Heuristic for the topological design of communication networks.

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Topological optimization of networks is a complex multi-constraint and multi-criterion optimisation problem in many real world fields (telecommunications, electricity distribution etc.). This paper describes an heuristic algorithm using Binary Decisions Diagrams (BDD) to solve the reliable communication network design problem (RCND) [2] [4]. The aim is to design a communication network topology with minimal cost that satisfies a given reliability constraint. We assume that a set of perfectly reliable nodes and their topology are given, along with a set of possible communication links that connect them. Each link has a known reliability and is assigned a cost. The relevant reliability metric is *all-terminal network reliability* defined as the probability that the network is still connected even if some links fail. A trade-off between the investments in the network and the quality of service provided to the users must be found. Both the RCND problem and the network reliability computation, have been proven to be NP-hard [1].

1 RCND problem

In this problem, a network topology with minimal cost must be found that satisfies the all-terminal network reliability constraint. A network is modeled by an undirected stochastic graph $G = (V, E, p, c)$ with V its vertex set (representing sites) and $E \subseteq V \times V$ its possible edge set (representing the $m = |E|$ candidate links). Each link can fail, statistically independently, with known probability. The failure probability of a link e_i is denoted q_i ($q_i \in [0, 1]$) and its reliability p_i ($p_i = 1 - q_i$). Hence, p represents a probabilistic vector $(p_1, \ldots, p_m)$. At each edge e_i is associated a cost c_i ($c = (c_1, \ldots, c_m)$). The objective function of this problem is stated as :

$$\text{Minimize } C(G') = \sum_{i=1}^{m} c_i x_i$$
$$\text{subject to : } R(G') \geq R_0$$

where :

- The boolean variable x_i ($x_i \in \{0, 1\}$) is equal to 1 if the edge e_i exists in the solution and 0 otherwise.
- c_i is the cost of the link $e_i \in E$ ($i \in \{1, \ldots, m\}$).
- $G' = (V, E')$ is a partial graph of G such that $E' = \{e_i \in E / x_i = 1\}$.
- $C(G')$ is the total cost of network G'.
- $R(G')$ is the network all-terminal reliability of G'.
- R_0 is the minimum reliability requirement.

The objective function is the sum of the total cost for all chosen network links. The RCND problem consists in finding a partial graph $G' = (V, E')$ of G ($E' \subseteq E$) with minimal cost such that $R(G') \geq R_0$.

2 All-terminal network reliability computation

Let X_i be the binary random variable "state of the link e_i in G", defined by $X_i = 1$ if link e_i is operational, and $X_i = 0$ if link e_i is down. $X = (X_1, \ldots, X_m)$ is the *random network state vector*. Given a graph state $\mathcal{G} = (x_1, x_2, \ldots, x_m)$ ($x_i \in \{0, 1\}$). The associated probability of $\mathcal{G}$ is defined as :

$$Pr(X = \mathcal{G}) = \prod_{i=1}^{m} (x_i.p_i + (1 - x_i).q_i)$$

By definition, the *network structure function* $\phi : \{0, 1\}^m \to \{0, 1\}$ is defined as follows :

$$\begin{cases} \phi(X) = 1 & \text{if } G_X \text{ is connected} \\ \phi(X) = 0 & \text{otherwise} \end{cases}$$

where G_X is the partial graph of G with edge set $E_X = \{e_i \in E, x_i = 1\}$.

The all-terminal network reliability is then given by :

$$R(G;p) = \sum_{\mathcal{G}} Pr(X = \mathcal{G}).\phi(\mathcal{G}) = Pr(\phi = 1)$$

We use the BDD-based algorithm, presented in [3], to compute $R(G;p)$. Based on BDD properties, the reliability of a same network for different values of p_i could be computed. The network reliability can be then re-evaluated without BDD construction each time the failure probability of one or many links is changed. As some of the links are more important than others in network reliability, we propose the following network importance measure for ranking the network links :

$$I_k^N = \frac{1}{c_k} * (R(G;p) - R(G/\ e_k \textit{ is down}; p)) = \frac{1}{c_k} * (Pr(\phi = 1) - Pr(\phi_{x_k=0} = 1))$$

I_k^N is the ratio of the degradation of the network reliability if link e_k is down and the cost of this link.

3 NDImprovement greedy algorithm

The NDImprovement greedy algorithm is divided into two main steps :

- Starting from a maximal theoretical network, the algorithm tries to lower the network reliability to R_0 by removing one edge (*i.e.* by fixing its functioning probability to 0) with the lower network importance measure. This process is applied until reaching R_0. A link with a functioning probability fixed to 0 is considered as not present in the solution.
- Starting from this first solution, we try to find a better solution by switching selected edges for discarded edges with lower cost.

Problem			**LS/NGA**		**NDImprovement**	
$\lvert V\rvert$	$\lvert E\rvert$	*Search space*	*Mean % from optimal*	Mean CPU sec. (P 133MHz)	*Mean % from optimal*	Mean CPU sec. (P 3GHz)
8	28	$2.68 * 10^8$	0.889	118.75	0.0224	0.12
9	36	$6.87 * 10^{10}$	1.050	203.38	0.0343	1.07
10	45	$3.15 * 10^{13}$	1.094	458.93	0.0303	9.65

Tableau 1 – Summary of our approach and comparison to LS/NGA

This algorithm has a running time of $O(m^3.B_w)$ where w presents the *linear-width* of the network and B_w is the number of partitions of w elements. Our test problems are taken from [2]. For all these test problems, NDImprovement gives always a better solution than the genetic algorithm LS/NGA [2] in term of nearness to optimality (Tab. 1).

4 Discussion and conclusion

In this paper, we have provided a framework for topological design of communication networks using the BDD structure. The NDImprovement algorithm gives good initial solution. We are currently working on a taboo search method in order to improve these results.

Références

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