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Rejection of unknown multiple narrow band disturbances - A direct adaptive control approach

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Abstract—This paper presents a methodology for the rejection of multiple narrow band disturbances using a direct adaptive feedback control approach and without requiring the use of an additional transducer. The direct adaptive control scheme is based on the internal model principle and the use of the Youla-Kucera parametrization of the controller which allows a direct updating of the internal model in the controller. The performance of the methodology is evaluated on the Bodson model for the case of simultaneous application of several unknown time varying sinusoidal disturbances.

Index Terms—direct adaptive control, internal model principle, Youla-Kucera parametrization, adaptive disturbance rejection, multiple narrow band disturbances

I. INTRODUCTION

One of the basic problems in control is the attenuation (rejection) of unknown disturbances without measuring them. The common framework is the assumption that the disturbance is the result of a white noise or a Dirac impulse passed through the "model of the disturbance". While in general one can assume a certain structure for such "model of disturbance", its parameters are unknown and may be time varying. This will require to use an adaptive approach. To be more specific, the disturbances considered are multiple narrow band or sinusoidal disturbances. These disturbances belong in fact to the class of "finite band disturbances". Furthermore for robustness reasons the disturbances should be located in the frequency domain within the regions where the plant has enough gain (see explanation in section III).

Solutions for this problem, provided that an "image" of the disturbance can be obtained by using an additional transducer, have been proposed by the signal processing community and a number of applications are reported ([9], [10], [5], [12]). However, these solutions (inspired by Widrow's technique for adaptive noise cancellation ([25])) ignore the possibilities offered by feedback control systems and require an additional transducer. The basic idea is that a "well located" transducer can provide a measurement, highly correlated with the unknown disturbance. This information is applied to the control input of the plant through an adaptive filter (in general a Finite Impulse Response - FIR) whose parameters are adapted such that the effect of the disturbance upon the output is minimized. The disadvantages of this approach are:

- It requires the use of an additional transducer.
- Difficult choice for the location of this transducer (it is probably the main disadvantage).
- It requires the adaptation of many parameters.

To achieve the rejection of the disturbance (at least asymptotically) without measuring it, a *feedback solution* can be considered. As mentioned earlier, the common framework is the assumption that the disturbance is the result of a white noise or a Dirac impulse passed through the "model of the disturbance"¹. The present paper will focus on the case where the plant model is known but the disturbance model is unknown, since this is the situation encountered in many applications. Among the various approaches considered for solving this problem, the following ones may be mentioned:

- 1) Use of the internal model principle ([11], [15], [4], [23], [24], [1], [2], [13], [14], [16]).
- 2) Use of an observer for the disturbance ([22], [8]).
- 3) Use of the "phase-locked" loop structure considered in communication systems ([7], [6]).

Of course, since the parameters of the disturbance model are unknown, all these approaches lead to an adaptive implementation which can be of *direct* or *indirect* type.

From the user point of view and taking into account the type of operation of existing adaptive disturbance compensation systems one has to consider two modes of operation of the adaptive schemes:

- *Self-tuning* operation (the adaptation procedure starts either on demand or when the performance is unsatisfactory and the current controller is frozen during the estimation/computation of the new controller parameters).
- *Adaptive* operation (the adaptation is performed continuously and the controller is updated at each sampling).

This paper explores the use of the internal model principle for the rejection of unknown time-varying multiple narrow band disturbances with the aim to obtain a direct adaptive control scheme.

Using the internal model principle, the controller should incorporate the model of the disturbance ([11], [15], [4], [23]). Therefore the rejection of unknown disturbances raises the problem of adapting the internal model of the controller and its re-design in real-time.

One way for solving this problem is to try to estimate in real time the model of the disturbance and re-compute the controller, which will incorporate the estimated model of the disturbance (as a pre-specified element of the controller). While the disturbance is unknown and its model needs to be

¹Throughout the paper it is assumed that the order of the disturbance model is known but the parameters of the model are unknown (the order can be estimated from data if necessary).

Fig. 1. Direct adaptive control scheme for rejection of unknown disturbances

estimated, one assumes that the model of the plant is known (obtained for example by identification). The estimation of the disturbance model can be done by using standard parameter estimation algorithms (see for example [19], [21]). This will lead to an indirect adaptive control scheme. The time consuming part of this approach is the redesign of the controller at each sampling time. The reason is that in many applications the plant model can be of very high dimension and despite that this model is constant, one has to re-compute the controller because a new internal model should be considered. This approach has been investigated in [7], [13], [14].

However, by considering the Youla-Kucera parametrization of the controller (known also as the Q -parametrization), it is possible to insert and adjust the internal model in the controller by adjusting the parameters of the Q polynomial (see figure 1). It comes out that in the presence of unknown disturbances it is possible to build a direct adaptive control scheme where the parameters of the Q polynomial are directly adapted in order to have the desired internal model without recomputing the controller (polynomials R_0 and S_0 in figure 1 remain unchanged). The number of the controller parameters to be directly adapted is roughly equal to the number of parameters of the denominator of the disturbance model. In other words, the size of the adaptation algorithm will depend upon the complexity of the disturbance model.

This paper focuses on the direct feedback adaptive control for the case of unknown and time-varying multiple narrow band disturbances. The direct adaptive control scheme to be presented ([16]) takes advantage of the Youla-Kucera parametrization for the computation of the controller. A related approach has been considered in [24] for an application to a chemical reactor but a theoretical analysis of the scheme is not provided. A related paper is also [1], [2] where the application field is the active noise control in an acoustic duct. However the case of rejection of unknown multiple narrow band disturbances has not been discussed in any of the above mentioned references.

The paper is organized as follows. Section II is dedicated to a brief review of the plant, disturbance and controller representation as well as of the Internal Model Principle. Some robustness issues are addressed in section III. The

direct adaptive control schemes for disturbance rejection is presented in section IV. Simulation results are presented in section V. Some concluding remarks are given in section VI.

II. PLANT REPRESENTATION AND CONTROLLER STRUCTURE

The structure of a linear time invariant discrete time model of the plant (used for controller design) is:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^*(z^{-1})}{A(z^{-1})}, \quad (1)$$

with:

$$\begin{aligned} d &= \text{the plant pure time delay in number of sampling periods;} \\ A &= 1 + a_1 z^{-1} + \dots + a_{n_A} z^{-n_A}; \\ B &= b_1 z^{-1} + \dots + b_{n_B} z^{-n_B} = q^{-1} B^*; \\ B^* &= b_1 + \dots + b_{n_B} z^{-n_B+1}, \end{aligned}$$

where $A(z^{-1})$, $B(z^{-1})$, $B^*(z^{-1})$ are polynomials in the complex variable z^{-1} and n_A , n_B and $n_B - 1$ represent their orders². The model of the plant may be obtained by system identification. Details on system identification of the plant models considered in this paper can be found in [20], [17].

Since in this paper we are focused on regulation, the controller to be designed is a RS-type polynomial controller ([18], [20]) - see also figure 1.

The output of the plant $y(t)$ and the input $u(t)$ may be written as:

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p_1(t); \quad (2)$$

$$S(q^{-1}) \cdot u(t) = -R(q^{-1}) \cdot y(t), \quad (3)$$

where q^{-1} is the delay (shift) operator ($x(t) = q^{-1}x(t+1)$) and $p_1(t)$ is the resulting additive disturbance on the output of the system. $R(z^{-1})$ and $S(z^{-1})$ are polynomials in z^{-1} having the orders n_R and n_S , respectively, with the following expressions:

$$R(z^{-1}) = r_0 + r_1 z^{-1} + \dots + r_{n_R} z^{-n_R} = R'(z^{-1}) \cdot H_R(z^{-1}) \quad (4)$$

$$S(z^{-1}) = 1 + s_1 z^{-1} + \dots + s_{n_S} z^{-n_S} = S'(z^{-1}) \cdot H_S(z^{-1}) \quad (5)$$

where H_R and H_S are pre-specified parts of the controller (used for example to incorporate the internal model of a disturbance or to open the loop at certain frequencies).

We define the following sensitivity functions:

- Output sensitivity function (the transfer function between the disturbance $p_1(t)$ and the output of the system $y(t)$):

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{P(z^{-1})}; \quad (6)$$

- Input sensitivity function (the transfer function between the disturbance $p_1(t)$ and the input of the system $u(t)$):

$$S_{up}(z^{-1}) = -\frac{A(z^{-1})R(z^{-1})}{P(z^{-1})}, \quad (7)$$

²The complex variable z^{-1} will be used for characterizing the system's behavior in the frequency domain and the delay operator q^{-1} will be used for describing the system's behavior in the time domain.

where

$$\begin{aligned} P(z^{-1}) &= A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \\ &= A(z^{-1})S'(z^{-1}) \cdot H_S(z^{-1}) \\ &\quad + z^{-d}B(z^{-1})R'(z^{-1}) \cdot H_R(z^{-1}) \end{aligned} \quad (8)$$

defines the poles of the closed loop. In pole placement design, $P(z^{-1})$ is the polynomial specifying the desired closed loop poles and the controller polynomials $R(z^{-1})$ and $S(z^{-1})$ are minimal degree solutions of (8) where the degrees of P , R and S are given by: $n_P \leq n_A + n_B + d - 1$, $n_S = n_B + d - 1$ and $n_R = n_A - 1$. Using the equations (2) and (3), one can write the output of the system as:

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p_1(t) = S_{yp}(q^{-1}) \cdot p_1(t) . \quad (9)$$

For more details on RS-type controllers and sensitivity functions see [20].

Suppose that $p_1(t)$ is a deterministic disturbance, so it can be written as

$$p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) , \quad (10)$$

where $\delta(t)$ is a Dirac impulse and $N_p(z^{-1})$, $D_p(z^{-1})$ are coprime polynomials in z^{-1} , of degrees n_{N_p} and n_{D_p} , respectively. In the case of stationary disturbances the roots of $D_p(z^{-1})$ are on the unit circle. The energy of the disturbance is essentially represented by D_p . The contribution of the terms of N_p is weak compared to the effect of D_p , so one can neglect the effect of N_p .

Internal Model Principle: *The effect of the disturbance given in (10) upon the output:*

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) , \quad (11)$$

where $D_p(z^{-1})$ is a polynomial with roots on the unit circle and $P(z^{-1})$ is an asymptotically stable polynomial, converges asymptotically towards zero if and only if the polynomial $S(z^{-1})$ in the RS controller has the form:

$$S(z^{-1}) = D_p(z^{-1})S'(z^{-1}) . \quad (12)$$

In other terms, the pre-specified part of $S(z^{-1})$ should be chosen as $H_S(z^{-1}) = D_p(z^{-1})$ and the controller is computed using (8), where P , D_p , A , B , H_R and d are given³.

Using the Youla-Kucera parametrization (Q-parametrization) of all stable controllers ([3], [23]), the controller polynomials $R(z^{-1})$ and $S(z^{-1})$ get the form:

$$R(z^{-1}) = R_0(z^{-1}) + A(z^{-1})Q(z^{-1}) ; \quad (13)$$

$$S(z^{-1}) = S_0(z^{-1}) - z^{-d}B(z^{-1})Q(z^{-1}) . \quad (14)$$

The (central) controller (R_0, S_0) can be computed by poles placement (but any other design technique can be used). Given the plant model (A, B, d) and the desired closed-loop poles P one has to solve:

$$P(z^{-1}) = A(z^{-1})S_0(z^{-1}) + z^{-d}B(z^{-1})R_0(z^{-1}) . \quad (15)$$

Equations (13) and (14) characterize the set of all stabilizable controllers assigning the closed loop poles as defined by $P(z^{-1})$ (it can be verified by simple calculations that the poles of the closed loop remain unchanged). For the purpose of this paper $Q(z^{-1})$ is considered to be a polynomial of the form:

$$Q(z^{-1}) = q_0 + q_1z^{-1} + \dots + q_{n_Q}z^{-n_Q} . \quad (16)$$

To compute $Q(z^{-1})$ in order that the controller incorporates the internal model of the disturbance one has to solve the diophantine equation:

$$S'(z^{-1})D_p(z^{-1}) + z^{-d}B(z^{-1})Q(z^{-1}) = S_0(z^{-1}) , \quad (17)$$

where $D_p(z^{-1})$, d , $B(z^{-1})$ and $S_0(z^{-1})$ are known and $S'(z^{-1})$ and $Q(z^{-1})$ are unknown. Equation (17) has a unique solution for $S'(z^{-1})$ et $Q(z^{-1})$ with: $n_{S_0} \leq n_{D_p} + n_B + d - 1$, $n_{S'} = n_B + d - 1$, $n_Q = n_{D_p} - 1$. One sees that the order n_Q of the polynomial Q depends upon the structure of the disturbance model.

III. ROBUSTNESS CONSIDERATIONS

As it is well known, the introduction of the internal model for the perfect rejection of the disturbance (asymptotically) will have as effect to raise the maximum value of the modulus of the output sensitivity function S_{yp} . This may lead to unacceptable values for the modulus and the delay margins if the controller design is not appropriately done (see [20]). As a consequence, a robust control design should be considered assuming that the model of the disturbance is known, in order to be sure that for all situations an acceptable modulus margin and delay margin are obtained.

On the other hand at the frequencies where perfect rejection of the disturbance is achieved one has $S_{yp}(e^{-j\omega}) = 0$ and

$$|S_{up}(e^{-j\omega})| = \left| \frac{A(e^{-j\omega})}{B(e^{-j\omega})} \right| . \quad (18)$$

Equation (18) corresponds to the inverse of the gain of the system to be controlled. The implication of equation (18) is that cancellation (or in general an important attenuation) of disturbances on the output should be done only in frequency regions where the system gain is large enough. If the gain of the controlled system is too low, $|S_{up}|$ will be large at these frequencies. Therefore, the robustness vs additive plant model uncertainties will be reduced and the stress on the actuator will become important. Equation (18) also implies that serious problems will occur if $B(z^{-1})$ has complex zeros close to the unit circle (stable or unstable zeros) at frequencies where an important attenuation of disturbances is required. It is mandatory to avoid attenuation of disturbances at these frequencies.

Since on one hand we would not like to react to very high frequency disturbances and on the other hand we would like to have a good robustness it is often wise to open the loop at $0.5f_s$ (f_s is the sampling frequency) by introducing a fixed part in the controller $H_R(q^{-1}) = 1 + q^{-1}$ (for details see [20] and section II).

³Of course it is assumed that D_p and B do not have common factors.

IV. DIRECT ADAPTIVE CONTROL FOR DISTURBANCE ATTENUATION

The objective is to find an estimation algorithm which will directly estimate the parameters of the internal model in the controller in the presence of an unknown disturbance (but of known structure) without modifying the closed loop poles. Clearly, the Q -parametrization is a potential option since modifications of the Q polynomial will not affect the closed loop poles. In order to build an estimation algorithm it is necessary to define an *error equation* which will reflect the difference between the optimal Q polynomial and its current value.

In [23], such an error equation is provided and it can be used for developing a direct adaptive control scheme. This idea has been used in [24], [1], [2], [16]. Using the Q -parametrization, the output of the system in the presence of a disturbance can be expressed as:

$$\begin{aligned} y(t) &= \frac{A(q^{-1})[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})]}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \\ &= \frac{S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})}{P(q^{-1})} \cdot w(t), \end{aligned} \quad (19)$$

where $w(t)$ is given by (see also figure 1):

$$w(t) = \frac{A(q^{-1})N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = A \cdot y(t) - q^{-d} \cdot B \cdot u(t). \quad (20)$$

In the time domain, the internal model principle can be interpreted as finding Q such that asymptotically $y(t)$ becomes zero. Assume that one has an estimation of $Q(q^{-1})$ at instant t , denoted $\hat{Q}(t, q^{-1})$. Define $\varepsilon^0(t+1)$ as the value of $y(t+1)$ obtained with $\hat{Q}(t, q^{-1})$. Using (19) one gets:

$$\varepsilon^0(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \hat{Q}(t, q^{-1}) w(t) \quad (21)$$

One can define now the *a posteriori* error (using $\hat{Q}(t+1, q^{-1})$) as:

$$\varepsilon(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \hat{Q}(t+1, q^{-1}) \cdot w(t) \quad (22)$$

Replacing $S_0(q^{-1})$ from the last equation by (17) one obtains

$$\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1, q^{-1})] \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} w(t) + v(t+1) \quad (23)$$

where

$$v(t) = \frac{S'(q^{-1})D_p(q^{-1})}{P(q^{-1})} \cdot w(t) = \frac{S'(q^{-1})A(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \delta(t)$$

is a signal which tends asymptotically towards zero.

Define the estimated polynomial $\hat{Q}(t, q^{-1})$ as: $\hat{Q}(t, q^{-1}) = \hat{q}_0(t) + \hat{q}_1(t)q^{-1} + \dots + \hat{q}_{n_Q}(t)q^{-n_Q}$ and the associated estimated parameter vector: $\hat{\theta}(t) = [\hat{q}_0(t) \hat{q}_1(t) \dots \hat{q}_{n_Q}(t)]^T$. Define the fixed parameter vector corresponding to the optimal value of the polynomial Q as: $\theta = [q_0 q_1 \dots q_{n_Q}]^T$. Denote:

$$w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) \quad (24)$$

and define the following observation vector:

$$\phi^T(t) = [w_2(t) \quad w_2(t-1) \dots w_2(t-n_Q)] \quad (25)$$

Equation (23) becomes

$$\varepsilon(t+1) = [\theta^T - \hat{\theta}^T(t+1)] \cdot \phi(t) + v(t+1) \quad (26)$$

One can remark that $\varepsilon(t)$ corresponds to an adaptation error ([18]).

From equation (21) one obtains the *a priori* adaptation error:

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t),$$

with

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1); \quad (27)$$

$$w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t); \quad (28)$$

$$w(t+1) = A(q^{-1}) \cdot y(t+1) - q^{-d}B^*(q^{-1}) \cdot u(t) \quad (29)$$

where $B(q^{-1})u(t+1) = B^*(q^{-1})u(t)$.

The *a posteriori* adaptation error is obtained from (22):

$$\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t).$$

For the estimation of the parameters of $\hat{Q}(t, q^{-1})$ the following parameter adaptation algorithm is used ([18]):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1); \quad (30)$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \phi^T(t)F(t)\phi(t)}; \quad (31)$$

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t); \quad (32)$$

$$F(t+1) = \frac{1}{\lambda_1(t)} \left[F(t) - \frac{F(t)\phi(t)\phi^T(t)F(t)}{\frac{\lambda_1(t)}{C} + \phi^T(t)F(t)\phi(t)} \right] \quad (33)$$

$$1 \geq \lambda_1(t) > 0; 0 \leq \lambda_2(t) < 2 \quad (34)$$

where $\lambda_1(t), \lambda_2(t)$ allow to obtain various profiles for the evolution of the adaption gain $F(t)$ (for details see [18], [20]).

In order to implement this methodology for disturbance rejection (see figure 1), it is supposed that the plant model $\frac{z^{-d}B(z^{-1})}{A(z^{-1})}$ is known (identified) and that it exists a controller $[R_0(z^{-1}), S_0(z^{-1})]$ which verifies the desired specifications in the absence of the disturbance. One also supposes that the degree n_Q of the polynomial $Q(z^{-1})$ is fixed, $n_Q = n_{D_p} - 1$, i.e. the structure of the disturbance is known.

The following procedure is applied at each sampling time for *adaptive* operation:

- 1) Get the measured output $y(t+1)$ and the applied control $u(t)$ to compute $w(t+1)$ using (29).
- 2) Compute $w_1(t+1)$ and $w_2(t)$ using (27) and (28) with P given by (15).
- 3) Estimate the Q -polynomial using the parametric adaptation algorithm (30) - (33).
- 4) Compute and apply the control (see figure 1):

$$S_0(q^{-1})u(t+1) = -R_0(q^{-1})y(t+1) - \hat{Q}(t, q^{-1})w(t+1) \quad (35)$$

For the *self tuning* operation of the adaptive scheme, the estimation of the Q - polynomial starts once the level of the output is over a defined threshold. A parameter adaptation algorithm (30)-(33) with *decreasing adaption gain* is used and the estimation is stopped when the adaption gain is below a pre-specified level⁴. During estimation of the new parameters, the controller is kept constant. The controller is updated once the estimation phase is finished. For a stability analysis of this scheme see [16].

V. A SIMULATED EXAMPLE

The rejection of unknown disturbance using the adaptive control scheme proposed in Section IV is illustrated for the case of a model taken from Bodson and Douglas (1997)[7] where $\dot{y}(t) = -a_0y(t) + b_0u(t) + v(t)$. The transfer function is:

$$H(s) = \frac{1}{0.01s + 1} \quad (36)$$

In our case the disturbance will be represented by a sum of two, respectively four sinusoids with time-varying frequency applied to the output of the plant, so we shall consider $n_{Dp} = 4$ and $n_{Dp} = 8$ that mean for Q - parametrization $n_Q = 3$ respectively $n_Q = 7$. For a realistic simulation we have considered a random noise signal $-0.1 \leq v(t) \leq 0.1$ applied on the output of system plant. The implementation of the controller is in discrete time. The selected sampling frequency is $f_s = 250\text{Hz}$. The nominal digital controller $[R_0, S_0]$ (without the internal model of the disturbance) has been designed using pole placement with calibration of sensitivity functions. The dominant poles for closed loop are a pair of complex poles fixed at the frequency 136.36 rad/sec ⁵ with a damping $\xi = 0.8$. The considered disturbances applied on the output of system have the form:

$$d_1(t) = 0.6 \sin(f_{11} 2\pi t) + 0.6 \sin(f_{12} 2\pi t) \quad (37)$$

$$d_2(t) = 0.6 \sin(f_{21} 2\pi t) + 0.6 \sin(f_{22} 2\pi t) + 0.6 \sin(f_{23} 2\pi t) + 0.6 \sin(f_{24} 2\pi t) \quad (38)$$

where the $f_{11}, f_{12}, f_{21}, f_{22}, f_{23}, f_{24}$ are the frequencies in Hz for different sinusoids.

The parameter adaptation algorithm used for rejection of those disturbances has a number of parameters according with the complexity of $d_1(t)$ and $d_2(t)$ (see above for details). For the case of two sinusoids we use the following simulation protocol:

- The system operates in close loop with the noise $v(t)$ applied all the time ($0 \leq t \leq 30$).
- For $1 \leq t < 10$ the disturbance $d_1(t) = 0.6 \sin(f_{11} \cdot 2\pi) + 0.6 \sin(f_{12} \cdot 2\pi)$ with $f_{11} = 10\text{Hz}$ and $f_{12} = 15\text{Hz}$ is applied (each sinusoid has magnitude 0.6).

⁴The magnitude of the adaptation gain gives an indication upon the variance of the parameter estimation error - see for example [18].

⁵The frequency 136.36 rad/sec (22 Hz) assures a step response in close loop almost equal to that in open loop

- For $10 \leq t < 20$ a change in frequencies occurs. The new frequencies are: $f_{11} = 5\text{Hz}$ respectively $f_{12} = 8\text{Hz}$.
- At $t = 20$, the first sum of sinusoids with $f_{11} = 10\text{Hz}$ and $f_{12} = 15\text{Hz}$ is applied again ($20 \leq t \leq 30$).

The estimation algorithm works permanently and the controller is recomputed at each sampling period. The method for parameter adaptation is forgetting variable factor switching to constant trace (for details see [18], [20]) and the degree of Q -polynomial is 3. The time domain responses in open loop and in closed loop are presented in figure 2. One can see that the adaptation transient is negligible when the disturbance characteristics are changing. The level of the residual output is close to the level of the noise. We can see

Fig. 2. Time domain results of adaptive rejection for a sum of 2 sinusoids (upper figure: open loop, lower figure: closed loop).

that the algorithm works well for both sums of sinusoids. The spectral densities of the output in open loop, respectively in closed loop using the adaptive rejection are presented in Fig.3. We remark that the magnitudes of the two frequencies are attenuated by 30 dB bringing the residual disturbance almost at the level of the noise (in the absence of noise the attenuation is almost 40 dB).

Fig. 3. Power spectral density for a sum of 2 sinusoids.

In the case of four sinusoids the simulation conditions are:

- The system operates in close loop with the noise $v(t)$ applied all the time ($0 \leq t \leq 30$).

- For $1 \leq t \leq 30$ the disturbance $d_2(t)$ with $f_{21} = 4\text{Hz}, f_{22} = 9\text{Hz}, f_{23} = 13\text{Hz}, f_{24} = 17\text{Hz}$ is applied.

The estimation algorithm works permanently and the controller is recomputed at each sampling period. The method for parameter adaptation is forgetting variable factor switching to constant trace and the degree of Q -polynomial is 7. The time domain responses in open loop and in closed loop are presented in figure 4. One can see that the residual disturbances are close to the level of the noise. Changes of frequencies lead also to negligible adaption transient (not shown in this figure). The spectral densities of the

Fig. 4. Time domain results of adaptive rejection for a sum of 4 sinusoids (upper figure: open loop, lower figure: closed loop).

output in open loop, respectively in closed loop using the adaptive rejection are presented in Fig. 5. We remark that the amplitudes for all frequencies are attenuated by almost 30 dB bringing the residual disturbance close to the level of the noise (in the absence of noise the attenuation is more than 35 dB).

Fig. 5. Power spectral density for adaptive rejection of a sum of 4 sinusoids.

VI. CONCLUSIONS

It was shown in this paper that the use of the internal model principle combined with the adaptation of the internal model implemented in a Youla - Kucera parametrized controller allows the development of a direct adaptive control

scheme for a very efficient rejection of the unknown multiple time varying narrow band disturbances without requiring the use of an additional transducer.

The results obtained in a realistic simulation including the presence of a measurement noise are very good both in terms of attenuation and adaption transients.

The approach presented extent the methodology already used in practice for active vibration control in the presence of a single narrow band disturbance.

For future research it will be interesting:

- to evaluate comparatively this approach with the other approaches in terms of complexity and performance.
- to include an anti windup procedure .
- to study the robustness of the scheme with respect to plant model uncertainties

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