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DECENTRALIZED AND AUTONOMOUS DESIGN FOR FDI/FTC OF NETWORKED CONTROL SYSTEMS

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Abstract: In this paper, the fault tolerant control problem is addressed in a networked framework. An isolation filter together with a fault compensation mechanism are proposed for FDI/FTC. Several design procedures are studied. First, in the centralized architecture, the inputs and outputs information used for FDI & FTC are collected and processed in a central node. Considering that sub-systems exchange information over the network, a decentralized architecture is then proposed for FDI/FTC. The autonomous capability of the decentralized design is also considered. But, whatever the design architecture is, fault detectability and isolability, are dependant on the available information. A method based on system structural analysis is then proposed, to provide fault detectability and fault isolability conditions, thus making possible fault tolerant control. Finally, we give an algorithm which allows to distribute a system into subsystems such that the problem of autonomous FDI is solvable for each of these subsystems.

Keywords: Networked Control Systems, Fault Diagnosis, Fault Tolerant Control, Decentralized Design, Structural analysis, Graph-theoretic approach.

1. INTRODUCTION

Networked Control systems (NCS) are in general composed of a large number of interconnected devices that exchange data through communication networks; examples include industrial automation, building supervision, automotive control, ... NCSs provide many advantages such as modular and flexible system design, fast implementation, distribution. But, some disadvantages such as loss of information, time delays in the data transmission may also have an effect on the control performances and should be taken into account in the design procedure. Since the early 70's, Fault Detection and Isolation (FDI) in dynamic system has received much attention from both theoretical and application viewpoints (Franck and Ding, 2000; Gertler, 1998; Chen and Patton, 1999). The fault tolerant control problem is a more recent research topic but there also various results on it (Zhang and Jiang, 2003). The task to be tackled in achieving

fault-tolerance is the design of a controller with suitable structure to guarantee satisfactory performance, not only when all control components are operational, but also in the case when instruments are operating under a faulty mode. If we compare with the development in FDI/FTC for small scale centralized systems, corresponding research for large scale interconnected systems is very limited. This is in particular the case for the NCS's systems which are considered here, such as the example represented on Figure 1. The FDI/FTC system is distributed over the network, and decomposed into subsystems, each of which being in charge of a local FDI/FTC unit. The information required by each unit from the others being transmitted via the networks, it is therefore important for fault tolerance to consider the network faults which can occur. A second new problem concerns complex systems, controlled through a network of controllers, each one dedicated to a subset of components, and communicating with

each others, because as the components are physically interconnected, the control system must take it into account. This is specifically important when a fault occurs, because this fault will propagate along the process, and isolated local diagnosis could lead to a wrong result. The theoretical problem is related to the distribution of diagnosis between several "diagnosers" and how to make them communicate (which information must they share? This study should take into account the plant physical structure, the components interactions, the algorithms complexity, etc. The aim of the paper is to study the global as well as the local fault detectability and isolability which represent the necessary conditions before a distributed implementation of a FDI/FTC system.

Figure 1. : Networked Control Systems

The paper is organized as follows. In the next section, we formulate general statements about different architectures for FDI/FTC: centralized, decentralized FDI, and autonomous distributed. The residual generation problem for FDI is addressed, considering designs based on the use of directional filters. The third section is dedicated to the structural analysis of each of the previous architectures. Using a graph-theoretic approach, we give necessary and sufficient conditions to the detectability and isolability of any chosen part of the fault vector for any of the three FDI/FTC schemes. Some concluding remarks end the paper.

2. PROBLEM STATEMENT

A classical way to achieve fault-tolerant control relies on supervised control where an FDI unit provides information about the location and time occurrence of any fault. Faults are compensated via an appropriate control law triggered according to diagnosis of the system (Noura *et al.*, 2000). Once the FDI module indicates which sensor or actuator is faulty, the fault magnitude can be estimated and a new control law is added to the nominal one to thwart the fault effect on the system.

Generation of residuals having directional properties in response to a particular faults is an attractive way for enhancing fault isolability. A fault detection filter is a special dynamic observer which generates directional residuals. It was first developed by Beard (Beard, 1971) and Jones (Jones, 1973) and later revisited by (Massoumnia, 1986) in the geometric framework and by White and Speyer (White and Speyer, 1987) in the context of eigenstructure assignment. Further improvements were suggested by Park and Rizzoni (Park and Rizzoni, 1994) and Liu and Si (Liu

and Si, 1997).

Let us suppose the system under study can be described by the following model :

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is state vector, $y(t) \in \mathbb{R}^m$ the output vector, $u(t) \in \mathbb{R}^p$ the input vector. E is faults distribution matrix and $f(t) \in \mathbb{R}^q$ is the fault vector. We also consider the observer:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - C\hat{x}(t)) \quad (2)$$

and residual generator:

$$r(t) = LC(x(t) - \hat{x}(t)) \quad (3)$$

The goal is to compute the gain K such that transfer matrix $W(s)$ from f to r is given by:

$$W(s) = LC(sI - (A - KC))^{-1}E = \left(\frac{1}{s}\right)I \quad (4)$$

where (4) ensures the diagonal structure of transfer from faults to residuals allowing the multiple faults isolation. We have:

$$\begin{aligned} W(s) &= LC(sI - (A - KC))^{-1}E \\ &= \sum_{k \geq 0} s^{-k-1} [\dots | LC(A - KC)^k E_i | \dots] \end{aligned} \quad (5)$$

where $\sum_{k \geq 0} s^{-k-1} LC(A - KC)^k E_i = s^{-1} LCE_i + s^{-2} LC(A - KC)E_i + s^{-3} LC(A - KC)^2 E_i + \dots$. Hence, if the observer's gain K satisfies the algebraic constraint:

$$(A - KC) [\dots | E_i | \dots] = (A - KC)E = 0 \quad (6)$$

then (5) gives:

$$W(s) = [\dots | LCE_i s^{-1} | \dots] = D \left(\frac{1}{s}\right)I \quad (7)$$

with $D = LCE$ and (4) is satisfied under $D = I$.

Solution to this problem makes use of the following theorem by Keller (J.-Y. Keller, 1999).

Theorem 1. Under $\text{rank}(D) = q$ which implies $q < m$, the solution of the eigenstructure assignment (6) can be parameterized according to

$$K = \omega\Pi + \bar{K}\Sigma \quad (8)$$

with $\Sigma = \beta(I - D\Pi)$, $\Pi = D^{-1}$ and $\omega = AE$.

where $\beta \in \mathbb{R}^{m-q,m}$ is an arbitrary matrix determined so that matrix Σ is of full rows rank and $\bar{K}$ is the reduced gain describing the remaining degree of freedom.

Considering theorem 1, we obtain :

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + \omega q(t) + \bar{K}\gamma(t) \quad (9)$$

Furthermore, it is easy to show that $\gamma(t) = \Sigma(y(t) - C\hat{x}(t))$ is decoupled from the faults, while $q(t) = \Pi(y(t) - C\hat{x}(t))$ depends on the fault. Hence, it is possible to give a fault free estimate of the actual state vector according to:

$$\bar{x}(t) = \begin{pmatrix} \Sigma C \\ \Pi C \end{pmatrix}^+ \begin{pmatrix} \Sigma \hat{x}(t) \\ \Pi C \hat{x}(t) + q(t) \end{pmatrix} \quad (10)$$

With respect to eq. (7) an estimation of the faults can be obtained according to $\hat{f}(t) = \dot{q}(t)$. For FTC, the goal

is to make the nominal system and the reconfigured system as closed as possible. Once the fault is isolated, the corresponding fault estimation and compensation module is switched on to reduce the fault effect on the system. The control signal applied to the system is given by

$$u(t) = -G\bar{x}(t) + u_{ad}(t) \quad (11)$$

and is computed such that the fault influence on to the system is compensated. In other terms, it must satisfy

$$Bu_{ad}(t) + E\hat{f}(t) = 0 \quad (12)$$

the solution of (12) can be obtained if $\text{rank}([B \ E]) = \text{rank}(E)$.

In this classical centralized design, FTC can be represented by Fig. 2 which clearly shows that all the available inputs and outputs information are collected and processed in a central node.

The aim of the paper is to develop a distributed FDI

Figure 2. : *Centralized architecture for FDI/FTC*

architecture for networked control systems. A distributed FDI system is one that consists of several localized FDI sub-systems, acting on a single subsystem, in order to achieve a global FDI objective.

Consider the structured linear system (Σ) (1). As such system is a distributed system, it is constituted of several subsystems (Σ_i) , $i = 1, \dots, N$. Each subsystem satisfies to a model of the form:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + \sum_{j=1, j \neq i}^N H_{i,j} x_j(t) + B_i u_i(t) + E_i f_i(t) \\ y_i(t) = C_i x_i(t) \end{cases} \quad (13)$$

where for $i = 1, \dots, N$, $x_i \in \mathbb{R}^{n_i}$ is the state vector of subsystem (Σ_i) , $y_i \in \mathbb{R}^{m_i}$ is the output vector of subsystem (Σ_i) , $u_i \in \mathbb{R}^{p_i}$ is the input vector of subsystem (Σ_i) , $f_i \in \mathbb{R}^{q_i}$ is the fault vector occurring on subsystem (Σ_i) .

Consider also the following decentralized residual generators for $i = 1, \dots, N$

$$\begin{cases} \dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + K_i (y_i(t) - C_i \hat{x}_i(t)) + \sum_{j=1, j \neq i}^N H_{i,j} \bar{x}_j(t) + B_i u_i(t) \\ r_i(t) = L_i (y_i(t) - C_i \hat{x}_i(t)) \end{cases}$$

where $\bar{x}_j(t)$ is a fault free state estimate, $r_i(t)$ the output of the filter and where $K_i \in \mathbb{R}^{n_i \times m_i}$ are unknown matrices that will be designed in order to fulfill local fault detection and isolation requirements.

If we consider that :

$$\lim_{t \rightarrow \infty} x_i(t) - \bar{x}_i(t) = 0, \quad i = 1, \dots, N \quad (14)$$

then the local estimation errors and residuals satisfy :

$$\begin{cases} \dot{\varepsilon}_i(t) = (A_i - K_i C_i) \varepsilon_i(t) + E_i f_i(t) \\ r_i(t) = L_i C_i \varepsilon_i(t) \end{cases} \quad i = 1, \dots, N$$

Thus, in order to deal with a decentralized design of the FDI/FTC approach, the following conditions should be satisfied :

$$\begin{aligned} \lim_{t \rightarrow \infty} (f_i - \hat{f}_i(\hat{x}_i, u_i, y_i, \bar{x}_j)) &= 0 \\ \lim_{t \rightarrow \infty} (x_i - \bar{x}_i(\hat{x}_i, u_i, y_i, \bar{x}_j)) &= 0 \quad \forall f_i(t) \end{aligned} \quad (15)$$

In this decentralized design, FTC can be represented by Fig. 3. In the decentralized design, each local

Figure 3. : *Decentralized networked FDI*

FDI/FTC unit requires information from the others. But when the network fails, these information are no longer available and the local FDI/FTC unit must be able to perform its work autonomously. Following the same idea as previously, a way to design a fully distributed autonomous FDI/FTC is to consider the following residual generator :

$$\begin{cases} \dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + K_i (y_i(t) - C_i \hat{x}_i(t)) + B_i u_i(t) \\ r_i(t) = L_i (y_i(t) - C_i \hat{x}_i(t)) \end{cases} \quad (16)$$

Fig. 4 gives an illustration of the distributed autonomous design. Finally, what is possible to do from

Figure 4. : *Distributed autonomous FDI*

the FDI/FTC point of views strongly depends on the fault detectability and isolability conditions.

3. STRUCTURAL ANALYSIS OF THE FAULT DETECTABILITY AND ISOLABILITY FOR DISTRIBUTED SYSTEMS

In this section, we give the necessary conditions for the solvability of the observer-based FDI for distributed systems in different configurations: centralized FDI, decentralized FDI and autonomous FDI. To do so, we use a graph theoretic approach, which is well-adapted to handle the structural properties of the studied systems and to deal with large scale systems. In many modeling problems, system matrices have a number of fixed zero entries determined by the physical structure while the remaining entries are not precisely known. To study the properties of these systems in spite of the poor knowledge we have on them, the idea is that we only keep the zero/nonzero entries in the state space matrices. Thus, we consider models where the fixed zeros are conserved while the

nonzero entries are replaced by free parameters. There is a huge amount of interesting works using this kind of models called structured models since their study requires a low computational burden which allows to deal with large scale systems. Many studies on structured systems are related to the graph-theoretic approach. Survey paper (Dion *et al.*, 2003) reviews the most significant results in this area. From these works, it results that the graph-theoretic approach provides simple and elegant solutions and so is very well suited to analyse large scale or/and uncertain systems.

3.1 Graphical representation of a structured linear system for FDI purpose

Consider structured linear system (1) which is constituted of several subsystems (Σ_i) , $i = 1, \dots, N$:

$$(\Sigma_i) \begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j=1, j \neq i}^N H_{i,j} x_j(t) + E_i f_i(t) \\ y_i(t) = C_i x_i(t) \end{cases} \quad (17)$$

where for $i = 1, \dots, N$, $x_i \in \mathbb{R}^{n_i}$ is the state vector of subsystem (Σ_i) , $y_i \in \mathbb{R}^{m_i}$ is the output vector of subsystem (Σ_i) , $u_i \in \mathbb{R}^{p_i}$ is the input vector of subsystem (Σ_i) , $f_i \in \mathbb{R}^{q_i}$ is the fault vector occurring on subsystem (Σ_i) . A_i , B_i , $H_{i,j}$, E_i , C_i represent matrices of appropriate dimensions whose elements are either fixed to zero or assumed to be free non-zero parameters $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_h)^T$. If all the non-zeros parameters λ_i are fixed, we obtain an admissible realization of structured system (Σ_Λ) . We say that a property is true generically if it is true for almost all realizations of the system or for almost all parameter values. To structured system (Σ_i) , we associate a digraph noted $\mathcal{G}(\Sigma_\Lambda)$ which is constituted by a vertex set $\mathcal{V}$ and an edge set

$$\mathcal{E}. \text{ More precisely, } \mathcal{V} = \bigcup_{i=1}^N (\mathbf{X}_i \cup \mathbf{U}_i \cup \mathbf{Y}_i \cup \mathbf{F}_i),$$

where $\mathbf{X}_i = \{x_{i,1}, \dots, x_{i,n_i}\}$ is the set of state vertices for subsystem i , $\mathbf{U}_i = \{u_{i,1}, \dots, u_{i,p_i}\}$ is the set of input vertices for subsystem i , $\mathbf{Y}_i = \{y_{i,1}, \dots, y_{i,m_i}\}$ is the set of output vertices for subsystem i and $\mathbf{F}_i = \{f_{i,1}, \dots, f_{i,q_i}\}$ is the set of failure vertices for subsystem i . The edge set is

$$\mathcal{E} = \bigcup_{i=1}^N (A_i\text{-edges} \cup B_i\text{-edges} \cup C_i\text{-edges} \cup E_i\text{-edges} \cup \bigcup_{j=1, j \neq i}^N H_{i,j}\text{-edges}), \text{ where}$$

$$\begin{aligned} A_i\text{-edges} &= \{(x_{i,j}, x_{i,k}) \mid A_i(k, j) \neq 0\}, \\ B_i\text{-edges} &= \{(u_{i,h}, x_{i,l}) \mid B_i(l, h) \neq 0\}, \\ H_{i,j}\text{-edges} &= \{(x_{j,l}, x_{i,h}) \mid H_{i,j}(h, l) \neq 0\}, \\ E_i\text{-edges} &= \{(f_{i,h}, x_{i,l}) \mid E_i(l, h) \neq 0\} \quad \text{and} \\ C_i\text{-edges} &= \{(x_{i,h}, y_{i,l}) \mid C_i(l, h) \neq 0\}. \end{aligned}$$

Here, $M(i, j)$ is the (i, j) th element of matrix M and (v_1, v_2) denotes a directed edge from vertex $v_1 \in \mathcal{V}$ to vertex $v_2 \in \mathcal{V}$. The following example illustrates the previous settings.

Example 2. Consider the structured subsystem 1 of a distributed system constituted of three subsys-

$$\begin{aligned} \text{tems: } A_1 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \times & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \times & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \times & 0 & 0 & 0 & 0 \\ 0 & 0 & \times & \times & 0 & 0 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} \times \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, E_1 = \begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ C_1 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \times & 0 \\ 0 & 0 & 0 & 0 & 0 & \times & \times \end{pmatrix}, H_{1,2} = H_{1,3} = 0, A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \times & \times & 0 & 0 & 0 & 0 \\ \times & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ B_2 &= \begin{pmatrix} \times & 0 \\ 0 & \times \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, C_2 = \begin{pmatrix} 0 & 0 & \times & 0 & 0 & 0 \\ 0 & 0 & \times & \times & 0 & 0 \\ 0 & 0 & 0 & 0 & \times & \times \end{pmatrix}, E_2 = \begin{pmatrix} 0 & 0 \\ \times & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \times \end{pmatrix}, H_{2,1} = \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \times & \times & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, H_{2,3} = 0, A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \times & 0 & 0 & 0 & 0 & 0 \\ \times & 0 & 0 & 0 & 0 & 0 \\ 0 & \times & 0 & 0 & 0 & 0 \\ 0 & 0 & \times & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \times \end{pmatrix}, B_3 = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \\ E &= \begin{pmatrix} \times \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, H_{3,1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \times & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, H_{3,2} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \times & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and} \\ C_3 &= \begin{pmatrix} 0 & 0 & 0 & \times & 0 & 0 \\ 0 & 0 & 0 & 0 & \times & \times \end{pmatrix}. \end{aligned}$$

Here, for the sake of simplicity, we represent each nonzero parameter λ_i of the structured system by the symbol $\times$. To such a model, we associate the digraph in figure 5.

Figure 5. Digraph associated to Example 2

We denote path $\mathbf{P}$ containing vertices $s_{r_0}, s_{r_1}, \dots, s_{r_i}$ by $\mathbf{P} = s_{r_0} \rightarrow s_{r_1} \rightarrow \dots \rightarrow s_{r_i}$ where $(s_{r_j}, s_{r_{j+1}}) \in \mathcal{E}$ for $j = 0, 1, \dots, i-1$. Some paths are disjoint if they have no common vertex. In the sequel, $\mathbf{V}_1$ and $\mathbf{V}_2$ denote two subsets of $\mathcal{V}$. A path $\mathbf{P}$ is said a $\mathbf{V}_1$ - $\mathbf{V}_2$ path if its begin vertex belongs to $\mathbf{V}_1$ and its end vertex belongs to $\mathbf{V}_2$. We denote by $\rho[\mathbf{V}_1, \mathbf{V}_2]$ the maximal number of disjoint $\mathbf{V}_1$ - $\mathbf{V}_2$ paths.

3.2 Centralized FDI schemes

The necessary and sufficient conditions for the solvability of the observer-based FDI problem are directly deduced from (Commault *et al.*, 2002).

Proposition 3. Consider structured system (Σ_Λ) represented by digraph $\mathcal{G}(\Sigma_\Lambda)$. The fault components asso-

ciated to any $\bar{\mathbf{F}} \subseteq \mathbf{F}$ are isolable and detectable using a centralized observer-based FDI scheme iff in $\mathcal{G}(\Sigma_\Lambda)$

$$\rho[\mathbf{F}, \mathbf{Y}] = \text{card}(\bar{\mathbf{F}}) + \rho[\mathbf{F} \setminus \bar{\mathbf{F}}, \mathbf{Y}]$$

In example 2, since $\rho[\mathbf{F}, \mathbf{Y}] = 6$ and $\text{card}(\mathbf{F}) = 6$, all the faults are detectable and isolable.

3.3 Decentralized FDI scheme

Assume now that, for some $j \in J \subseteq \{1, 2, \dots, i-1, i+1, \dots, N\}$, all the magnitude of measurement Y_j and input U_j are transmitted via the network to subsystem (Σ_i) .

Proposition 4. Consider structured system (Σ_Λ) represented by digraph $\mathcal{G}(\Sigma)$. For subsystem i , the fault components associated to any $\bar{\mathbf{F}} \subseteq \mathbf{F}$ are generically detectable and isolable using a decentralized observer-based FDI scheme if and only if

$$\rho\left[\left(\bigcup_{j \in J \cup \{i\}} \mathbf{U}_j\right) \cup \mathbf{F}, \bigcup_{j \in J \cup \{i\}} \mathbf{Y}_j\right] = \text{card}(\bar{\mathbf{F}}) + \rho\left[\left(\bigcup_{j \in J \cup \{i\}} \mathbf{U}_j\right) \cup (\mathbf{F} \setminus \bar{\mathbf{F}}), \bigcup_{j \in J \cup \{i\}} \mathbf{Y}_j\right] \quad (18)$$

3.4 Decentralized autonomous FDI scheme

Now, we consider that no information from subsystems (Σ_j) , $j \neq i$ arrive to subsystem (Σ_i) .

Proposition 5. Consider structured system (Σ_Λ) represented by digraph $\mathcal{G}(\Sigma)$. For subsystem i , the fault components associated to any $\bar{\mathbf{F}} \subseteq \mathbf{F}_i$ are detectable and isolable using a decentralized observer-based FDI scheme iff

$$\rho\left[\left(\bigcup_{j \neq i} \mathbf{U}_j\right) \cup \mathbf{F}, \mathbf{Y}_i\right] = \text{card}(\bar{\mathbf{F}}) + \rho\left[\left(\bigcup_{j \neq i} \mathbf{U}_j\right) \cup (\mathbf{F} \setminus \bar{\mathbf{F}}), \mathbf{Y}_i\right]$$

Considering example 2, we can do the following computations:

• Subsystem 1:

for $J = \{2\}$ or for $J = \{3\}$: $\forall \bar{\mathbf{F}} \subseteq \mathbf{F}_1$,

$$\rho\left[\left(\bigcup_{j \in J \cup \{i\}} \mathbf{U}_j\right) \cup \mathbf{F}, \bigcup_{j \in J \cup \{i\}} \mathbf{Y}_j\right] < \text{card}(\bar{\mathbf{F}}) + \rho\left[\left(\bigcup_{j \in J \cup \{i\}} \mathbf{U}_j\right) \cup (\mathbf{F} \setminus \bar{\mathbf{F}}), \bigcup_{j \in J \cup \{i\}} \mathbf{Y}_j\right]. \quad \text{Nevertheless,}$$

for $J = \{2, 3\}$, condition (18) is satisfied for every $\bar{\mathbf{F}} \subseteq \mathbf{F}_1$. So the faults of subsystem 1 can be detectable and isolable using distributed FDI scheme only since we consider that all the inputs and output measurement of subsystems 2 and 3 are transmitted via the network to subsystem 1.

• Subsystem 2: $\rho\left[\bigcup_{j=1,2} \mathbf{U}_j \cup \mathbf{F}, \mathbf{Y}_2\right] = 3$ and

$\rho\left[\bigcup_{j=1,2} \mathbf{U}_j \cup \mathbf{F} \setminus \mathbf{F}_2, \mathbf{Y}_2\right] = 1$. Consequently, for this subsystem, all the faults are generically detectable and isolable using an autonomous decentralized observer-based FDI scheme.

• Subsystem 3: f_3 is generically detectable and isolable using a decentralized observer-based FDI scheme only if the input and output measurements of subsystem 2 are transmitted to subsystem 3.

4. DISTRIBUTION OF FDI NODES

In this last paragraph, using the previous results, the objective is to determine a distribution of the FDI task on local nodes such that the problem of autonomous local observer based FDI problem is generically solvable for each of these nodes.

Consider a global linear system (1), where A, B, C and E are structured matrices of appropriate dimensions. The objective is not exactly to obtain a subdivision of the global system into subsystems but to obtain a certain number of Control/FDI nodes such that each of them is able locally to detect and isolate a part of the faults occurring on the system.

As previously, we define a global digraph associated to system (Σ_Λ) noted $\mathcal{G}_g(\Sigma_\Lambda)$. It is constituted by a vertex set $\mathcal{V}_g$ and an edge set $\mathcal{E}_g$ i.e. $\mathcal{G}(\Sigma_\Lambda) = (\mathcal{V}_g, \mathcal{E}_g)$. More precisely, $\mathcal{V}_g = \mathbf{X} \cup \mathbf{U} \cup \mathbf{Y} \cup \mathbf{F}$ and the edge set is $\mathcal{E}_g = A\text{-edges} \cup B\text{-edges} \cup E\text{-edges} \cup C\text{-edges}$. We define for each $\mathbf{V}_Y \subseteq \mathbf{Y}$ functions α as:

$$\alpha: 2^Y \rightarrow 2^F$$

$$\mathbf{V}_Y \mapsto \alpha(\mathbf{V}_Y) = \{\mathbf{f}_i \in \mathbf{F}, \text{ such that } \rho[\mathbf{f}_i, \mathbf{V}_Y] \neq 0\}$$

The digraph $\mathcal{G}_g(\Sigma_\Lambda)$ associated to the system of example 2 is depicted in figure 6. Note that the distribution

Figure 6. Digraph associated to Example 2

of the FDI nodes has a solution only if the observer-based FDI problem is solvable for the centralized scheme. Therefore, we assume that $\rho[\mathbf{F}, \mathbf{Y}] = \text{card}(\mathbf{F})$. From Proposition 5, a subsystem defined by vertex subsets $\mathbf{V}_Y \subseteq \mathbf{Y}$ and $\mathbf{V}_U \subseteq \mathbf{U}$ is autonomous in a FDI context iff

$$\rho[\mathbf{F} \cup (\mathbf{U} \setminus \mathbf{V}_U), \mathbf{V}_Y] = \text{card}(\alpha(\mathbf{V}_Y)) + \rho[\mathbf{U} \setminus \mathbf{V}_U, \mathbf{V}_Y]$$

To establish this equality, we assume that the FDI node defined by the pair $(\mathbf{V}_U, \mathbf{V}_Y)$ must detect and isolate all the fault components included in $\mathbf{V}_Y$. Moreover, the subdivision of a subsystem defined by vertex subsets $\mathbf{V}_Y \subseteq \mathbf{Y}$ and $\mathbf{V}_U \subseteq \mathbf{U}$ is acceptable only if the rest of the system is also autonomous in a FDI context

$$\rho[\mathbf{F} \cup \mathbf{V}_U, \mathbf{Y} \setminus \mathbf{V}_Y] = \text{card}(\alpha(\mathbf{Y} \setminus \mathbf{V}_Y)) + \rho[\mathbf{V}_U, \mathbf{Y} \setminus \mathbf{V}_Y]$$

We give hereafter an algorithm which allows to compute the set of vertex subsets $\Delta \subseteq 2^{U \cup Y}$.

Algorithm 1.

Initialisation: $\mathbf{Z} = \mathbf{U} \cup \mathbf{Y}$, $\Delta = \emptyset$.

```

for  $\kappa = 1$  to  $\text{card}(\mathbf{Z})$ 
  for  $j = 1$  to  $\binom{\text{card}(\mathbf{Z})}{\kappa}$ 
    let  $\Omega_{\kappa,j}$  be the  $j^{\text{th}}$   $\kappa$ -subset of  $\mathbf{Z}$ 
    and let  $\mathbf{V}_U \stackrel{\text{def}}{=} \Omega_{\kappa,j} \cap \mathbf{U}$ ,  $\mathbf{V}_Y \stackrel{\text{def}}{=} \Omega_{\kappa,j} \cap \mathbf{Y}$ 
    if the following conditions hold
      C1 :  $\Delta \cap \{\Omega_{\kappa,j}\} = \emptyset$ 
      C2 :  $\rho[\mathbf{F} \cup \mathbf{U} \setminus \mathbf{V}_U, \mathbf{V}_Y] = \text{card}(\alpha(\mathbf{V}_Y)) + \rho[(\mathbf{U} \setminus \mathbf{V}_U), \mathbf{V}_Y]$ 
      and
      C3 :  $\rho[\mathbf{F} \cup \mathbf{V}_U, \mathbf{Y} \setminus \mathbf{V}_Y] = \text{card}(\alpha(\mathbf{Y} \setminus \mathbf{V}_Y)) + \rho[\mathbf{V}_U, \mathbf{Y} \setminus \mathbf{V}_Y]$ 
    Then  $\Delta = \Delta \cup \{\Omega_{\kappa,j}\} \cup \{\mathbf{Z} \setminus \{\Omega_{\kappa,j}\}\}$ 
end

```

Proposition 6. Consider structured linear system (Σ_Λ) represented by digraph $\mathcal{G}_1(\Sigma_\Lambda)$. Consider a subdivision of the system into N subsystems defined by the pairs of vertex subsets $(\mathbf{U}_i, \mathbf{Y}_i)$, $i = 1, \dots, N$. The problem of autonomous observer based FDI problem is generically solvable for each of these subsystems iff:

- $\bigcup_{i=1}^N \mathbf{U}_i = \mathbf{U}$, $\bigcup_{i=1}^N \mathbf{Y}_i = \mathbf{Y}$,
- $\forall i \in \{1, \dots, N\}$ and $\forall j \in \{1, \dots, i\}$, we must have $\mathbf{U}_i \cap \mathbf{U}_j = \emptyset$ and $\mathbf{Y}_i \cap \mathbf{Y}_j = \emptyset$,
- $\forall i \in \{1, \dots, N\}$, $\mathbf{U}_i \cup \mathbf{Y}_i \in \Delta$.

In the case of the system defined in example (2), Algorithm 1 allows to compute set of vertex subsets

$$\Delta = \{ \{y_3\}, \{u_2\}, \{u_4\}, \{u_5\}, \{u_6\}, \{u_1, u_2, u_3, u_4, u_5, u_6, y_1, y_2, y_4, y_5, y_6, y_7\}, \{u_1, u_3, u_4, u_5, u_6, y_1, y_2, y_3, y_4, y_5, y_6, y_7\}, \{u_1, u_2, u_3, u_5, u_6, y_1, y_2, y_3, y_4, y_5, y_6, y_7\}, \{u_1, u_2, u_3, u_4, u_6, y_1, y_2, y_3, y_4, y_5, y_6, y_7\}, \{u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3, y_4, y_5, y_6, y_7\}, \{u_1, u_2, u_3, u_4, u_5, u_6, y_1, y_2, y_3, y_4, y_5, y_6, y_7\} \}$$

This set makes an inventory of all the possibilities for distributing the control/FDI nodes. The algorithm described above is not optimized and the computational point of view is not treated in this paper.

5. CONCLUSION

An important problem that must be considered when dealing with control over network, is the distribution of the FDI and FTC algorithms. An alternative approach to the classical centralized design for FDI/FTC has been studied in this paper. For that purpose, a residual generator with fixed directional properties has been considered. We have shown, that under some conditions, it is possible to distribute among several local processing units the residual generator and the resulting fault compensation mechanism. The fault detectability and isolability conditions for each configuration (centralized, distributed and autonomous NCS)

have also been studied, considering a structural analysis approach and an algorithm which allows us to distribute the FDI task on local autonomous nodes is provided. In future works, information delays will be also considered in the design.

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