The Argument against Relation Instances in *The Principles of Mathematics*

Bertrand Russell's argument against relation instances in *The Principles of Mathematics* (chapter IV, § 55) is so well-known that it is a paradigmatic argument in analytical metaphysics.[1] But because his argument is paradigmatic now, we don't pay attention to the reasoning framework in which Russell inserts it initially: "[The] argument for this thesis [that if relations are particularized, the *same* relation cannot hold between different set of relata] is extremely curt and is mixed with an analysis of unity constituting a relation complex, or what Russell called at this time a 'proposition'."[2] And on account of this lack of attention, the aim of Russell's argument is not really understood. For instance J. Winslade claims that Russell wants to reject nominalism about universals[3]. The purpose of the following paper is to argue against these interpretations which are unwarranted. The criticism of the relation instances is not only "mixed" with an analysis of the propositional unity, but is really part of Russell's analysis. This paper emphasizes that Russell doesn't argue against nominalism but against F. H. Bradley's idealism. In order to justify this interpretation, we should examine two issues. Why does Russell use an argument about relation instantiation in his analysis of the propositional unity? And why shouldn't relations be instantiated?

I

Everything is a term in *The Principles of Mathematics*: things, predicates and relations are entities which "possess[ ] of all the properties commonly assigned to substances or substantives. Every term, to begin with, is a logical subject […]. Again every term is immutable and indestructible."[4] But whereas things are terms *stricto sensu*, which fulfil only one logical function, concepts, that is, predicates and relations, fulfil several logical functions. Predicates could be assertions or logical subjects; and relations could be relations actually relating or logical subjects. When relations are relating relations, they unify propositional constituents in one entity, the proposition. But because everything is a term, we don't understand the unity of this entity.

Like F. H. Bradley, Russell fails to explain the complex unity. "The arrangement of given facts into relations and qualities may be necessary in practice, but it is theoretically

[1] See D. W. Mertz, *Moderate Realism and its Logic*, Yale University Press, 1996, pp. 174-192.

[2] *Ibid.*, p. 180.

[3] "Russell's Theory of Relations", in E. D. Klemke (ed.), *Essays on Bertrand Russell*, p. 96.

[4] *The Principles of Mathematics*, W. V. Norton Company, 1996, p. 44.

unintelligible. The reality, so characterized, is not true reality, but is appearance."[5] Bradley understands that like him Russell fails to account for this unity. And so he couldn't accept that Russell maintains his realist approach to relations.

> Mr. Russell's main position has remained to myself incomprehensible. On the one side I am led to think that he defends a strict pluralism, for which nothing is admissible beyond simple terms and external relations. On the other side Mr. Russell seems to assert emphatically, and to use throughout, ideas which such a pluralism surely must repudiate. He throughout stands upon unities which are complex and which cannot be analysed into terms and relations. These two positions to my mind are irreconcilable, since the second, as I understand it, contradicts the first flatly.[6]

The independence of all the terms prevents us from understanding their unification in a propositional complex. Russell has great difficulty in accounting for the relational nature of the proposition in the chapter IV of *The Principles of Mathematics.* To account for this we should analyze the proposition in its terms and therefore we lose the proposition as entity. So while we preserve the relational nature of the proposition, we don't manage to analyze the proposition in its terms.

This problem arises when Russell undertakes to characterize relations, terms which are indicated with verbs, and to distinguish them from both things and entities which are indicated with adjectives. Let's look at the chain of ideas about predicates and relations in the chapter IV. In paragraph 48, Russell clearly distinguishes things from predicates. "Socrates is a thing, because Socrates can never occur otherwise than as a term in a proposition: Socrates is not capable of that curious twofold use which is involved in *human* and *humanity*."[7] Unlike things, predicates can be logical subject and assertion. Then he devotes paragraphs 49 to 51 to proving that the predicate doesn't change its nature in these two uses: it is the same term but one which is not in the same (external) relations with the other terms. The predicate specificity is not of a substantial but of a functional nature. "Terms which are concepts differ from those which are not, not in respect of self-subsistence, but in virtue of the fact that, in certain true or false propositions, they occur in a manner which is different in an indefinable way from the manner in which subjects or terms of relations occur."[8] The adjective always indicates an entity. Russell proves this with two arguments. Firstly, the sentences which claim that adjectives couldn't be subject are self-contradictory. Asserting something about predicates is *ipso facto* making them subjects and asserting their substantial nature. Secondly,

[5] F. H. Bradley, *Appearance and Reality*, Oxford University Press, 1969, p. 21

[6] *Mind*, volume 19, 1910, new series, p. 179.

[7] *Op.cit.*, p. 45.

[8] *Ibid.*, p. 46.

like the other concepts, predicates have identity, which characterizes entities, but a special identity, which is both conceptual and numerical.[9]

In paragraphs 52 to 55, Russell applies himself "to discuss the verb, and to find marks by which it is distinguished from the adjective."[10] He begins to explain that, like predicates, relations have two uses: they could be logical subjects and relations relating. If we summarize the paragraphs 52 to 55: in paragraph 52, Russell uses the same argument about relations as about predicates. The sentences are self-contradictory which claim that relations couldn't be subjects. When the relation is a logical subject, the whole proposition is changed into a logical subject. And this change involves a problem: the assertion and unification capacity of the relation is lost. By becoming logical subject, a proposition seems to change nature. Is the proposition still a complex *term*, if "the contradiction which was to have been avoided, of an entity which cannot be made a logical subject, appears to have here become inevitable"?[11] The paragraph 53 deals with the predication relation and attempts to resolve this issue: does the verb "to be" express a relation? In paragraph 54 Russell characterizes the twofold function of the relation: the relation as abstract relation and as relation actually relating. Lastly, in paragraph 55 Bertrand Russell proves that relations cannot be instantiated.

Given the main ideas of the chapter IV, we are now in position to examine their logic in depth. Why does Russell analyse the subject-predicate proposition here? Is this analysis a digression only? Is the reasoning about the non-instantiation a sheer description about relations or does it play a great role with respect to the whole chapter IV? What is the meaning of Russell's thoughts on these pages?

II

At the beginning of paragraph 55 Russell introduces the issue of relation instances:

> It may be doubted whether the general concept *difference* occurs at all in the proposition "*A* differs from *B*", or whether there is not rather a specific difference of *A* and *B*, and another specific difference of *C* and *D*, which are respectively affirmed in "*A* differs from " and "*C* differs from *D*." In this way, *difference* becomes a class-concept of which there are as many instances as there are pairs of different terms; and the instances may be said, in platonic phrase, to partake of the nature of difference. As this point is quite vital in the theory of relations, it may be well to dwell upon it.[12]

He pursues two aims with this issue. First he wants to contend that relations don't have to be reduced to predicates. He emphasizes that relations have to be distinguished from predicates.

[9] *Ibid.*, pp. 46-47.

[10] *Ibid.*, p. 47.

[11] *Ibid.*, p. 48.

[12] *Ibid.*, p. 50.

He begins his reasoning about relations in this way: "It remains to discuss the verb, and to find marks by which it is distinguished from the adjective."[13] And he concludes in the same way: "Verbs do not, like adjectives, have instances, but are identical in all the cases of their occurrence."[14] Instantiation constitutes the criterion for distinguishing relations from predicates.[15] Unlike predicates relations are not particularized.

But this is not the ultimate aim of paragraph 55. Above all, Russell thinks of the relation instantiation as a way for settling the problem of the propositional unity. This a crucial, but not widely appreciated, point. In paragraph 54, he shows that the analysis ruins the propositional unity, and in paragraph 55, he looks for a solution. This solution would be relation instances. But why would relation instances assure the propositional unity?

Russell claims that particularizing relations makes them class-concepts, that is, predicates. He defines the class in an extensional way. The class-concept is the intensional point of view on the class: it denotes class. "A class is a certain combination of terms, a class-concept is closely akin to a predicate, and the terms whose combination forms the class are determined by the class-concept."[16] But how can making relations class-concepts resolve the problem of the propositional unity? Is the problem resolved only if the class-concept, or the predicate, is not independent and should be related to the subject? But doesn't this solution imply an ontology which Russell refuses? But he nevertheless devotes a very long paragraph (§ 55) to this solution. Why doesn't he turn down this solution if it is based on this ontology? Doesn't it indicate that Russell gets caught up in this ontology which only recognizes the subject-predicate proposition and denies the predicate independence?

The ambiguity of the subject-predicate proposition in the chapter IV guides us toward this interpretation. Russell devotes the whole paragraph to its analysis before he attempts to resolve the problem of the propositional unity. "It may be asked whether everything that, in the logical sense we are concerned with, is a verb, expresses a relation or not."[17] In paragraph 48, he contends that intransitive verbs symbolically indicate relations, "a definite relation to an indefinite relatum,"[18] while it doesn't grammatically seem to indicate a relation. The nature

[13] *Ibid.* , p. 47.

[14] *Ibid.* , p. 52.

[15] We disagree William J. Winslade, *op.cit.* p. 96.

[16] *Op. cit.* , p. 55.

[17] *Ibid.* , p. 49.

[18] *Ibid.*, p. 44.

of "to be" which is used as copula in the tradition of subject predicate proposition is more delicate.[19] In paragraph 48 Russell writes that verbs "are always or almost always relations".[20] What verbs could not indicate relations? Transitive verbs clearly express relations and in spite of the grammar, intransitive verbs also express them. The copula remains. Moreover in the same paragraph Russell emphasized the copula as verb: "Thus we shall say that 'Socrates is a human' is a proposition having only one term; of the remaining components of the proposition, one is the verb, the other is a *predicate*."[21] So the paragraph is about the copula.

His thoughts about "to be" are confused. Let us quote the complete paragraph 53 which is symptomatic of this confusion.

> It may be asked whether everything that, in the logical sense we are concerned with, is a verb, expresses a relation or not. It seems plain that, if we were right in holding that "Socrates is a human" is a proposition having only one term, the *is* in this proposition cannot express a relation in the ordinary sense. In fact, subject-predicate propositions are distinguished by just this non-relational character. Nevertheless, a relation between Socrates and humanity is certainly *implied*, and it is very difficult to conceive the proposition as expressing no relation at all. We may perhaps say that it is a relation, although to be regarded as an assertion concerning the referent. A similar remark may apply to the proposition "*A* is", which holds of every term without exception. The *is* here is quite different from the *is* in "Socrates is human"; it may be regarded as complex, and as really predicating Being of *A*. In this way, the true logical verb in a proposition may be always regarded as asserting a relation. But it is so hard to know exactly what is meant by *relation* that the whole question is in danger of becoming purely verbal.[22]

What is a relation in the ordinary sense? "A relation between two terms is a concept which occurs in a proposition in which the interchange of the two terms gives a different proposition."[23] Relations relates one term from which they proceed, the referent, to another to which they proceed, the relatum.

In predication there is not relatum, not even an indeterminate relatum, like in the case of intransitive verbs. That's the reason why the subject-predicate proposition only contains a

[19] Peter Hylton incidentally remarks: " Russell sometimes suggests that there is a third element in a proposition of this form, corresponding to the copula 'is'; but his position on this seems to remain vague or agnostic", in Richard Rorty, J. B. Schneewind and Quentin Skinner (ed.), *Philosophy in History*, Cambridge University Press, 1984, p. 376.

[20] *Op. cit.*, p. 44.

[21] *Ibid.*, p. 45.

[22] *Ibid.*, p. 49.

[23] *Ibid.* , p. 95.

"pseudo-relation of subject to predicate"[24]. But maintaining that it expresses a pseudo-relation is actually admitting that it is a relational proposition.

In paragraph 53 Russell asserts two contradictory claims: the subject-predicate proposition shouldn't express a relation and however it should express this type of relation: "it is very difficult to conceive the proposition as expressing no relation at all". Why it is so difficult? Firstly, on account of the logical-grammatical parallelism: "it must be admitted, I think, that every word occurring in a sentence must have *some* meaning"[25]. Secondly, because the predicate and its subject both are entities which are ontologically distinct. So a relation should relate them each other to constitute a genuine proposition. Propositions are complex unities, that is, entities composed of several distinct terms. If there is no relation between the subject and the predicate, either the subject-predicate proposition is only an aggregate without unity and is not a genuine proposition, or the predicate is in the subject and the subject-predicate proposition is not a proposition, since the subject and the predicate should be distinct in order to form a proposition.

That is why in the proposition "Socrates is human" "a relation between Socrates and humanity is certainly *implied*", that is to say that another proposition, which contains a relation among its constituents, could proceed from this proposition: "Socrates has humanity". But Russell muddies the waters when he writes: "The relation which occurs in the second type (Socrates has humanity) is characterized completely by the fact that it implies and is implied by a proposition with only one term, in which the other term has become a predicate."[26] In which is the subject-predicate proposition explained here? This reasoning leads to a defeating circularity. The subject-predicate proposition should be explained with another proposition. But the latter should be explained with the former! And Russell claims that "the two propositions can be clearly distinguished, and it is important to the theory of classes that this should be done."[27]

Why does Russell preserve the subject-predicate proposition while its nature is unintelligible? Why doesn't he accept that there is no subject-predicate proposition and that the grammatical predicative form indicates a relational proposition, as he claimed it in

[24] *Ibid.*, p. 95.

[25] *Ibid.*, p. 42.

[26] *Ibid.*, p. 55.

[27] *Ibid.*, p. 54.

1899?[28] In 1903, in contrast, he maintains a position which seems archaïc: "Logically, the fundamental relation is that of subject and predicate […] a relation which […] is peculiar in that the relatum cannot be regarded as a term in the proposition."[29] By writing that "the relatum couldn't be regarded as a term in the proposition", Russell nevertheless acknowledged that the subject-predicate contained a relatum. What is this term?

Russell was eventually able to resolve the problem of the subject-predicate proposition with his theory of denoting. The relatum is not contained in the subject-predicate proposition, but what the proposition contains is the concept which indicates the term. "But the fact that description is possible — that we are able, by the employment of concepts, to designate a thing between some concepts and some terms, in virtue of which such concepts inherently and logically *denote* such terms. It is this sense of denoting which is here in question. This notion lies at the bottom (I think) of all theories of substance, of the subject-predicate logic, and of the opposition between things and ideas, discursive thought and immediate perception."[30] The proposition "Socrates is a man" entirely exhibits the subject-predicate proposition "Socrates is human".

But as "the sense of '*A* is a-man' […] is very like identity"[31], couldn't be said that the sense of the subject-predicate proposition is almost the same that the sense of the identity proposition? The subject-predicate proposition would equal the identity proposition. Indeed they partake of the characteristic of not being really relational: "identity must be admitted, and the difficulty as to the two terms of a relation must be met by a sheer denial that two different terms are necessary. There must always be a referent and a relatum, but these need not be distinct; and where identity is affirmed, they are not so."[32] In the subject-predicate

[28] "Whether there are, fundamentally, any propositions of subject and predicate may be doubted. Where the subject is not a *thing*, as in 'red is a colour', the judgment is primarily one of inclusion in a class, and whether classes must be constituted by common predicates is a difficult question. When the subject is a *thing*, as in 'The chair is red', or better, 'this is red', the proposition seems generally reducible to the existence of one or more qualities in a temporal or spatio-temporal place. Since occupation of a place is formally a one-sided relation, like the relation of subject and predicate, there is no prima facie bar to a reduction of one to the other. In any case, there are two one-sided relations, i.e. occupation of a part time and of a part space." In "The Classification of Relations" in *The Collected Papers of Bertrand Russell, volume 2. Philosophical Papers 1896-1899*, London, Unwin Hyman, pp. 145-146.

[29] *Op. cit.*, p. 77.

[30] *Ibid.*, p. 53.

[31] *Ibid.*, p. 64.

[32] *Ibid*., p. 64.

proposition, the relation would relate the subject to itself and that is why the relation wouldn't appear in the sentence. We could conclude that in one way Russell maintains the copula "is", which could be reduced to the "is" of identity.

Russell's reasoning about the subject-predicate proposition is not an insignificant digression in chapter IV. The subject-predicate proposition should help us to understand the propositional unity. Russell makes the subject-predicate proposition the fundamental proposition because it would be the simplest. "The simplest of propositions are those in which one predicate occurs otherwise than as a term, and there is only one term of which the predicate in question is asserted. Such propositions may be called subject-predicate propositions".[33] This assertion is surprising. How could the subject-predicate proposition be the simplest, when it expresses a relation without containing a relatum? How should this simplicity be understood? The subject-predicate proposition is the simplest because it only contains one term. And on account of it its unity wouldn't be problematic.

The paper "Meinong's Theory of Complexes and Assumptions" (1904) clearly points out that the subject-predicate proposition is not problematic for Russell at this time. Russell excludes the relation from the proposition. He agrees with Meinong, who asserts that "the concomitance of complexes and relations cannot be maintained universally, since a complex may be thought without any presentation of the relation".[34] If Russell approves this thesis, it is because he understands every proposition by means of the subject-predicate pattern. Meinong explains his thesis with a subject-predicate proposition "the cross is red", and Russell accounts for it with genuine relational propositions. Russell argues for this thesis with the famous argument of F. H. Bradley: if the relation is as term in a proposition, it should be related to its relata with relations which should be themselves related to their relata with relations and so on. This reasoning leads to a vicious endless regress. [35] This argument points out that the relation couldn't be in the proposition either as abstract relation or as particularized relation. By excluding the relation from the proposition, the difficulty in understanding the propositional unity would be removed. But it is a flawed solution. Russell analyzes two propositions: "A is the father of B" and "fatherhood holds between A and B". Unlike the latter, the former wouldn't be about fatherhood. The fatherhood relation wouldn't be in the former. But isn't it a grammatical sleight of hand which hides the relation, because

[33] *Ibid.*, p. 54.

[34] "Meinong's Theory of Complexes and Assumptions", in *The Bertrand Russell 's Collected Papers, volume 4. Foundations of Logic, 1903-1905*, London and New York, Routledge, 1994, p. 456.

[35] *Ibid.*, p. 456.

the copula counts for nothing? In the second proposition there are three terms *stricto sensu*: fatherhood, A, and B. And there is a new relation, "to hold between". This relation really is in the proposition; it is the relation relating of the proposition and fatherhood is as subject. In what sense is this relation relating? Russell seems to generalize to any proposition a conclusion which is drawn from his investigation about subject-predicate propositions. "In some sense which it would be very desirable to define, a relational proposition seems to be *about* its terms, in a way in which it is not *about* relation."[36] Through the subject-predicate proposition he turns out the relation whose role in the unification of propositions remains mysterious.

Why does he examine the subject-predicate proposition when he reasons about the propositional unity? He asserts that predicates are particularized, instantiated. If relations could be particularized, they are like predicates. And if they can be like predicates, any proposition is a subject-predicate proposition. Now as he confusedly conceives the subject-predicate proposition as the pattern of the propositional unity, the propositional unity is no longer problematic.

III

Let us analyze in depth the argument against relation instances. Why shouldn't the relations be instantiated? While this argument is widely used about particulars and universals, Russell has another aim. He sees it as a realistic argument against Bradley's idealism. He argues for the relation non instantiation against Bradley, who concludes that understanding the relation as an independent entity in the proposition leads to a vicious endless regress and makes the propositional unity unintelligible. But Bradley's reasoning doesn't mean that he argues for internal relations or for relation instances, as Russell interprets it. [37] Russell's argument shows the flaws in his interpretation of Bradley. Russell grounds his criticism on this argument because he confuses the independence of relations with their non-instantiation. And he therefore confuses their instantiation with their dependence, which he translates into a theory of internal relations.

[36] *Ibid*., p. 456.
[37] See F. H. Bradley, "Relations", in *Collected Essays*, Freeport, New York, Books For Librairies Press, 1968, p. 643.

Universals are particularized if besides universals and particulars there are instances of universals. In 1903 the predicate particularization is obvious. [38] In the paper "Do Differences Differ?" Russell examines the relation instantiation in depth. While the previous texts conceive the instantiation from a spatio-temporal pattern, in this paper Russell asks this question in the respect of the role played by the relation in the proposition. He is interested in a specific relation, difference, because "mere difference *per se* appears to be the bare *minimum* of a relation, being in fact a precondition of almost all relations."[39]

He wants to understand what the concept of difference is in the complexes "Red differs from blue" and "Identity differs from difference". "Does the difference between red and blue differ from the difference between identity and difference? […] Both differences agree in being difference. This, in some sense, is obvious; but to discover the exact sense is difficult. That there is some abstract concept *difference* of which, in Platonic phrase, both differences partake, is sufficiently shown by the plural. But it is far from easy to specify the relation of *a* difference to difference."[40] He examines three ways of defining the relation of difference to differences. Firstly, difference is in the complex as itself, it is not particularized, so difference is the same in all its occurrences. Secondly difference is in complexes. Difference is a complex notion which associates the difference and a specific quality which distinguishes the difference in this complex from the difference in another complex. Finally, it is not the difference itself which is in the complex but a particularized difference, peculiar to this complex. And particularized differences are related to the difference by means of a relation of inclusion in a class. [41]

As in "Do Differences Differ?", in paragraph 55 of *The Principles of Mathematics*, Russell refers to Platonic partaking but in a more precise sense than in the draft of 1900, where the notion of partaking is the vague notion which expresses that difference is somehow in the complexes. In 1903 partaking characterizes the relation of difference to differences as particularization, instantiation. "In this way, *difference* becomes a class-concept of which there are as many instances as there are pairs of terms; and the instances may be said, in Platonic phrase, to partake of the nature of difference."[42] And as in 1900, Russell considers

[38] See Thomas R. Foster, "Russell on Particularized Relations", *in Russell: the Journal of Bertrand Russell Archives, new series, volume 3, n° 2, winter 1983-1984*, pp. 129-143.

[39] *The Principles of Mathematics*, p. 172.

[40] *Op. cit.*, p. 555.

[41] *Ibid*., p. 555.

[42] *Op. cit.* , p. 50.

the three same possible relations of difference to differences. He analyzes these three possibilities in the same way in both texts. The analysis of difference as notion composed of abstract difference and a quality peculiar to a specific complex, is referred to the analysis of the two other possibilities, because it combines them.

Although these texts resemble each other in a few respects, they draw opposing conclusions. In 1903 Russell concludes that difference is not particularized, it is itself in complexes. So it is not a class-concept. In 1900 he claims, however, that it is not an abstract relation which relates terms, but a specific relation peculiar to a complex.

> The doctrine in question may be extended to all relations. Any relation which actually relates two terms must be incapable of relating any others; thus there is only one proposition in which it is related. Between two relations of the same class there is numerical diversity, as between two points or two colours; also each has to some one single pair of terms (and to no others) the relation of relating them, which again must of course be unique in each case.[43]

Why do these texts contradict each other? In 1900 Russell refuses the non-instantiation of relations. The thesis of non-instantiation leads Russell to analyze the proposition "A differs from B" as A, difference, B. The analysis breaks the propositional unity. While in 1903 this loss of unity is the starting-point of the reasoning, in 1900 it is an argument against the abstract relation in the complex. Another relation R should relate difference to A and B, so that the unity is not lost. But because this new relation R is not instantiated, is an abstract relation, one more relation R' should relate R to difference, A and B. This reasoning leads to an endless regress which is vicious because it prevents us from understanding the meaning of the proposition. Russell uses the Bradley's argument with which Bradley argued against relation realism:

> 'There is a relation *C*, in which *A* and *B* stand; and it appears with both of them.' But here again we have made no progress. The relation *C* has been admitted different from *A* and *B*, and no longer is predicated of them. Something, however, seems to be said of this relation *C*, and said, again, of *A* and *B*. And this something is not to be the ascription of one to the other. If so, it would appear to be another relation, *D*, in which *C*, on one side, and, on the other side, *A* and *B*, stand. But such a makeshift leads at once to the infinite process. The new relation D can be predicated in no way of *C*, or of *A* and *B*; and hence we must have recourse to a fresh relation, *E*, which comes between *D* and whatever we had before. But this must lead to another *F*; and so on, indefinitely[44].

[43] *Do Differences Differ?*, p. 557.

[44] *Appearance and Reality*, pp. 17-18.

In 1900, on account of this vicious endless regress, Russell claims that relations should be instantiated. In 1903 he draws the opposite conclusion. This endless regress doesn't argue against the relation as abstract relation in the proposition. It argues against a way of explaining the propositional unity by relating the relation to the terms which it relates, with another relation.

> And we saw that the attempt to avoid the failure of analysis by including in the *meaning* of "*A* differs from *B*" the relations of difference to *A* and *B* was vain. This attempt, in fact, leads to an endless process of the inadmissible kind; for we shall have to include the relations of the said relation to *A* and *B* and difference, and so on, and this continually increasing complexity we are supposed to be only analysing the *meaning* of our original proposition this argument establishes a point of very great importance, namely, that when a relation holds between two terms, the relation and the terms, and so on *ad infinitum*, though all implied by the proposition affirming the original relation, form no part of the *meaning* of this proposition.[45]

Russell comments on his reasoning in this way: "But the above argument does not suffice to prove that the relation of *A* and *B* cannot be abstract difference". But does he render harmless the argument by making it an argument against a certain way of restoring the propositional unity, and not really an argument about the relation particularization? The meaning of the argument is as follows: because analyzing proposition by introducing relations between terms and their relation into the proposition meaning leads an endless regress, the propositional unity shouldn't be explained in this way. These additional relations cannot be part of the proposition. They are only implied by the proposition. But this reasoning is purely dogmatic and doesn't prove what it is supposed to prove.

Russell maintains his idea of proposition as a unity of independent terms at all costs, without succeeding in accounting for propositional unity. He doesn't cogently criticize Bradley's argument. On the one hand, he cannot accept the idealist conclusion which Bradley draws from his incapacity to understand propositional unity, and on the other hand he doesn't suggest a genuine answer to the issue of propositional unity. He seems not to have moved an inch since he observed in paragraph 53 that: "the true logical verb in a proposition may be always regarded as asserting a relation. But it is so hard to know exactly what is meant by *relation* that the whole question is in danger of becoming purely verbal."[46] What would be a proposition which loses his relating power when it is no longer in the proposition? How could we assert that it is a genuine term which possesses a specific logical function? Russell doesn't resolve this problem.

[45] *The Principles of Mathematics*, p. 51.

[46] *Ibid.*, p. 49.

In 1903, in order to show that relations are not instantiated, he criticizes the thesis that there are relation instances. Why can't this thesis account for the propositional unity? Russell uses two arguments. Firstly: "For, even if the difference between of A and B be absolutely peculiar to A and B, still the three terms A, B, difference of A from B, do not reconstitute the proposition "A differs from B", any more than A and B and difference did."[47] The particularization doesn't resolve the issue of the propositional unity any better than does the non-instantiation. And so why does Russell prefer the latter? Because it avoids an ontological increase? No, rather because the thesis of particularization leads to the internal relation theory, and rejects the relation independence.

In 1900 Russell maintains that there are relation instances, because with this particularization the relation keeps its relying power in the proposition as analysed. For instance, the proposition "A differs from B" is analyzed as A, the difference between A and B, and B. Only a relation which explicitly refers to its relatum and its referent in its meaning is a genuine relation. Difference in the abstract is only a class-concept and is not a relation which possesses a relying power. "Difference in the abstract relates nothing, but is related to differences as *Point* to points."[48] Then the relation instantiation poses the problem of specific relation independence. How can specific relations be independent and be genuine terms if their meaning is only understood with terms which they relate? Specific relations could be complex terms. But the issue which they should resolve recurs in regard to themselves. How would these complex terms be composed? Of the relation and the terms which it relates. How is the relation related to its terms? So we understand why the problem of relation instantiation is a "quite vital [point] in the theory of relations"[49]. The nature of relation as term hangs in the balance. Unfortunately, neither of these opposing theses justify the claim that relations are genuine terms. If relations are not instantiated, we don't understand why relations would possess a specific logical function which distinguishes them from things and predicates. And if relations are instantiated, relations depend on the terms that they relate.

Secondly, Russell argues against relation instances that difference cannot be a class-concept. To belong to the same class, the class of difference, differences should be related to difference in the same way.

[47] *Ibid.*, p. 51.

[48] *Do Differences Differ*? , p. 557.

[49] *Op. cit.*, p. 50.

> But the most general way in which two terms can have something in common is by both having a given relation to a given term. Hence if no two pairs of terms can have the same relation, it follows that no two terms can have anything in common, and hence different differences will not be any definable sense *instances* of difference.[50]

For example, there are three instances of difference: the difference between A and B (1), the difference between B and C (2) and the difference between C and D (3). What do these instances have in common? They are instances of difference: (1) instantiates difference, (2) instantiates difference, and (3) instantiates difference. If we accept the thesis of relation instantiation, instantiation, which is a relation, should be instantiated too. But if instantiation is instantiated, there are no difference instances, since (1), (2) and (3) couldn't have the same relation to difference.

As F. Rodriguez-Consuegra has shown, Russell in this example uses the principle of abstraction: "At this point Russell applies the principle of abstraction: to have something in common is nothing but to be in a certain relation to a given term"[51] "And it seems plain that, even if differences did differ, they would still have to have something in common. But the most general way in which two terms have something in common is by both having a given relation to a given term."[52] To differ differences should have something in common, that is, they should instantiate difference. This common property, being an instance of difference, expresses being in the same relation to a third term, the difference. Russell proves that the theory of relation instances is inconsistent with its own base: it makes the principle of abstraction impossible and therefore refutes itself. The principle of abstraction, which defines instantiation, is based on the identity of relation to a third term: instantiation is only possible if the relation of instantiation is not particularized.

This analysis proves that Russell is not here concerned with the issues about the nature of universals as a priority. He uses a conception of instantiation based on the principle of abstraction to argue against the idealist thesis which, according to Russell, leads to the claim that the relation non-instantiation implies an idealist conclusion. His remark about the nature of universals is of secondary importance. He draws from this argument a digression about universals in a footnote:

> The above argument appears to prove that Mr Moore's theory of universals with numerically diverse instances in his paper on Identity (*Proceedings of the Aristotelian Society*, 1900-1901) must not be

[50] *Ibid.*, p. 51.

[51] Francisco Rodriguez-Consuegra, *The Mathematical Philosophy of Bertrand Russell: Origins and Development*, Basel, Boston, Berlin, Birkhäuser Verlag, 1991, p. 218.

[52] *Op. cit.*, p. 51.

> applied to all concepts. The relation of an instance to its universal, at any rate, must be actually and numerically the same in all cases where it occurs.[53]

Establishing whether some universals are instantiated is not the ultimate issue. But Russell takes advantage of his reasoning against idealism to emphasize its consequences with regard to another dispute which is in itself an important one. Should every universal be instantiated? Russell remarks that only predicates should be instantiated. Because relations cannot be instantiated, predicates can be instantiated, predicate instances can partake of the same relation to predicates that they instantiate.

IV

The argument against relation instantiation shows how the conception of the proposition in *The Principles of Mathematics* is flawed. To explain that the proposition is a term, possesses unity, Russell comes to examine the thesis of relation instantiation. But the instantiation of relations cannot resolve the problem of propositional unity since the instantiation of relations is inconsistent with the notion of complex term. So he rejects this method of examining the propositional unity. Part of this examination of propositional unity is a criticism of F. H. Bradley. Russell believes that Bradley claims relation instances because Bradley shows that relation independence leads to an endless regress. And that is why Russell defends the opposing thesis: relations shouldn't be instantiated. But this thesis does no better in accounting for propositional unity.

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[53] *Op. cit.*, p. 51-52.