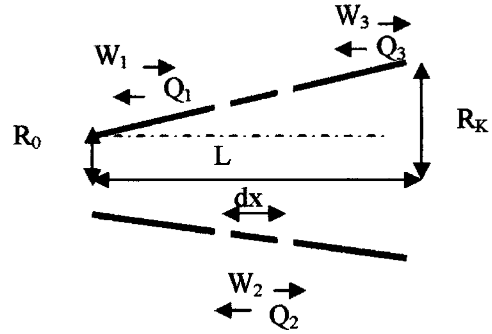

FIG. 9. Addition of several cones with different transmission probabilities.

by associating several cones of transmission probability in the direct W and back Q directions [see Fig. 9]. For each passage of molecules, one enters the collision number on the surface of the length element dx while applying, according to the direction, Eq. (19) or (20). The probabilities W_2 and Q_2 tend towards 1 and by simulating a flow and a capacity of pumping, we get a total collision number according to x, for an element dx,

$$N_{\text{choc}} = N\frac{2dx\tan(\alpha)}{R(x)}\underbrace{\left\{f(\alpha)\left(\frac{2-W_3}{W_3}\right)+\left(\frac{1-W_3}{W_3}\right)+\frac{Q_3}{W_3}\frac{(1-\gamma)[2f(\alpha)+1]}{\gamma}\right\}}_{h}, \tag{21}$$

with $Q_3/W_3=R^2(x)/R_k^2$, W_3 is of course a function of x and can be expressed on the basis of Eqs. (6) and (7) by

$$\frac{1}{W_3}=1+\frac{[R(x)+R_k]}{\frac{16}{3}R_k^2k[L-x/R(x)]}(L-x)$$

$$\text{and } R(x)=R_0+x\tan(\alpha),$$

the factor k varies slightly according to L/R_0 (see Fig. 2), for an angle of 40°, k varies from 4.5 to 5.3 for, respectively, a $L/R_0=1$ and $L/R_0=10$. We can roughly express this variation as an equation. For a direct cone of 40°, $k(t=L/R_0)\approx(4.5+2.65t+2.72t2)/(1+0.56t+0.5t2)$.

1. Calculation of the surface distribution by the method known as the extension of Oatley

By estimating that the number of shocks per unit time on the surface of the cone length dx is $\Gamma=(1/4)V_mnS(dx)$ (with n molecular density), the surface distribution of pressure is expressed as

$$P_{S_1}(x)\approx\frac{QN_{\text{choc}}}{(1/4)V_mS(dx)N}=\frac{Q\sin(\alpha)}{(1/4)V_m\pi R(x)^2}h. \tag{22}$$

2. Calculation of the surface distribution by the traditional method (Knudsen)

As in the case of the tube, by deriving the $Q=\delta CdP$ equation (with δC intrinsic cone conductance given by Knudsen and for a length dx) and while taking $P_{s_1}(0)$ as initial condition (obtained by the Oatley method), this distribution of surface pressure can be written as follows:

$$P_{S_2}(x)=\int\frac{-Q}{(2/3)V_m\pi k}\frac{dx}{R(x)^3}=\frac{Q}{(4/3)V_m\pi k\tan(\alpha)}\left[\frac{1}{R(x)^2}-\frac{1}{R_0^2}\right]+P_{S_1}(0). \tag{23}$$

3. Extension of the Oatley method for a cone in the back direction

In the same manner, we can determine the surface pressure distribution for a cone in the back direction, with, in this case, $R(x)=R_k-x\tan(\alpha)$, a corrective factor $k'=k/[1+(16/3)k\tan(\alpha)]$ and with a collision number,

$$N_{\text{choc}-R}=N\frac{2dx\tan(\alpha)}{R(x)}\left\{f(\alpha)\left(\frac{1-Q_3}{Q_3}\right)+\left[\frac{1+f(\alpha)}{Q_3}\right]+\frac{W_3}{Q_3}\frac{(1-\gamma)(2-f(\alpha)+1)}{\gamma}\right\}. \tag{24}$$

It is obvious that this is only an approximation which does not take into account the beam effects (the changes of the spatial distribution of the molecules for the different passages). According to Fig. 10, one can note that Eq. (23) derived from the conductance calculated by Knudsen is not valid, or only for one correction factor k equal to 1.

In this case, as in the extension of the Oatley method, we obtain a relatively good approximation of the surface pressure compared to simulation by Monte Carlo. This approximation improves when the pumping speed decreases. Figure 10 shows the influence of a pump on the surface pressure distribution at the cone exit. This pump will modify the

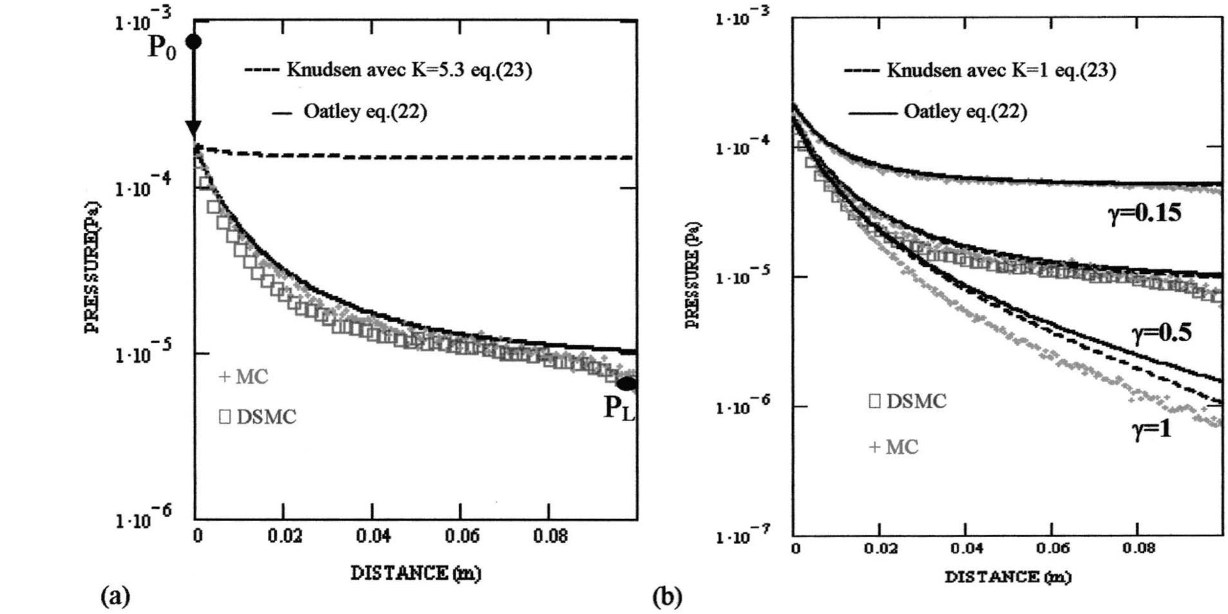

FIG. 10. Surface pressure distribution for conical conductance of "direct" direction for nitrogen at 20 °C, with a flux 2.8×10^{-5} Pa m^3 s^{-1}, for different sticking coefficients, for $L/R_0=10$ with $r=1$ cm and $\alpha=40.28°$.

spatial distribution of the molecules at the cone exit. In the case of the large sticking coefficient, the spatial distribution of the molecules at the cone exit is not Lambert's law. For the back cone, the approximation by the extension of the Oatley method is satisfactory and that for all pumping speeds (Fig. 11). The good agreement between these two methods encourages to consider the spatial distribution on the exit tube as Lambert's law. The Knudsen equation is not very sensitive to the correction factor k and greatly diverges when approaching downstream opening and for high pumping capacities.

V. COMPARISON BETWEEN MEASUREMENT AND SIMULATIONS

The experimental setup (see Fig. 3) is formed by the junction of a back cone and a direct cone. It is necessary for the extension of the Oatley method to determine the sticking coefficients corresponding to a turbo pumping speed of $S_0=125$ l s^{-1}, for the second cone $\gamma=S_0/C_{e2}=0.135$ and for the first cone $\gamma1=0.768$ first [with $C_{A,2}=798$ l s^{-1} for $k=4.8$ and $1/(C_{e1}\gamma1)=1/125-1/C_{e2}+1/C_{A,2}+1/C_{e1}$]. By the Knudsen method, it is enough to fix the pressure at the entry of the first cone or the exit of the second with

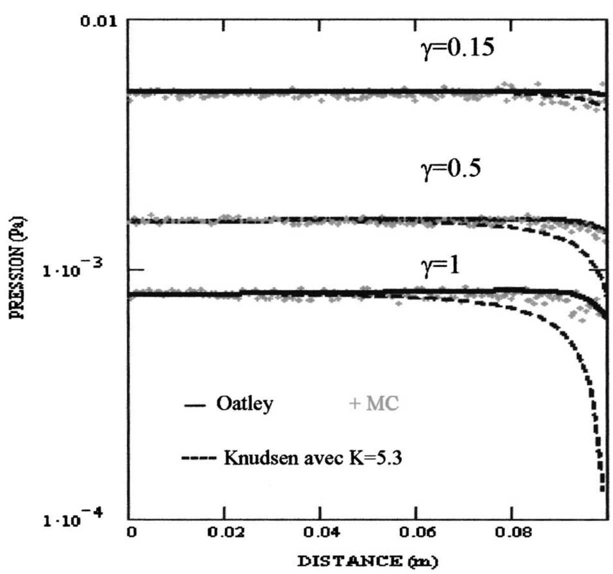

FIG. 11. Surface pressure distribution for conical conductance of "back" direction for nitrogen at 20 °C, with a flux 2.8×10^{-5} Pa m^3 s^{-1}, for different sticking coefficients, for $L/R_0=10$ with $r=1$ cm and $\alpha=40.28°$.

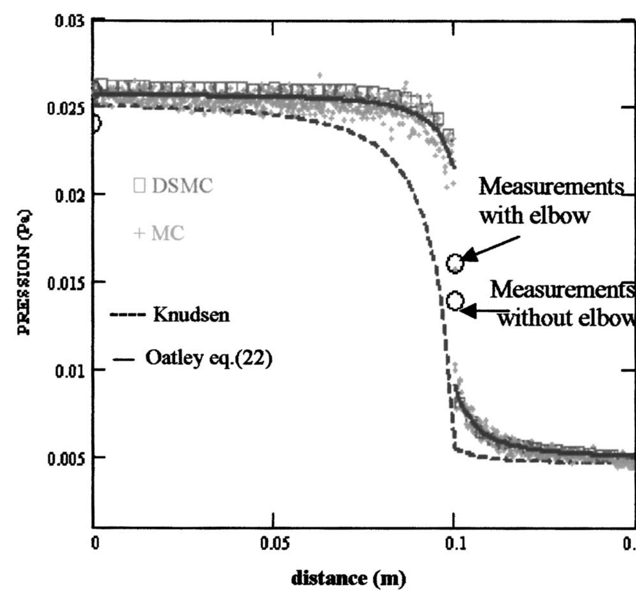

FIG. 12. Surface pressure distribution on the two conical tubes of experimental device for a pumping speed of 125 l/s.

$P_1 \approx Q/(1/S_0 - 1/C_{e2} + 1/C_{A,2} + 1/C_{R,1})$ or $P_0 = Q/(1/S_0 - 1/C_{e2})$. For simulations by Monte Carlo, we simulate the two cones and as well as the turbo pump at the system exit as a unique system and we take as sticking coefficient of $\gamma = 0.135$.

Figure 12 shows a good correspondence between simulation by Monte Carlo and Oatley. A small variation is noted with the points of measurements taken without a prepumping elbow but remains within the error margin. The pressure recorded by the two spinning rotor gauges seems to validate in our case the simulations methods and consequently to confirm the correction factors k.

However, the formula for the surface pressure distribution derived from the Knudsen conductance with the adequate k factors diverges to give a pressure at $x = 0.1$ m of 5.5×10^{-3} Pa instead of a measured pressure of 1.4×10^{-2} Pa and a simulated pressure of 1.55×10^{-2} Pa.

VI. CONCLUSION

The Monte Carlo method made it possible to determine the correction factor k to apply to the Knudsen conductance formula in the case of the cone. In addition, the extension of the Oatley method gives a good approximation of the surface pressure distribution for tubes or cones taking into account a pump at the exit of the conductance. This distribution is all the more sensitive since the lengths are short and the pumping capacities are important. This can, therefore, have a great influence on the vacuum metrology. Our measurements seem to support our simulations but they need to be improved and extended to other geometries.

ACKNOWLEDGMENT

The author would like to thank Dr. T. Garvey for improving the English of the manuscript.

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