

# Light transmission by subwavelength square coaxial aperture arrays in metallic films

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**Abstract:** Using Fourier Modal Method, we study the enhanced transmission exhibited by arrays of square coaxial apertures in a metallic film. The calculated transmission spectrum is in good agreement with FDTD calculations. We show that the enhanced transmission can be explained considering a few guided modes of a coaxial waveguide.

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**OCIS codes:** (050.1120) Apertures, (240.6680) Surface plasmons, (230.7370) Waveguides

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## 1 Introduction

Nowadays, opticians are greatly interested in structures that exhibit anomalous effects because they have potential applications in novel photonic devices. The extraordinary

enhanced transmission by subwavelength metallic hole arrays is one such phenomenon. Since the publication of Ebbesen *et al.*<sup>1</sup>, many experimental and theoretical studies were carried out in order to determine the physical origin of the observed enhanced transmission. Three kinds of explanations were proposed since, relating the enhanced transmission to the excitation of surface plasmons,<sup>2,3</sup> to a Fabry-Pérot cavity behavior of the holes,<sup>4,5</sup> or explaining the transmission in terms of dynamical diffraction.<sup>6,7</sup> It is now established that both horizontal and vertical resonances play a role in the extraordinary transmission<sup>8</sup>. It is then of importance to characterize and to understand the electromagnetic behavior of the channel through which the light propagates inside the metallic film<sup>10</sup>. Recently, numerical simulations have shown that a transmission as high as 80 % can be obtained with annular apertures<sup>11</sup>. The aim of the present communication is to study the spectral response of metallic films with square coaxial aperture. Those structures are similar to the above mentioned ones from the electromagnetic point of view. Since the aperture dimensions are of the order of magnitude of the wavelength a rigorous electromagnetic theory is necessary to analyze the behavior of such structures. Although the FDTD method allows to calculate rigorously the reflection and transmission of a plane wave by a periodical structure in the resonance domain, the Fourier Modal Method gives a more physical insight in the present resonant phenomenon<sup>12</sup>. The diffraction problem is reduced to the search of eigenvalues and eigenvectors of a particular matrix. It permits to calculate the effective index of the modes of the coaxial aperture and the coupling of these modes with the reflected and transmitted order.

## 2 Statement of the problem

Let us consider a metallic film deposited on a glass substrate with an engraved periodic structure of square coaxial apertures ( see Fig. 1).

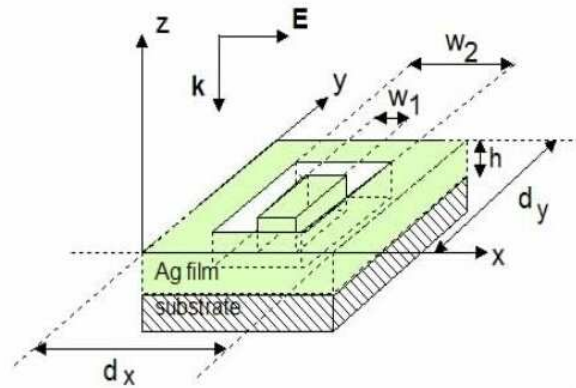


Fig. 1. Coaxial square aperture in a metallic film

The refractive index of the metal is described by a simple free-electron Drude model with a plasma frequency  $\omega_p = 1.374 \times 10^{16} \text{ s}^{-1}$  and a relaxation time  $\tau = 0.3 \times 10^{-14} \text{ s}$ . The periods are  $d_x$  in the x-direction and  $d_y$  in the y-direction. The width and the position of an aperture are controlled by two parameters  $w_1$  and  $w_2$  (see Fig.1). Lastly the thickness is denoted by  $h$ . The structure is illuminated in vacuum, under normal incidence, by a monochromatic linearly polarized plane wave, with a wavelength  $\lambda$ , a wavenumber  $k = 2\pi/\lambda$  and a time dependence  $\exp(i\omega t)$ . Our goal is to calculate and to understand the reflection and transmission spectrum of this structure with the help of

the Fourier Modal Method. In the layer, any component  $F$  of the electric or magnetic field can indeed be expressed as a superposition of eigenmodes:

$$F(x, y, z) = \sum_{mnq} (A_q^+ \exp(-ik\gamma_q z) + A_q^- \exp(ik\gamma_q(z - h))) F_{mnq} e_{mn}(x, y)$$

with

$$e_{mn}(x, y) = \exp\left(-i\frac{2m\pi x}{d_x}\right) \exp\left(-i\frac{2n\pi y}{d_y}\right)$$

where  $m$  and  $n$  are integers such that  $-M \leq m \leq M$  and  $-N \leq n \leq N$ . The integers  $M$  and  $N$  describe the truncation scheme. The matrix from which eigenvalues and eigenvectors are calculated is then of rank  $2(2M + 1)(2N + 1)$ .  $A_q^+$  and  $A_q^-$  are the unknown complex amplitudes of the upward and downward propagating or decaying waves. Our numerical code includes the correct factorization rules derived by Li<sup>9</sup>, our personal parametric formulation, and the S matrix approach for writing the boundary conditions. Although it is not the scope of this paper, it should be emphasised that the above mentioned numerical tools are of great importance to obtain reliable and converged results. In order to compare the Fourier modal method and the FDTD that was used by Baida and Van Labeke<sup>11</sup> we have calculated the transmission spectrum of a structure with the following parameters:  $w_1 = 105 \text{ nm}$ ,  $w_2 = 155 \text{ nm}$ ,  $d_x = d_y = 300 \text{ nm}$ ,  $h = 150 \text{ nm}$ ,  $n_s = 1.45$ .

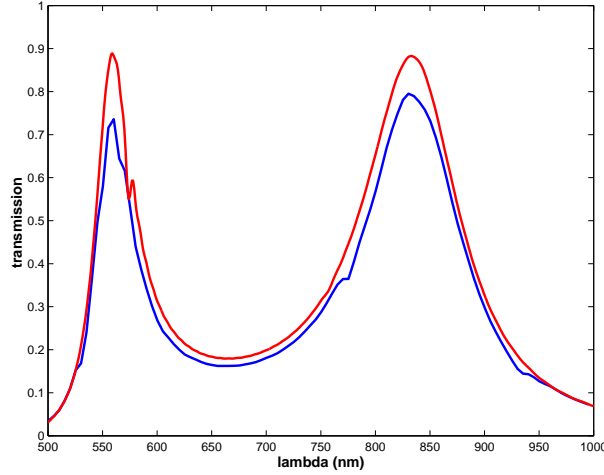


Fig. 2. Transmission of a square coaxial aperture calculated with the FDTD (blue line) and the Fourier Modal Method (red line)

It can be seen that both methods give resonances at the same place even though a small difference is observed in their intensity.

### 3 Discussion

#### 3.1 Analysis of the mode

Our goal is to analyze the enhanced transmission using the guided modes of the coaxial apertures. Since we consider a metallic medium, an aperture is very weakly coupled with its neighbours. A mode for the entire structure thus corresponds to the excitation of a particular mode of a sole aperture in every aperture. Indeed, the eigenvalues and the fields corresponding to an eigenmode do not change when the distance between holes

varies. Hence we may make no distinction between the modes supported by the structure and the modes of a sole coaxial aperture and the eigenvalues  $\gamma_q$  give an immediate access to the effective index of each mode.

Because of the metal all the propagating constants are complex but some of them can be considered as guided modes with low losses. For the considered structure we have found that there were three such modes, two of them being degenerated due to the square symmetry. The numerically obtained dispersion relations are plotted on Fig. 3 and Fig. 4.

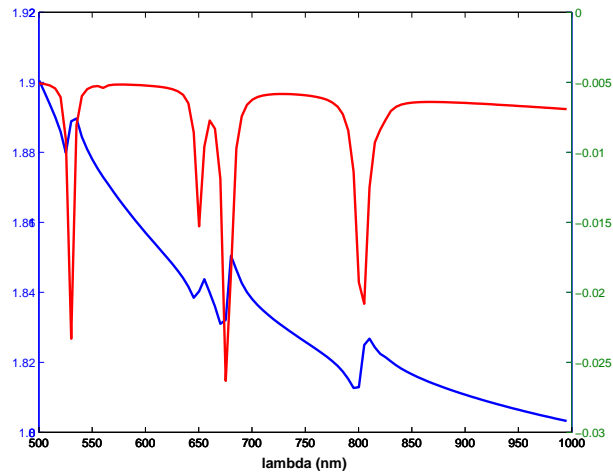


Fig. 3. dispersion curves of the first mode. blue line: real part. red line: imaginary part. The presence of dips is probably due to the right angle corners.

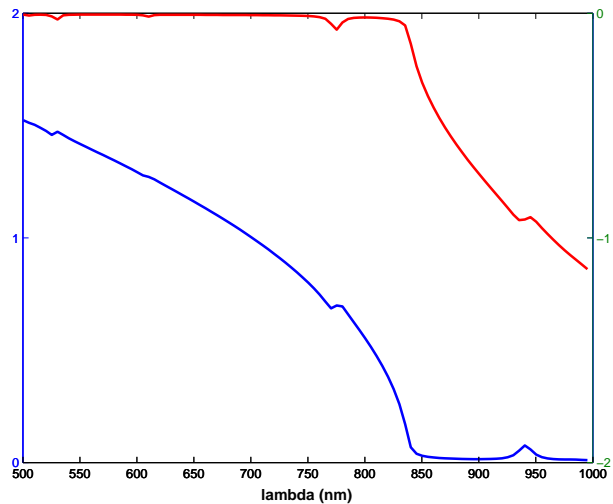


Fig. 4. dispersion curves of the second mode. blue line: real part. red line: imaginary part. The presence of dips is probably due to the right angle corners.

Figure 5 and Fig. 6 show the map of the modulus of the transverse electric field of the first and second modes.

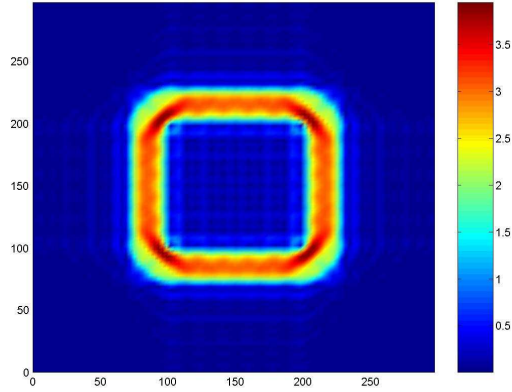


Fig. 5. Modulus of the transverse electric field of the first guided mode.

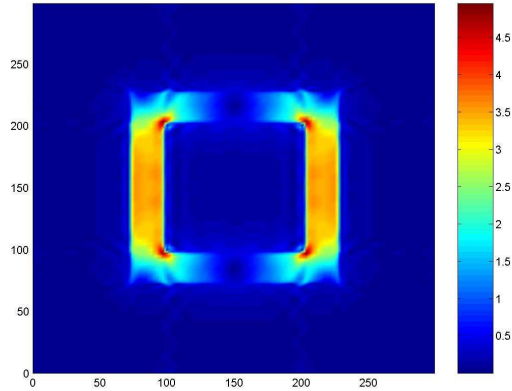


Fig. 6. Modulus of the transverse electric field of the second guided mode.

The mode whose effective index has the largest real part and the lowest imaginary part corresponds to the TEM mode of the same coaxial structure with perfect conducting walls. This mode is characterized by an electric field normal to the walls and has no cut-off. In the present case, it is not strictly speaking a TEM mode since its effective index is greater than one. However, when the width of the aperture becomes larger, the coupling between the opposite sides of the coaxial waveguide diminishes resulting in a lower effective index. The two other guided modes have a cut-off around  $\lambda = 845 \text{ nm}$ .

### 3.2 Analysis of the coupling of the modes to free radiation

We have shown the existence of attenuated guided modes. In the present section we are interested in the way they can be excited by an incident plane wave. The S matrix approach is a very appropriate tool for such an analysis. Let us consider the particular wavelength  $\lambda = 558 \text{ nm}$  where a resonance occurs. Figure 7 and Fig. 8 show the twenty first calculated modal coefficients corresponding to the upwards and downwards waves

inside the aperture when the film is illuminated by a  $x$  polarized plane wave under normal incidence.

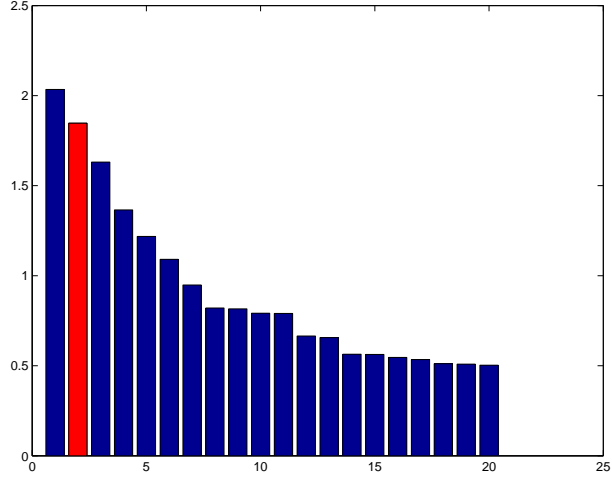


Fig. 7. The twenty first modal amplitude coefficients inside the coaxial on the upper face. The red bar corresponds to an attenuated guided wave.

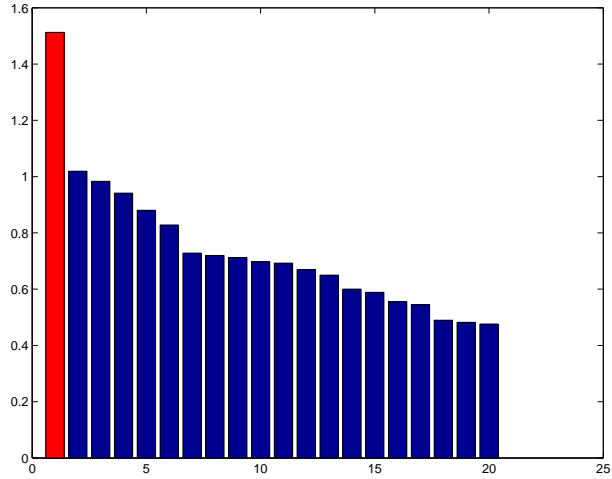


Fig. 8. The twenty first modal amplitude coefficients inside the coaxial on the lower face. The red bar corresponds to an attenuated guided wave.

For convenience, they have been sorted in decreasing order. In order to get some physical information from this spectrum analysis, we have carefully normalized all the eigenvectors. It should be noted that the above coefficients are calculated on the interface where the corresponding wave has been excited. By considering the eigenvalues, ie the normalized propagating constants, one can easily deduce which kind of mode is excited. Figure 9 represents the location in the complex plane of the prograding constant associated to the modal amplitude of Fig. 7 . In Fig. 7, the first and the third mode are a degenerated mode whose the imaginary part of the effective index is as high as 22.8. In the present case, we can conclude that the mode responsible for the resonant transmis-

sion is the attenuated guided mode that matches the polarization of the incident wave. This mode has an effective index of  $1.39 - 0.006i$ .

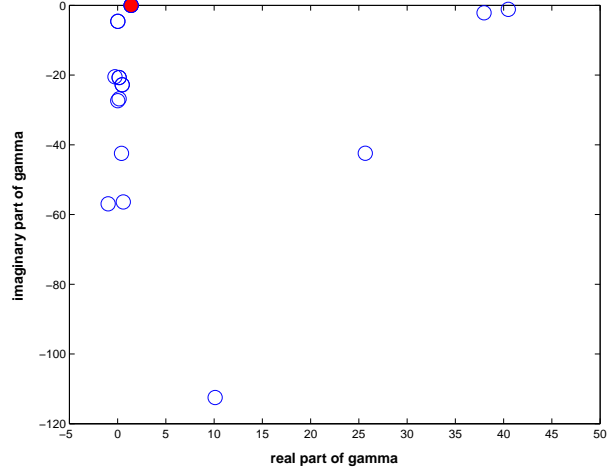


Fig. 9. The ten first complex propagating constants associated to the ten first modes that are excited on the upper face inside the coaxial waveguide. The red one corresponds to an attenuated guide wave, its value is:  $\gamma = 1.39 - 0.006i$ .

#### 4 Conclusion

We have numerically studied the spectral response of subwavelength coaxial apertures. We have calculated the propagating constants of the modes supported by a square coaxial waveguide. Some of them correspond to attenuated guided modes. However, the excitation of such modes is only possible when the incident wave matches the mode profile. Due to the electric properties of metals at optical wavelengths, the dispersion relations of the modes of the transmission channel are very specific and very different from those of the same channel with perfectly conducting walls. This preliminary study paves the way for future investigations in order to engineer the modes and their excitation for applications.