\omega)] = \arg[D(\omega)] \quad (10)$$

Moreover, $\arg[D(\omega)]$ must be $2\pi$-periodic:

$$\arg[D(\omega_1)] = \arg[D(\omega_1 + 2\pi)]\ ,\ \forall\omega_1 \quad (11)$$

Hence, following eqs. (10) and (11), we must have in particular:

$$\arg[D(\omega)]\Big|_{\omega=-\pi} = \arg[D(\omega)]\Big|_{\omega=\pi} = 0 \quad (12)$$

Since

$$\arg[D(\omega)]\Big|_{\omega=\pi} = \int_0^\pi \arg'[D(\omega)]d\omega \quad (13)$$

we conclude with eq.(12) that the phase derivative must be a zero mean function. So, we must normalize the phase derivative as follows:

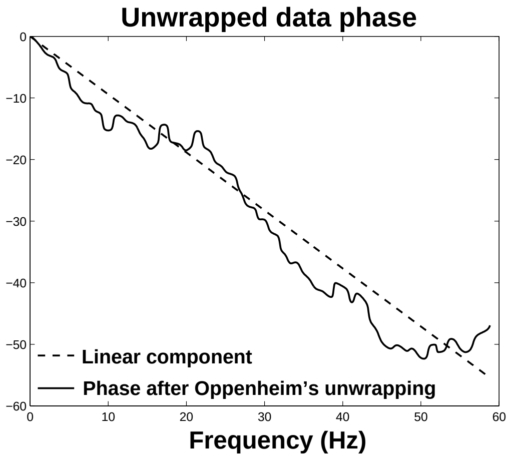
$$\overline{\arg'[D(\omega)]} = \arg'[D(\omega)] - r_d \quad (14)$$

with $r_d = \dfrac{1}{\pi}\displaystyle\int_0^\pi \arg'[D(\omega)]d\omega$.

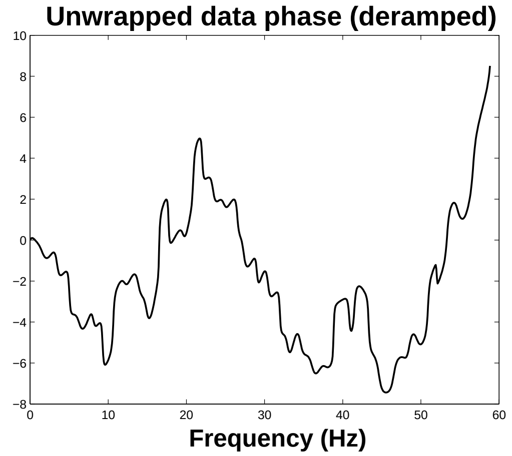
The normalization in eq.(14) is equivalent to suppress the linear component in the unwrappped phase:

$$\overline{\arg[D(\omega)]} = \arg[D(\omega)] - r_d\omega \tag{15}$$

The unwrapping and deramping of the data phase is hence required before applying the phase averaging in eq. (6). Figure 2 shows an illustration of the deramping process: figure 2(a) represents the unwrapped phase of a data and the corresponding linear component superimposed on the phase; figure 2(b) shows the deramped data phase.



(a) Phase unwrapped by Oppenheim's method and corresponding linear component.



(b) Data phase obtained by suppressing the linear component.

**Fig. 2**. Unwrapping then deramping the data phase.

## 3. SIMULATIONS AND WAVELET ESTIMATION RESULTS

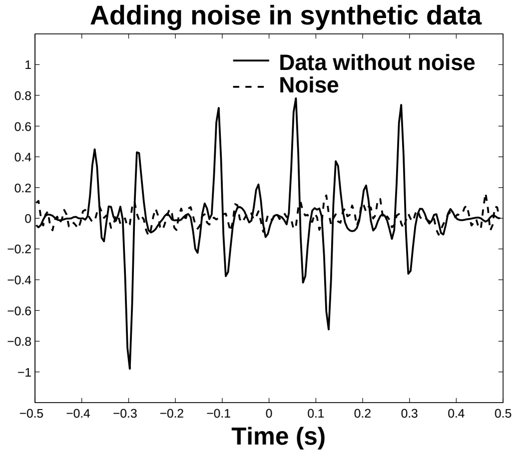
For wavelet estimation, we have considered $N_s$ highly uncorrelated noisy traces $\{d_s(t)\}_{j=1..N_s}$ of $N_t$ time samples ($N_s = 750, N_t = 199$). Each trace is expressed as $d_s(t) = [w * r_s](t) + b_s(t)$ where $w(t)$, $r_s(t)$ and $b_s(t)$ are the actual emitted wavelet, the earth response and a zero-mean gaussian noise with variance $\sigma_b^2 = 5\ 10^{-3}$. The adding noise $b_s(t)$ superimposed to the noiseless data, is highlighted in figure 3(a) in the case of one trace, the resulting noisy trace is represented in figure 3(b). The first ten highly uncorrelated noisy traces are shown in figure 3(c). The previous synthetic case can be obtained from real highly correlated data of size $(M_s \times M_t)$ with $M_s < N_s, M_t > N_t$, by picking up $N_s$ traces of $N_t$ time samples randomly in the real data array.

Figure 4(a) illustrates the wavelet magnitude estimate obtained by the above averaging method: the estimate is very close to the theoretical wavelet magnitude $|W(f)|$ until about the frequency $f = 40$ Hz, and deviates slightly beyond it. From the previous estimate, we have calculated the DSP that allows to incorporate a bandpass filtering on the data for recovering the wavelet phase. The cut-off frequencies of the passband filter are fixed at $\Delta_{dB} = -35$ dB from the maximum of the DSP. Figure 4(b) illustrates the wavelet phase estimates obtained by the above averaging method, after the phase unwrapping methods and deramping: (1) globally, the phase estimate is better after Oppenheim's phase unwrapping than the one after Matlab phase unwrapping, (2) the estimate values are inconsistent from frequency $f = 30$ Hz: at this frequency, the signal energy decreases by about 6 dB. Figure 4(c) shows the temporal wavelet estimates resulting from the frequency wavelet estimation in figure 4(b): we can notice that the wavelet estimate obtained after Oppenheim's phase unwrapping is better than the one after Matlab phase unwrapping, especially around the time $t = 0.02$ s: this is due to a better wavelet phase estimate obtained after Oppenheim's phase unwrapping (see figure 4(b)).

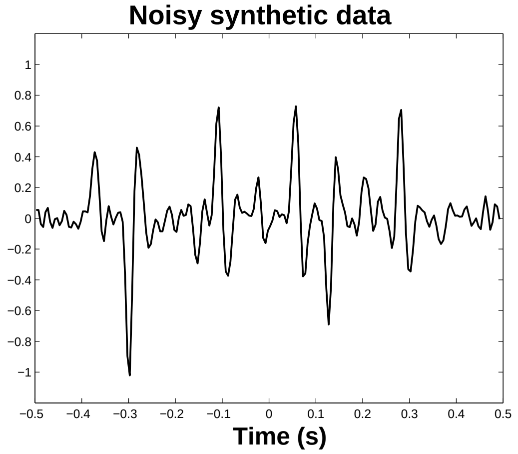
Figure 5(a) represents a trace whereas the figures 5(b) and (c) illustrate estimates of the earth's response obtained by deconvolving the data in figure 5(a), with the Wiener inverse filter $G(f) = \dfrac{\hat{W}^*(f)}{|\hat{W}(f)|^2 + f_n}$, where $\hat{W}(f)$ is the wavelet estimate obtained by LSA, $f_n$ is a noise factor whose value that we have chosen is $f_n = 10^{-3}\left[|\hat{W}(f)|^2\right]_{\max}$. Comparing with the actual earth response in figure 6, we can notice that the earth response reconstruction is better in figure 5(c) (after Oppenheim's phase unwrapping) than in figure 5(b) (after Matlab phase unwrapping): indeed, around the times $t = \{-0.3, -0.1, 0.06, 0.14, 0.3\}$, we can observe that the secondary peaks are weaker in figure 5(c) than in figure 5(b).

## 4. CONCLUSION

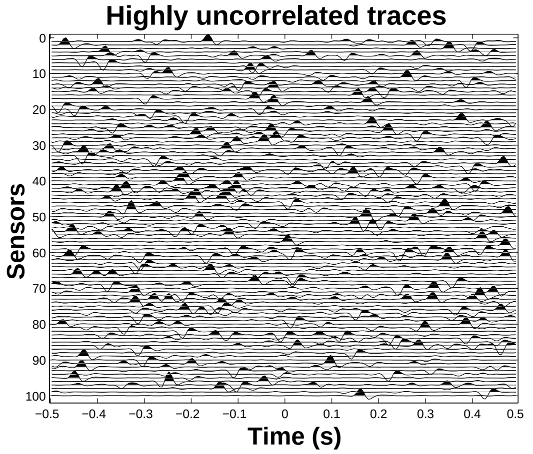
Log Spectral Averaging is a useful tool for estimating the source wavelet. The method is based on stationary wavelet and spatially variable earth's response assumptions. An important advantage of Log Spectral Averaging is that it is not restricted to minimum-phase wavelets since the method can estimate the magnitude and the phase of the actual wavelet. For synthetic noisy data recorded by a number of sensors large enough, we have shown that the Log Spectral Averag-



(a) Noiseless data and noise superimposed.

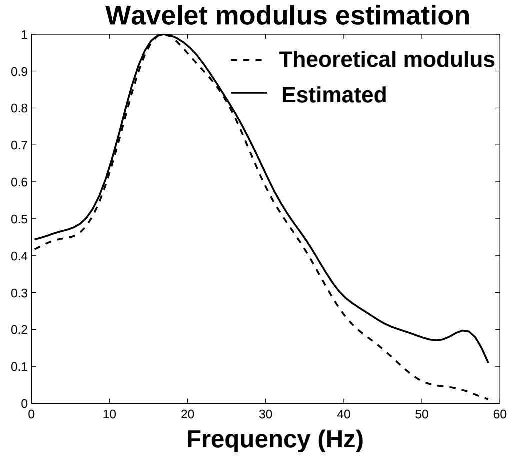


(b) Signal resulting of the addition of the noiseless data and noise in figure (a).

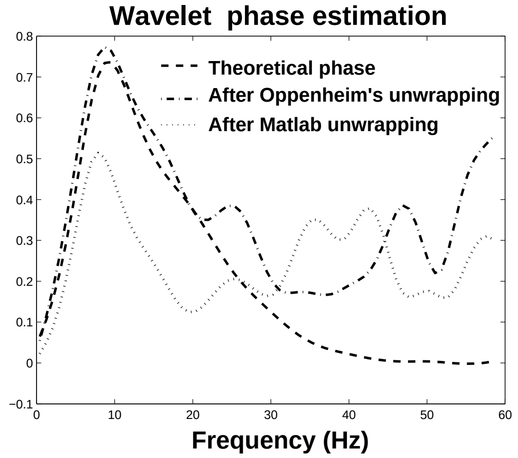


(c) Highly uncorrelated noisy data.

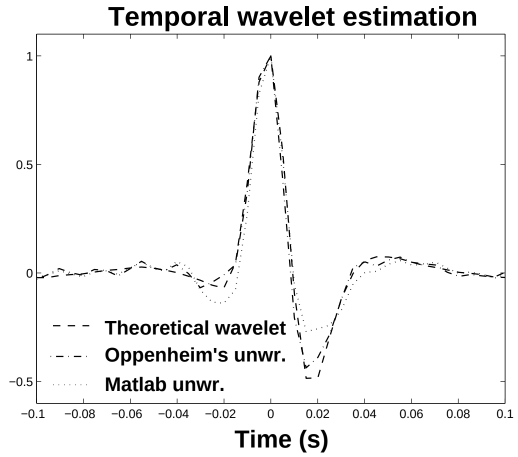
**Fig. 3**. Synthesis of highly uncorrelated noisy data for wavelet estimation and earth response reconstruction.



(a) Estimation of the wavelet modulus.

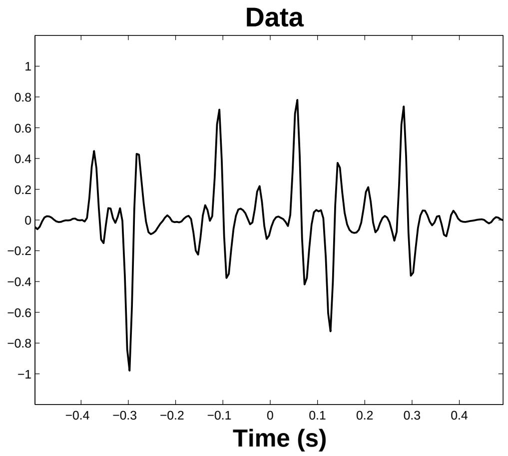


(b) Estimation of the wavelet phase after Matlab unwrapping routine and Oppenheim's unwrapping method.

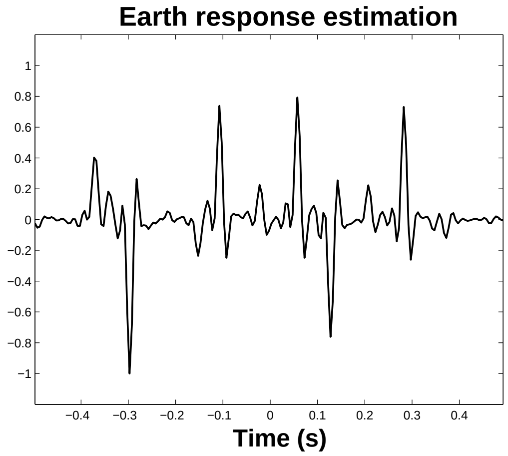


(c) Temporal wavelet estimates obtained from modulus and phase estimates in (a) and (b).

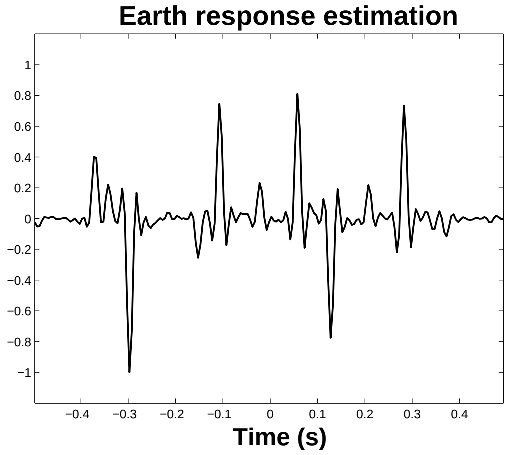
**Fig. 4**. Estimation of the modulus and the phase of the wavelet by averaging method.



(a) Data to be deconvolved.



(b) Earth response estimate obtained from wavelet spectrum estimation in figure 4 (after Matlab unwrapping method).



(c) Earth response estimate obtained from wavelet spectrum estimation in figure 4 (after Oppenheim's unwrapping method).

**Fig. 5**. Earth response reconstruction by Wiener filtering.

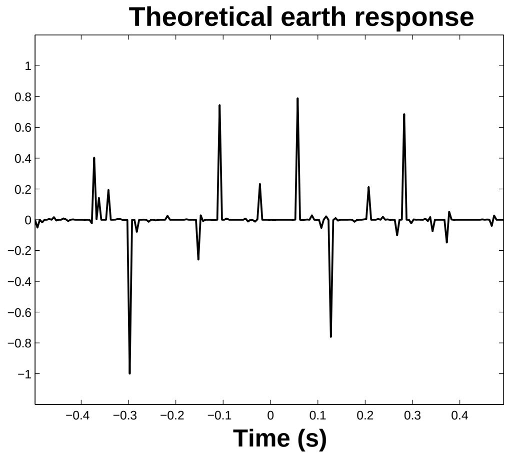


**Fig. 6**. Theoretical earth response corresponding to figures 5(b) and (c).

ing provides a good wavelet magnitude estimate and it can evaluate well enough the actual wavelet phase if the phase of the data is unwrapped and then deramped. Nevertheless, the quality of the wavelet phase estimation seems to depend on the choice of the phase unwrapping method: in particular, the wavelet phase estimation can be improved by chosing a more efficient phase unwrapping method.

Once the wavelet estimate is available, it is used to construct an inverse filter for deconvolving the data and obtaining the earth's response. Performances of the above method have to be deeply evaluated in function of Signal-to-Noise Ratio. In addition, for real data it will be useful in terms of computation time, to incorporate a bandpass filtering (filter determined when estimating the wavelet) and deconvolution into one operation.

## 5. ACKNOWLEDGMENT

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## 6. REFERENCES

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