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A General Equilibrium Analysis of Emission Allowances

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A General Equilibrium Analysis of Emission Allowances

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Abstract

Each Party of the Kyoto Protocol on Climate Change must achieve quantified greenhouse gases emission reduction. One of the major policy instrument to be used to comply with these commitments is the opening of an emission allowances market. This paper analyzes, in the general equilibrium framework, the effects of the opening of such a market on the economic equilibrium.

Keywords General Equilibrium Theory, emission allowances, general pricing rules, sensitivity.

1 Introduction

In order to promote a sustainable development, the Member States of the European Union (EU) signed the Kyoto Protocol to the United Nations Framework Convention on Climate Changes. The 15 Member States that made up the EU until 1 May 2004 are committed to reducing their emissions of greenhouse gases by 8 % from 1990 levels by the end of 2012. In order to meet this target, the European Commission built up the Emission Trading Scheme, launched on 1 January 2005. The UE Emission Trading Scheme is based on the idea that creating a price for CO₂ emission through the establishment of a market for emission allowances is the most cost-effective policy instrument for EU Member States to meet their Kyoto commitments. An emission allowance represents the right to emit one ton of CO₂. Member States have agreed on national allocation plans which give each firm in the scheme an individual endowment in emission allowances. The limitation on the number of emission

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allowances allocated, corresponding to the EU Members States obligations in the Kyoto Protocol, creates the scarcity needed for a market to emerge.

This paper proposes a study of the EU Emission Trading Scheme in the general equilibrium framework. We consider an initial economy, representing the EU before the opening of the Emission Trading Scheme, and study the impact of the introduction of the emission allowance market on the economic equilibrium. We choose for the initial economy a model with general pricing rules. This gives room for increasing returns to scale and various pricing behaviors, such as marginal (cost) pricing, average pricing, etc. The emission allowance market is then open with the following characteristics. Each firm is freely allocated with an amount of emission allowances corresponding to its objectives; to compensate for the CO₂ emissions due to its production choice y , each firm is set to hold an amount $f(y)$ of emission allowances. The firms which CO₂ emissions are below their objectives are able to sell their excess emission allowances. The firms facing difficulties in meeting their objectives are able to change their pricing behavior, or to buy the extra emission allowance they need, or to choose a combination of the two.

Our main issue consists in establishing the existence of an equilibrium in the economy with emission allowances. This would raise no difficulties if we could posit general assumptions on the economy with allowances. However, we are concerned that in order to have any economic relevance the existence result should rely only on assumptions on the initial economy. This requirement makes the problem untrivial. Moreover, the opening of a new market is a type of perturbation that hasn't been studied yet, to our knowledge, in general equilibrium theory. In order to deal with the problem, we use an approach inspired by the "Walras tâtonnement": everything goes as if a price was announced for the emission allowance; an equilibrium would then be determined on the initial markets, and the allowance price would be modified until the correspondent demand in emission allowances equals the amount the authority in charge of pollution regulation is willing to offer. Mathematically, this amounts to study a class of economy perturbed by the allowance price. We can then exhibit conditions on the initial economy and a set of endowments in emission allowances leading to an equilibrium in the economy with emission allowances. Indeed, the perturbations induced by the opening of the additional market of emission allowance on the markets of the initial economy can be sum up by a continuous modification of the firms pricing rules. We thus can use a particularity of the sufficient conditions for the existence of equilibria given in Bonnisseau and Jamin (2005), namely that these conditions are stable up to continuous perturbations. We therefore introduce the class of perturbed economies, which differs from the initial economy only in the firms behaviors: their pricing rules are perturbed by an exogenous price for the emission allowance. We show that a perturbed economy admits an equilibrium, provided that the allowance price is such that a survival and a revenue assumption are verified. Finally, there is an equilibrium in the economies with

emission allowance as soon as the amount of emission allowances supplied by the authorities entails an allowance's market price such that the preceding holds.

We then focus, following an approach similar to Jouini's [see Jouini (1991)], on the sensitivity of the equilibria with regards to the allowance price. Under classical regularity assumptions on the agent's behavior, we show that there is a continuous path from the equilibria of the initial economy to the equilibria with emission allowances. We then focus on the influence of the price of the emission allowance on the CO₂ emission level in order to determine whether the opening of an emission allowance market gives any incentive for pollution reduction. In fact, this holds only under very restrictive convexity assumptions.

This paper is organized as follows. We first present the model, the initial economy and the economy with emission allowances, and the Assumptions on the primitive data that will be maintained throughout. We refer to Appendix 1 for a justification of our modeling choice for the firms pricing behaviors in the economy with emission allowance. Then, through the study of the perturbed economies, we exhibit economies with emission allowances admitting equilibria with non zero allowance prices. We finally address the problem of sensitivity of these equilibria with regards to the allowance price, and discuss the problem of sensitivity of the global demand in emission allowances with regards to the allowance price.

2 The model

2.1 The initial economy

We consider an economy with a finite number ℓ of commodities. We take \mathbb{R}^ℓ for commodity space and $\mathcal{H} = \{p \in \mathbb{R}^\ell \mid p \cdot e = 1\}$ for price space, where $e = (1, \dots, 1) \in \mathbb{R}^\ell$ is a reference commodity bundle. We denote by $\omega \in \mathbb{R}^\ell$ the vector of total initial endowments of the economy.

We consider a finite number m of consumers in the economy, indexed by i , $1 \leq i \leq m$. The subset of all possible consumption plans for consumer i , given his physical constraints, is \mathbb{R}_+^ℓ . The tastes of this consumer are described by a binary preference relation¹ \preceq_i on \mathbb{R}_+^ℓ . There is a finite number n of firms in the economy, indexed by j , $1 \leq j \leq n$. The technological possibilities of firm j are represented by its production set $Y_j \subset \mathbb{R}^\ell$. Firm j is set to follow a general pricing rule φ_j , a correspondence from ∂Y_j to \mathcal{H} , that is, the price vector $p \in \mathcal{H}$ is acceptable for firm j given the production plan $y_j \in \partial Y_j$ if and only if $p \in \varphi_j(y_j)$. Finally, the total initial endowments and the profits or

¹ We define $x \prec_i x'$ by $[x \preceq_i x'$ and not $x' \preceq_i x]$.

losses of the firms are allocated among the consumers according to the wealth functions r_i from $\mathcal{H} \times \mathbb{R}^n$ to \mathbb{R} , that is, for the price vector $p \in \mathcal{H}$ and the production allocation $(y_j) \in \prod_{j=1}^n \partial Y_j$, the profit (or loss) of firm j is given by $\pi_j(p, y_j) = p \cdot y_j$, and the wealth of consumer i is $r_i(p, (\pi_j(p, (y_j))))$.

The initial economy is thus a collection

$$\mathcal{E} = \left(\mathbb{R}^\ell, \omega, (\preceq_i, r_i)_{i=1}^m, (Y_j, \varphi_j)_{j=1}^n \right),$$

and equilibria of the initial economy are defined as follows.

Definition 1 *An equilibrium of the economy \mathcal{E} is a collection $(\hat{p}, (\hat{x}_i), (\hat{y}_j))$ in $\mathcal{H} \times (\mathbb{R}^\ell)^m \times (\mathbb{R}^\ell)^n$ satisfying:*

- (a) *for every i , \hat{x}_i is a greater element for \preceq_i in the budget set $B_i(\hat{p}, (\hat{y}_j)) := \{x_i \in \mathbb{R}_+^\ell \mid \hat{p} \cdot x_i \leq r_i(\hat{p}, (\hat{p} \cdot \hat{y}_j))\}$;*
- (b) *for every j , $\hat{y}_j \in \partial Y_j$ and $\hat{p} \in \varphi_j(\hat{y}_j)$;*
- (c) $\sum_{i=1}^m \hat{x}_i = \sum_{j=1}^n \hat{y}_j + \omega$.

For every $t \geq 0$, we let

$$A_t := \left\{ (y_j) \in \prod_{j=1}^n \partial Y_j \mid \sum_{j=1}^n y_j + \omega + te \in \mathbb{R}_+^\ell \right\}$$

be the set of *t-attainable production allocations* of the economy \mathcal{E} , that is, the set of production allocations that become attainable when t units of the reference commodity bundle e are added to the total initial endowments in the economy \mathcal{E} . A_0 clearly denotes the set of *attainable production allocation* of the economy \mathcal{E} .

We also let

$$PE := \left\{ (p, (y_j)) \in \mathcal{H} \times \prod_{j=1}^n \partial Y_j \mid p \in \cap_{j=1}^n \varphi_j(y_j) \right\}$$

be the set of *production equilibria*, a production equilibrium being an element $(p, (y_j)) \in \mathcal{H} \times \prod_{j=1}^n \partial Y_j$ satisfying condition (b) of Definition 1, and

$$APE := \left\{ (p, (y_j)) \in PE \mid (y_j) \in A_0 \right\}$$

be the set of *attainable production equilibria*.

Let us finally state several assumptions on the primitive data of the initial economy that will be maintained throughout this paper.

Assumption (C) *For every i , \preceq_i is a continuous, convex and non-satiated preorder on \mathbb{R}_+^ℓ , and r_i is a continuous function on $\mathcal{H} \times \mathbb{R}^n$ such that, for every $(p, (\pi_j)) \in \mathcal{H} \times \mathbb{R}^n$, $\sum_{i=1}^m r_i(p, (\pi_j)) = p \cdot \omega + \sum_{j=1}^n \pi_j$.*

Assumption (P) For every j , Y_j is a closed subset of \mathbb{R}^ℓ allowing for inaction ($0 \in Y_j$) and free-disposal ($Y_j - \mathbb{R}_+^\ell \subset Y_j$).

Note that, from Lemma 5.1 in Bonnisseau and Cornet (1988), under Assumption (P) the boundaries of the production sets are homeomorphic to e^\perp . Consequently, for all j , the set ∂Y_j will be implicitly be endowed with the \mathcal{C}^1 manifold structure of dimension $\ell - 1$ induced by this homeomorphism.

Assumption (PR) For every j , φ_j is an upper hemi-continuous correspondence from ∂Y_j into \mathcal{H} with nonempty, convex, compact values. Furthermore, for every $(y_j) \in A_0$, $\prod_{j=1}^n \varphi_j(y_j) \subset (\mathbb{R}_+^\ell)^n$.

2.2 The economy with emission allowances

We now add a market to the ℓ markets of the initial economy \mathcal{E} , the market of emission allowances. There are now $\ell + 1$ commodities: the ℓ commodities of the initial economy \mathcal{E} and the commodity “emission allowance”. We let $b = (b_j) \in (\mathbb{R}_+)^n$ be the initial endowments in emission allowances.

Each firm is set to be in possession of an amount of emission allowances compensating the level of CO₂ emission induced by its production process. We shall suppose that these amounts are well determined by the production plans, i.e., for every j the authorities lay down a rule f_j , a function from ∂Y_j to \mathbb{R} , eventually taking into consideration the particular environmental situation of firm j , that associates with every production plan $y_j \in \partial Y_j$ the amount $f_j(y_j)$ of emission allowances firm j has to be in possession of to put the production process y_j in motion. Furthermore, we shall suppose that the firms are not entitled to produce emission allowances. We summarize this situation by letting

$$Y_j^* = \left\{ (y_j, \tau) \in Y_j \times \mathbb{R} \mid \tau \leq -f_j(y_j) \right\} = (Y_j \times \mathbb{R}) \cap \text{hypo}(-f_j),$$

be the production set of firm j in the economy with emission allowances.

The opening of the emission allowance market also induces a modification of the firms pricing behavior on the ℓ first markets. We consider that these modifications are represented by the applications δ_j , from ∂Y_j to e^\perp , where e^\perp denote the orthogonal space² to the reference commodity bundle e . That is, the price vector $(p, q) \in \mathcal{H} \times \mathbb{R}$ is acceptable for firm j given the production plan $(y_j, -f_j(y_j)) \in \partial Y_j^*$ if and only if there exists a price vector $\hat{p}_j \in \varphi_j(y_j)$ such that $p = \hat{p}_j + q\delta_j(y_j)$. This modeling choice for the perturbation of the firms pricing behavior, as we’ll see in Appendix 1, is motivated by the particular case where the firms follow the marginal pricing rule. The pricing rule is thus

² $e^\perp = \{p \in \mathbb{R}^\ell \mid p \cdot e = 0\}$.

the correspondence φ_j^* from ∂Y_j^* to $\mathcal{H} \times \mathbb{R}$ defined by:

$$\varphi_j^*(y_j, -f_j(y_j)) = \left\{ (p_j + q\delta_j(y_j), q) \mid p_j \in \varphi_j(y_j), q \in \mathbb{R}_+ \right\}.$$

We shall posit the following regularity condition on the pricing perturbations δ_j .

Assumption (PR*) For every j , δ_j is a differentiable mapping from ∂Y_j to e^\perp .

As for the consumers, in the economy with emission allowances their consumption set is $\mathbb{R}_+^{\ell+1}$ and we suppose that they are indifferent with the emission allowance: for every i , the preference relation \preceq_i^* is the binary relation induced³ by \preceq_i on $\mathbb{R}^{\ell+1}$. Let us now come to the allocation of the total wealth among the consumers. The modifications of their revenue is due to the modification of the price and of the firms profits. For the price vector $(p, q) \in \mathcal{H} \times \mathbb{R}$ and the production allocation $((y_j, -f_j(y_j))) \in \prod_{j=1}^n \partial Y_j^*$, the profit (or loss) of firm j is given by $\pi_j^*(p, q, (y_j, -f_j(y_j))) = p \cdot y_j + q(b_j - f_j(y_j))$, and the wealth of consumer i is $r_i(p, (\pi_j^*(p, q, (y_j, -f_j(y_j))))$.

An economy with emission allowances associated with the initial economy \mathcal{E} is thus a collection

$$\mathcal{E}^*(b) = \left(\mathbb{R}^{\ell+1}, \omega, (\preceq_i^*, r_i)_{i=1}^m, (Y_j^*, \varphi_j^*, b_j)_{j=1}^n \right),$$

and the equilibria of this economy, called *equilibria with emission allowances*, are defined as follows.

Definition 2 An equilibrium of the economy with emission allowances $\mathcal{E}^*(b)$ is a collection $(p^*, q^*, (x_i^*), (y_j^*))$ in $\mathcal{H} \times \mathbb{R} \times (\mathbb{R}^\ell)^m \times (\mathbb{R}^\ell)^n$ satisfying:

- (a*) for every i , x_i^* is a greater element for \preceq_i in the budget set $B_i^*(p^*, q^*, (y_j^*)) := \{x_i \in \mathbb{R}_+^\ell \mid p^* \cdot x_i \leq r_i(p^*, (p^* \cdot y_j^* + q(b_j - f_j(y_j^*))))\}$;
- (b*) for every j , $y_j^* \in \partial Y_j$ and $(p^*, q^*) \in \varphi_j^*(y_j^*)$;
- (c*) $\sum_{i=1}^m x_i^* = \sum_{j=1}^n y_j^* + \omega$ and $\sum_{j=1}^n f_j(y_j^*) = \sum_{j=1}^n b_j$.

Given the previous notations and definitions, the following result is straightforward.

Proposition 1 An element $(\hat{p}, (\hat{x}_i), (\hat{y}_j))$ in $\mathcal{H} \times (\mathbb{R}^\ell)^m \times (\mathbb{R}^\ell)^n$ is an equilibrium of the initial economy \mathcal{E} if and only if $(\hat{p}, 0, (\hat{x}_i), (\hat{y}_j))$ is an equilibrium of the economy with tradable emission allowances $\mathcal{E}^*(b)$ satisfying $\sum_{j=1}^n f_j(\hat{y}_j) = \sum_{j=1}^n b_j$.

This means that if the economy is supplied with a total endowment in emission allowances that equals its needs at the initial situation, then the choices of

³ \preceq_i^* is defined on $\mathbb{R}_+^\ell \times \mathbb{R}_+$ by $[(x_i, \chi_i) \preceq_i^* (x'_i, \chi'_i)] \Leftrightarrow [x_i \preceq_i x'_i]$

the consumers and producers won't be modified as the commodity "emission allowance" is free. However, we are interested in finding conditions on the total amount $\sum_{j=1}^n b_j$ of emission allowances available under which the introduction of a market of emission allowance in an economy at equilibrium may lead to an equilibrium with emission allowances entailing a reduction of the total CO₂ emission.

3 Existence of equilibria

Proposition 1 shows that there will be no pollution reduction if the commodity "emission allowance" has a zero market price. Our approach consists in analyzing the perturbations induced by a modification of the price of the emission allowance on the initial economy to establish the existence of equilibria with emission allowance. This approach entitles us to prove the existence of equilibria by positing assumptions only on the initial economy. Moreover this approach is slightly different from the usual one, according to which we should study the perturbations induced by a modification of the initial endowment in emission allowances in the economy with emission allowances. Actually, the classical results [notably those in Jouini (1991)] could not be applied in our framework, as some crucial assumptions are not satisfied. Indeed, a free-disposal type condition is necessary to tackle the non-convexity of the production sets. However, the opening of the emission allowance market calls into question the free-disposal principle. Consider for example the case where there are two commodities in the initial economy and a firm, whose production set is given by:

$$Y_j = \left\{ (y_j^1, y_j^2) \in \mathbb{R}^2 \mid y_j^1 \leq 0 \text{ and } y_j^2 \leq -y_j^1 \right\}.$$

If firm j is associated with the function f_j defined by $f_j(y_j) = \sqrt{|y_j^1|}$, then firm j production set in the economy with emission allowance is:

$$Y_j^* = \left\{ (y_j^1, y_j^2, \tau) \in \mathbb{R}^3 \mid y_j^1 \leq 0, y_j^2 \leq -y_j^1 \text{ and } \tau \leq -\sqrt{|y_j^1|} \right\}.$$

But this production set Y_j^* doesn't satisfies any free-disposal type condition $Y_j^* - D \subset Y_j^*$, for any closed, convex, pointed cone D in $\mathbb{R}^{\ell+1}$ with a nonempty interior.

Our approach is inspired by the "Walras tâtonnement": everything goes as if a price was announced for the emission allowance; an equilibrium would then be determined on the ℓ first markets, and the allowance price would be modified until the correspondent demand in emission allowances equals the amount the authority in charge of pollution regulation is willing to offer. This approach is justified by the fact that the consumption and production plans are entirely determined on the ℓ first markets and because the emission allowance

price is the determinant of the influence of the emission allowance market on the agents. Moreover this approach is also relevant when the allowances are auctioned.

3.1 Perturbed economies

We shall denote by $\mathcal{E}^q(\beta)$ the perturbed economy associated with the initial economy \mathcal{E} , the price $q \in \mathbb{R}$ of the emission allowance, and the proportions β_j ($\beta_j \in \mathbb{R}_+$ for every j and $\sum_{j=1}^n \beta_j = 1$) of the total amount of emission allowances allocated to each firm. The characteristics of this economy are as follows.

The commodity space is \mathbb{R}^ℓ , and the price space \mathcal{H} ; $\omega \in \mathbb{R}^\ell$ is the vector of total initial endowments. For every i , consumer i is characterized by his consumption set, equal to \mathbb{R}_+^ℓ , his preferences \preceq_i , his initial endowments $\omega_i \in \mathbb{R}^\ell$ and his wealth function r_i from $\mathcal{H} \times \mathbb{R}^n$ to \mathbb{R} .

For every j , the production set of firm j is Y_j , and it follows the pricing rule φ_j^q , the correspondence from ∂Y_j to \mathcal{H} defined by

$$\varphi_j^q(y_j) = \left\{ p_j + q\delta_j(y_j) \mid p_j \in \varphi_j(y_j) \right\}.$$

Note that, if the Assumptions (C) and (P) are satisfied in the initial economy, then they still hold true in the perturbed economy $\mathcal{E}^q(\beta)$ for every $q \in \mathbb{R}$. Furthermore, Assumptions (PR) and (PR*) imply the following property for the pricing rules φ_j^q of the firms perturbed by the emission allowance market (see the proof in Appendix 2).

Lemma 1 *Under Assumption (PR) and (PR*), for every $q \in \mathbb{R}$ and every j , φ_j^q is an upper hemi-continuous correspondence from ∂Y_j into \mathcal{H} with nonempty, convex, compact values. Furthermore, if the set A_0 of attainable production allocations is compact, then there exists a convex, compact subset S^q of \mathcal{H} , containing $\mathcal{H} \cap \mathbb{R}_+^\ell$, such that $\prod_{j=1}^n \varphi_j^q(y_j) \subset (S^q)^n$ for every $(y_j) \in A_0$.*

This means that, under Assumption (PR*), for every $q \in \mathbb{R}$ the perturbed pricing rules φ_j^q are similar to the initial pricing rules φ_j in the sense of Assumption (PR).

Finally, everything goes as if the total amount of emission allowances $\sum_{j=1}^n f_j(y_j)$ required for the firms to set in motion any production plan $(y_j) \in \prod_{j=1}^n \partial Y_j$ was allocated among the firms according to the same distribution than in the economy with emission allowances: given a production plan $(y_j) \in \prod_{j=1}^n \partial Y_j$, for every j , firm j is endowed with $\beta_j \sum_{j=1}^n f_j(y_j)$ emission allowances, where

β_j is the proportion of the total endowment in emission allowances freely allowed to firm j in the economy with emission allowances $\mathcal{E}^*(b)$. Hence, for the price vector $p \in \mathcal{H}$ and the production plans $(y_j) \in \prod_{j=1}^n \partial Y_j$, for every j , the profit (or loss) of firm j in the perturbed economy $\mathcal{E}^q(\beta)$ is given by $\pi^q(p, (y_j)) = p \cdot y_j + q(\beta_j \sum_{j=1}^n f_j(y_j) - f_j(y_j))$, and the wealth of consumer i is $r_i(p, (\pi_j^q(p, (y_j))))$.

We now give the definition of an equilibrium of a perturbed economy.

Definition 3 *An equilibrium of the perturbed economy $\mathcal{E}^q(\beta)$ is a collection $(p^q, (y_j^q), (x_i^q))$ in $\mathcal{H} \times (\mathbb{R}^\ell)^n \times (\mathbb{R}^\ell)^m$ satisfying:*

- (a^q) *for every i , x_i^q is a greater element for \preceq_i in the budget set $B_i^q(p^q, (y_j^q)) := \{x_i \in \mathbb{R}_+^\ell \mid p^q \cdot x_i \leq r_i(p^q, (p \cdot y_j + q(\beta_j \sum_{j=1}^n f_j(y_j) - f_j(y_j))))\}$;*
- (b^q) *for every j , $y_j^q \in \partial Y_j$ and $p^q \in \varphi_j^q(y_j^q)$;*
- (c^q) $\sum_{i=1}^m x_i^q = \sum_{j=1}^n y_j^q + \omega$.

Recalling that $\sum_{j=1}^n \beta_j = 1$, Definition 3 leads to the following proposition:

Proposition 2 *An element $(p^q, (x_i^q), (y_j^q))$ in $\mathcal{H} \times (\mathbb{R}^\ell)^m \times (\mathbb{R}^\ell)^n$ is an equilibrium of the perturbed economy $\mathcal{E}^q(\beta)$ if and only if $(p^q, q, (x_i^q), (y_j^q))$ is an equilibrium of the economy with emission allowances $\mathcal{E}^*(b)$ satisfying $b_j = \beta_j \sum_{j=1}^n f_j(y_j^q)$ for every j .*

Remark that any perturbed economy $\mathcal{E}^0(\beta)$ associated with the economy \mathcal{E} coincides with the initial economy \mathcal{E} , hence has the same set of equilibria. Furthermore, in any perturbed economy associated with the initial economy \mathcal{E} , for every $t \geq 0$, the t -attainable set is exactly the t -attainable set of the initial economy. However, if $q \neq 0$, the pricing rules, hence the set of production equilibria are perturbed. For every $q \in \mathbb{R}$, we let:

$$PE^q = \left\{ (p, (y_j)) \in \mathcal{H} \times \prod_{j=1}^n \partial Y_j \mid p \in \bigcap_{j=1}^n \varphi_j^q(y_j) \right\}$$

be the set of q -production equilibria, a q -production equilibrium being an element $(p, (y_j))$ in $\mathcal{H} \times \prod_{j=1}^n \partial Y_j$ satisfying condition (b^q) of Definition 3, and

$$APE^q = \left\{ (p, (y_j)) \in PE^q \mid (y_j) \in A_0 \right\}$$

be the set of attainable q -production equilibria. We clearly have $PE^0 = PE$ and $APE^0 = APE$.

3.2 Equilibria in perturbed economies

To determine the existence of equilibria in a perturbed economy, hence equilibria with emission allowances, we shall use Theorem 1 in Bonnisseau and Jamin

(2005). According to this result, a perturbed economy $\mathcal{E}^q(\beta)$, associated with the initial economy and the allowance price $q \in \mathbb{R}$, has an equilibrium under Assumptions (C), (P), (PR), (PR*) and the following Assumptions (R^q) and (BLS^q), depending on the allowance price q via the perturbation of the firms pricing behavior.

Assumption (R^q) For every i , $r_i(p, (p \cdot y_j + q(\beta_j \sum_{j=1}^n f_j(y_j) - f_j(y_j)))) > 0$ for every $(p, (y_j)) \in APE^q$.

Before presenting the second assumption, let us recall that, following Bonnisseau and Cornet (1988), under Assumption (P) there exist an homeomorphism $\Lambda : (e^\perp)^n \rightarrow \prod_{j=1}^n \partial Y_j$ and a Lipschitz continuous function $\theta : (e^\perp)^n \rightarrow \mathbb{R}$ such that, for every $t \geq 0$, $\Lambda^{-1}(A_t) = \{s \in (e^\perp)^n \mid \theta(s) \leq t\}$. When A_t is nonempty and compact, we then let $\Theta(t) = \max\{\theta(s) \mid s \in \text{co}\Lambda^{-1}(A_t)\}$ [see Lemma 2 in Bonnisseau and Jamin (2005)].

Assumption (BLS^q) There exists a real number $t_0 \geq 0$ such that $A_{\Theta(t_0)}$ is nonempty and bounded, and:

(BS^q) for every $t \in [0, t_0[$ and every $(p, (y_j)) \in PE^q$, if $(y_j) \in A_t$ then $p \cdot (\sum_{j=1}^n y_j + \omega + te) > 0$;
(BL^q) for every $t \in [t_0, \Theta(t_0)]$, every $(p, (y_j)) \in S \times A_t$ and every $(p_j) \in \prod_{j=1}^n \varphi_j^q(y_j)$, if $p \cdot (\sum_{j=1}^n y_j + \omega + te) = 0$, then there exists $(\hat{y}_j^q) \in A_{t_0}$ such that $\sum_{j=1}^n (p_j - p) \cdot (y_j - \hat{y}_j^q) > 0$.

The particularity of these last two conditions lies in their stability with regard to the perturbations induced by the emission allowance market. Namely, if they are satisfied in a perturbed economy $\mathcal{E}^{q_0}(\beta)$ associated with the initial economy \mathcal{E} and the allowance price $q_0 \in \mathbb{R}$, satisfying Assumptions (C), (P), (PR) and (PR*), then they still hold true in perturbed economies $\mathcal{E}^q(\beta)$ associated with an allowance price q close enough to q_0 . In other words, if we let U be the set defined by:

$$U = \left\{ q \in \mathbb{R} \mid \text{Assumptions (R}^q) \text{ and (BLS}^q) \text{ hold true} \right\},$$

then the set $\{|q - q_0| \mid q \in U\}$ contains a neighborhood of 0. Let $Q(q_0)$ denotes its upper bound. This property lies on the following regularity results on the pricing rules and the set of production equilibria (see the proof in Appendix 2).

Lemma 2 Under Assumptions (PR) and (PR*), for every j , the correspondence that associates the set $\varphi_j^q(y_j)$ with every $(q, y_j) \in \mathbb{R} \times \partial Y_j$ is upper hemi-continuous on $\mathbb{R} \times \partial Y_j$.

Lemma 3 For every $q_0 \in \mathbb{R}$, under Assumptions (PR), (PR*) and (BLS^{q_0}), the correspondence that associates the set $\{(p, (y_j)) \in PE^q \mid (y_j) \in A_t\}$ with every $(q, t) \in \mathbb{R} \times \mathbb{R}_+$ is upper hemi-continuous on $\mathbb{R} \times [0, \Theta(t_0)]$.

We can now establish the following “stability” result.

Proposition 3 *Under the Assumptions (C), (P), (PR), (PR*), (R^{q0}) and (BLS^{q0}), $Q(q_0) > 0$, and there is an equilibrium in every perturbed economy $\mathcal{E}^q(\beta)$ associated with the initial economy and for every allowance price q satisfying $|q - q_0| < Q(q_0)$.*

Proof: Suppose first that Assumption (R^q) is not satisfied for $|q - q_0|$ small enough. Then there exist three sequences $\{q^\nu\}_\nu \subset \mathbb{R}$, $\{i^\nu\}_\nu \subset \{1, \dots, m\}$ and $\{(p^\nu, (y_j^\nu))\}_\nu \subset S \times \prod_{j=1}^n \partial Y_j$, such that $r_{i^\nu}(p^\nu, (p^\nu \cdot y_j^\nu + q^\nu(\beta_j \sum_{j=1}^n f_j(y_j^\nu) - f_j(y_j^\nu)))) \leq 0$ with $(p^\nu, (y_j^\nu)) \in APE^{q^\nu}$ for every $\nu \in \mathbb{N}^*$, and $\{q^\nu\}_\nu$ converges to q_0 . We can suppose without any loss of generality that $\{i^\nu\}_\nu$ is constant, say equal to i and, from Lemma 3, that $\{(p^\nu, (y_j^\nu))\}_\nu$ converges to some $(p, (y_j)) \in APE^{q_0}$. Finally, passing to the limit in $r_{i^\nu}(p^\nu, (p^\nu \cdot y_j^\nu + q^\nu(\beta_j \sum_{j=1}^n f_j(y_j^\nu) - f_j(y_j^\nu)))) \leq 0$, we deduce that $r_i(p, (p \cdot y_j + q_0(\beta_j \sum_{j=1}^n f_j(y_j) - f_j(y_j)))) \leq 0$, which contradicts Assumption (R^{q0}).

Suppose then that condition (BS^q) of Assumption (BLS^q) is not satisfied for $|q - q_0|$ small enough. Then, there exist three sequences $\{q^\nu\}_\nu \subset \mathbb{R}$, $\{t^\nu\}_\nu \subset [0, t_0[$ and $\{(p^\nu, (y_j^\nu))\}_\nu \subset S \times \prod_{j=1}^n \partial Y_j$, such that $p^\nu \cdot (\sum_{j=1}^n y_j^\nu + \omega + t^\nu e) \leq 0$ with $(p^\nu, (y_j^\nu)) \in PE^{q^\nu}$ and $(y_j^\nu) \in A_{t^\nu}$ for every ν , and $\{q^\nu\}_\nu$ converges to q_0 . We can suppose without any loss of generality that $\{t^\nu\}_\nu$ converges to some $t \in [0, t_0]$ and, from Lemma 3 that $\{(p^\nu, (y_j^\nu))\}_\nu$ converges to some $(p, (y_j)) \in PE^{q_0}$ such that $(y_j) \in A_t$. Finally, passing to the limit in $p^\nu \cdot (\sum_{j=1}^n y_j^\nu + \omega + t^\nu e) \leq 0$, we deduce that $p \cdot (\sum_{j=1}^n y_j + \omega + te) \leq 0$, which contradicts part (BS^{q0}) (if $t < t_0$) or part (BL^{q0}) (if $t = t_0$) of Assumption (BLS^{q0}).

Suppose finally that condition (BL^q) of Assumption (BLS^q) is not satisfied for $|q - q_0|$ small enough. Then there exist four sequences $\{t^\nu\}_\nu \subset [t_0, \Theta(t_0)]$, $\{p^\nu\}_\nu \subset S$, $\{(y_j^\nu)\}_\nu \subset \prod_{j=1}^n \partial Y_j$, $\{(p_j^\nu)\}_\nu \subset \mathcal{H}^n$ and $\{q^\nu\}_\nu \subset \mathbb{R}$ converging to 0 such that, for every ν , $(y_j^\nu) \in A_{t^\nu}$, $(p_j^\nu) \in \prod_{j=1}^n \varphi_j^{q^\nu}(y_j^\nu)$, $p^\nu \cdot (\sum_{j=1}^n y_j^\nu + \omega + t^\nu e) = 0$ and $\sum_{j=1}^n (p_j^\nu - p^\nu) \cdot (y_j^\nu - \hat{y}_j) \leq 0$ for every $(\hat{y}_j) \in A_0$. We can suppose without any loss of generality that $\{t^\nu\}_\nu$ converges to some $t \in [t_0, \Theta(t_0)]$, that $\{p^\nu\}_\nu$ converges to some $p \in S$ and that $\{(y_j^\nu)\}_\nu$ converges to some $(y_j) \in A_t$ since A_t is compact under Assumption (BLS⁰). From Lemma 2 we can also suppose that $\{(p_j^\nu)\}_\nu$ converges to some $(p_j) \in \prod_{j=1}^n \varphi_j(y_j)$. Finally, passing to the limit in $p^\nu \cdot (\sum_{j=1}^n y_j^\nu + \omega + t^\nu e) = 0$ and $\sum_{j=1}^n (p_j^\nu - p^\nu) \cdot (y_j^\nu - \hat{y}_j) \leq 0$ for every $(\hat{y}_j) \in A_0$ we get $p \cdot (\sum_{j=1}^n y_j + \omega + te) = 0$ and $\sum_{j=1}^n (p_j - p) \cdot (y_j - \hat{y}_j) \leq 0$ for every $(\hat{y}_j) \in A_0$, which contradicts part (BL^{q0}) of Assumption (BLS^{q0}).

Consequently, the set $\{|q - q_0| \mid q \in U\}$ is nonempty and its least upper bound $Q(q_0)$ is positive. Finally, following Bonnisseau and Jamin (2005), a perturbed economy $\mathcal{E}^q(\beta)$ associated with the initial economy \mathcal{E} and a price q of the emission allowance satisfying $|q - q_0| < Q$ has an equilibrium. \square

Finally, if we let Γ be the correspondence from \mathbb{R} into \mathbb{R} defined by:

$$\Gamma(q) = \left\{ \sum_{j=1}^n f_j(y_j) \mid (p, (y_j), (x_i)) \text{ is an equilibrium of } \mathcal{E}^q(f, \beta, \delta) \right\},$$

then Proposition 3 ensures that the values of this correspondence are nonempty on a neighborhood of U .

3.3 Equilibria with emission allowances

To ensure the existence of equilibria in perturbed economies, the least we can suppose is that the initial economy \mathcal{E} has an equilibrium. We shall thus suppose that the Assumptions (C), (P), (PR) are satisfied, together with the Assumptions (R⁰) and (BLS⁰). Then, since any perturbed economy $\mathcal{E}^0(\beta)$ coincides with the initial economy \mathcal{E} , we can deduce from Proposition 3 that, for q close enough to 0, there is an equilibrium in the perturbed economies $\mathcal{E}^q(\beta)$. Using Proposition 2 then leads to the following existence result, in the economy with emission allowances:

Theorem 1 *Under the Assumptions (C), (P), (PR), (PR*), (R⁰) and (BLS⁰), there exists $Q_0 > 0$ such that, for every $B \in \{\Gamma(q) \mid |q| < Q_0\}$, the economy with emission allowance $\mathcal{E}^*(\beta B)$ has an equilibrium.*

Theorem 1 proposes conditions on the initial economy and on the endowments in emission allowance under which the additional market of emission allowance can be open and lead to an economic equilibrium.

4 Sensitivity

We posit in this section some regularity conditions in order to study the sensitivity of the equilibria with respect to the emission allowance price.

4.1 Sensitivity of the equilibria

In order to study the influence of the allowance price on the firms production choices, we need further assumptions to ensure that the agents behaviors are completely determined by the prices. Following an idea of Jouini [see Jouini (1991)], we will therefore suppose from now on that the initial economy \mathcal{E} , seen as a perturbed economy $\mathcal{E}^0(\beta)$, is *locally price parametrized* in the following sense.

Definition 4 A perturbed economy $\mathcal{E}^q(\beta)$ is a locally price parametrized (LPP^q) economy if, for every equilibrium $(p^q, (y_j^q), (x_i^q))$ of $\mathcal{E}^q(\beta)$,

- (1) the behavior of the consumers can be sum up by a differentiable demand mapping D defined on a neighborhood of $(p^q, (y_j^q))$ with values in e^\perp and its partial derivative with respect to p is onto;
- (2) for every j , the mapping ϕ_j that associates $\varphi_j^q(y_j)$ with (q', y_j) is differentiable on a neighborhood of $(q, (y_j^q))$ and, for every j , its partial derivative with respect to y_j is onto.

In a LPP^q economy, we can define an excess demand mapping in a neighborhood of (p^q, q) , where p^q is an equilibrium price vector. Indeed, let $(p^q, (y_j^q), (x_i^q))$ be an equilibrium of an LPP^q economy $\mathcal{E}^q(\beta)$. The regularity condition on the perturbed pricing rules allows us to apply the Implicit Function Theorem, for every j , to the mapping that associates $\phi_j(q', y_j) - p$ with every (p, q', y_j) on a neighborhood of (p^q, q, y_j^q) : there exists a neighborhood $V(p^q)$ of p^q , a neighborhood V^q of q , a neighborhood $V_j(y_j^q)$ of y_j^q and a continuous mapping ψ_j^q , from $V(p^q) \times V^q$ to $V_j(y_j^q)$ such that $\phi_j^q(q', y_j) = p$ if and only if $\psi_j^q(p, q') = y_j$. We moreover have:

$$\frac{\partial \psi_j^q}{\partial (p, q)}(p^q, q) = - \left(\frac{\partial \phi_j^q}{\partial y_j}(q, y_j^q) \right)^{-1} \begin{pmatrix} -I & \delta_j(y_j^q) \end{pmatrix},$$

where I denotes the $(\ell - 1)$ th order identity matrix. Consequently, in every LPP^q economy $\mathcal{E}^q(\beta)$ the excess demand is totally determined by the vector price in the neighborhood of any equilibrium vector price p^q . Let us denote by Z^q this excess demand mapping, defined locally by

$$Z^q(p) = D(p, (\psi_j^q(p, q))) - \sum_{j=1}^n \psi_j^q(p, q) - \omega.$$

We shall posit a regularity condition on these local excess demand mappings. This condition is technical, but is satisfied under the classical gross substitute assumption and generalized demand law.

Assumption (DZ^q) The economy $\mathcal{E}^q(\beta)$ is locally price parametrized and, for every equilibrium vector price p^q , the mapping Z that associates $Z^q(p)$ with every (p, q') is differentiable on a neighborhood of (p^q, q) , and its partial derivative with respect to p is onto.

Note that if the initial economy \mathcal{E} , seen as a perturbed economy $\mathcal{E}^0(\beta)$, satisfies Assumption (DZ⁰) in addition to the Assumptions (C), (P), (PR), (PR*), (R⁰) and (BLS⁰), then there exists a neighborhood \mathcal{V} of 0 such that $\mathcal{V} \subset] - Q_0, Q_0[$ and, for every $q \in \mathcal{V}$, the perturbed economy $\mathcal{E}^q(\beta)$ has at least one equilibrium and satisfies Assumption (DZ^q).

The following result finally establishes the existence, locally, of a continuous path from the equilibria in the initial economy to equilibria of the perturbed economy.

Theorem 2 *Under the Assumptions (C), (P), (PR), (PR*), (R⁰), (BLS⁰) and (DZ⁰), there exists $\nu \in \mathbb{N}^*$ such that, for every $q \in \mathcal{V}$, the perturbed economy $\mathcal{E}^q(\beta)$ has the same number ν of equilibria than the initial economy \mathcal{E} , and there exists ν differentiable mappings g^1, \dots, g^ν defined on \mathcal{V} such that, for every $q \in \mathcal{V}$, $\{g^1(q), \dots, g^\nu(q)\}$ is the set of equilibrium price vectors of $\mathcal{E}^q(\beta)$. Moreover, for every $k = 1, \dots, \nu$, the function g^k is a solution on \mathcal{V} of the following differential equation:*

$$(g^k)'(q) = - \left(\frac{\partial Z}{\partial p}(g^k(q), q) \right)^{-1} \frac{\partial Z}{\partial q}(g^k(q), q)$$

with initial condition $g^k(0) = p_k^0$, where p_k^0 is the k th equilibrium vector price of the initial economy.

Proof: Let p^0 be an equilibrium price vector of the initial economy \mathcal{E} . Assumption (DZ⁰) allows us to apply the Implicit Function Theorem to the mapping \bar{Z} on $V(p^0) \times V^0$: there exists a continuous mapping ζ^q , from V_0 to $V(p^0)$ such that $\zeta^0(q) = p$ if and only if $Z(p, q) = 0$. Moreover, we have:

$$(\zeta^0)'(0) = - \left(\frac{\partial Z}{\partial p}(p^0, 0) \right)^{-1} \frac{\partial Z}{\partial q}(p^0, 0).$$

To sum up, $p \in V(p^0)$ is an equilibrium vector price of the initial economy \mathcal{E} if and only if $p = \zeta^0(0)$. This implies local unicity of the equilibria, and since the equilibrium price vectors of the initial economy \mathcal{E} lie in the compact set S , there exists $\nu \in \mathbb{N}^*$ such that the set of equilibrium price vectors of the initial economy \mathcal{E} can be written $\{p_k^0 \mid k = 1, \dots, \nu\}$.

Hence, for every $q \in V^0 \cap \mathcal{V}$ and every k , $p_k^q = \zeta_k^0(q)$ is an equilibrium of the LPP^q economy $\mathcal{E}^q(\beta)$ since $\bar{Z}(\zeta_k^0(q), q) = 0$. Since $q \in \mathcal{V}$, the perturbed economy $\mathcal{E}^q(\beta)$ satisfies Assumption (DZ^q) and we can apply the Implicit Function Theorem to the mapping Z on $V(p_k^q) \times V^q$. Moreover, from the continuity of the mapping Z , we deduce that $\{p_k^q \mid k = 1, \dots, \nu\}$ is the set of equilibrium price vectors of the perturbed economy $\mathcal{E}^q(\beta)$.

Let us now consider the mapping g_k defined on \mathcal{V} by $g_k(q) = \zeta_k^q(q)$, where the mappings ζ_k^q are obtained by applying the Implicit Function Theorem as above. From the properties of the ζ_k^q , for every $q \in \mathcal{V}$, the mapping g_k coincides with ζ_k^q on a neighborhood of q , hence g_k is differentiable on \mathcal{V} and, for every $q \in \mathcal{V}$,

$$g_k'(q) = - \left(\frac{\partial Z}{\partial p}(g_k(q), q) \right)^{-1} \frac{\partial Z}{\partial q}(g_k(q), q).$$

□

4.2 Sensitivity of the total demand in emission allowances

Thanks to Theorem 2, we are now able to determine the sensitivity of the total demand in emission allowances with respect to the allowance price.

Let us omit the subscript k for the sake of clarity. For every $j = 1, \dots, n$, if we let y_j be the mapping that associates with every $q \in \mathcal{V}$ the equilibrium production plan of firm j associated with the equilibrium price vector $g(q)$, i.e. the mapping defined on \mathcal{V} by $y_j(q) = \psi_j^q(g(q), q)$, then we have :

$$y'_j(q) = - \left(\frac{\partial \phi_j^q}{\partial y_j}(q, y_j(q)) \right)^{-1} (-g'(q) + \delta_j(y_j(q))).$$

Suppose now that, for every j , the function f_j that associates with every production choice $y_j \in \partial Y_j$ the amount $f_j(y_j)$ of emission allowances firm j has to be in possession of is differentiable. If we denote by F the function that associates with every $q \in \mathcal{V}$ the total demand in emission allowance at equilibrium in the perturbed economy $\mathcal{E}^q(\beta)$, i.e. the mapping defined on \mathcal{V} by $F = \sum_{j=1}^n f_j \circ y_j$, then we have:

$$F'(0) = - \sum_{j=1}^n \nabla f_j(y_j) \left(\frac{\partial \phi_j}{\partial y_j}(\hat{p}, \hat{y}_j) \right)^{-1} \left(\left(\frac{\partial Z}{\partial p}(\hat{p}, 0) \right)^{-1} \frac{\partial Z}{\partial q}(\hat{p}, 0) + \delta_j(\hat{y}_j) \right),$$

where $\hat{p} = g(0)$ is the equilibrium vector price in the initial economy and, for every j , \hat{y}_j is firm j production plan corresponding to \hat{p} , i.e. $\hat{y}_j = y_j(0)$.

Let us now remark that an infinitesimal perturbation of the allowance price induces an infinitesimal perturbation of the production plans on the frontiers of the production sets which clearly doesn't affect the profits in a neighborhood of $q = 0$. Consequently, we have:

$$\frac{\partial Z}{\partial q}(\hat{p}, 0) = - \sum_{j=1}^n \frac{\partial \psi_j^0}{\partial q}(\hat{p}, 0) = \sum_{j=1}^n \left(\frac{\partial \phi_j}{\partial y_j}(\hat{p}, \hat{y}_j) \right)^{-1} \delta_j(\hat{y}_j).$$

Finally, we can write that $F'(0) = -{}^t \nabla f(\hat{y}) (ABA + A) \delta(\hat{y})$ with

$$\begin{aligned} \nabla f(\hat{y}) &= (\nabla f_1(\hat{y}_1), \dots, \nabla f_n(\hat{y}_n)), \\ \delta(\hat{y}) &= (\delta_1(\hat{y}_1), \dots, \delta_n(\hat{y}_n)), \\ A &= \begin{pmatrix} \left(\frac{\partial \phi_1}{\partial y_1}(\hat{p}, \hat{y}_1) \right)^{-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \left(\frac{\partial \phi_n}{\partial y_n}(\hat{p}, \hat{y}_n) \right)^{-1} \end{pmatrix}, \end{aligned}$$

and

$$B = \begin{pmatrix} \left(\frac{\partial Z}{\partial p}(\hat{p}, 0)\right)^{-1} & \dots & \left(\frac{\partial Z}{\partial p}(\hat{p}, 0)\right)^{-1} \\ \vdots & & \vdots \\ \left(\frac{\partial Z}{\partial p}(\hat{p}, 0)\right)^{-1} & \dots & \left(\frac{\partial Z}{\partial p}(\hat{p}, 0)\right)^{-1} \end{pmatrix}.$$

We refer to Appendix 1 for a simplified form of the derivative of the total demand in emission allowance in the case where the firms have a marginal pricing behavior.

To determine whether the emission allowance price has any influence on the CO₂ emission level, i.e. whether the total demand in emission allowance is sensible at all to the allowance price, we should prove that $F'(0) \neq 0$, i.e. that $(ABA + A)\delta(\hat{y}) \notin (\nabla f(\hat{y}))^\perp$ from the calculus above.

Proposition 4 *Under the condition that*

$$\sup_{(\sigma_j) \in (e^\perp)^n} \frac{\left\| \left(\frac{\partial Z}{\partial p}(\hat{p}, 0)\right)^{-1} \sum_{j=1}^n \left(\frac{\partial \phi_j}{\partial y_j}(\hat{p}, \hat{y}_j)\right)^{-1} \sigma_j \right\|}{\sum_{j=1}^n \|\sigma_j\|} < \frac{1}{n},$$

we have $(ABA + A)\delta(\hat{y}) \notin (\nabla f(\hat{y}))^\perp$ for almost every δ .

Proof: When the matrix $(ABA + A)$ is invertible the condition $(ABA + A)\delta(\hat{y}) \notin (\nabla f(\hat{y}))^\perp$ is equivalent to $\delta(\hat{y}) \notin (ABA + A)^{-1} \left((\nabla f(\hat{y}))^\perp \right)$. Since $(ABA + A)^{-1} \left((\nabla f(\hat{y}))^\perp \right)$ is a hyperplane, this last condition holds true for almost every δ , that is to say for almost every perturbation of pricing rules. Consequently, if $\|BA\| < 1$ (we consider the norm induced by $\|\cdot\|_1$), then the matrix $(ABA + A)$ is invertible since A is invertible under Assumption (DZ⁰), and the preceding holds. \square

Given a price vector p for the ℓ commodities of the initial economy, as a firm undergoes a perturbation σ_j of its pricing rule, it modifies its production plan y_j ; markets then clear thanks to a modification of the market vector price p . The condition above says that the variation of the market vector price is lower than the average perturbation on the pricing rules. This is a very large generalization of the idea that demand and supply move in opposite directions with respect to the price vector.

In the last subsections, we give a more precise result in the case of marginal pricing and under strong convexity assumptions ; we also present some examples of pathological situations occurring when those assumptions do not hold.

4.3 CO₂ emission reduction

In the case where all the firms have a marginal pricing behavior, one has (see Appendix 1):

$$F'(0) = -{}^t\nabla f(\hat{y}) (ABA + A) \nabla f(\hat{y}).$$

We then have,

Proposition 5 *If the production sets are strictly convex and if there exist a representative utility maximizing consumer then $F'(0)$ is strictly negative and the demand for permits decreases with the permit price in the neighborhood of 0.*

Proof: For sake of clarity we denote $\Delta = \partial_p D$ and $A_j = (\frac{\partial \phi_j}{\partial y_j})^{-1}$, those differentials being evaluated at the initial equilibrium. One then has $\frac{\partial Z}{\partial q}(\hat{p}, 0) = \Delta - \sum_{j=1}^n A_j$. Now, if D is the demand of a representative consumer, at equilibrium Δ equals the slutsky matrix as there is no wealth effect and is then negative semi-definite (Kihlstrom et al. (1976)). As moreover we have assumed that the partial differential of the demand is inversible, it is negative definite.

We then introduce the following auxiliary objects:

$$B(t) = \begin{pmatrix} (t\Delta - \sum_{j=1}^n A_j)^{-1} & \cdots & (t\Delta - \sum_{j=1}^n A_j)^{-1} \\ \vdots & & \vdots \\ (t\Delta - \sum_{j=1}^n A_j)^{-1} & \cdots & (t\Delta - \sum_{j=1}^n A_j)^{-1} \end{pmatrix}$$

and

$$I(t) = -\delta(AB(t)A + A)\delta = -\left(\sum_{j=1}^n \delta_j A_j \delta_j + \sum_{j=1}^n \delta_j A_j (t\Delta - \sum_{j=1}^n A_j)^{-1} \sum_{j=1}^n A_j \delta_j\right)$$

We then notice that $F'(0) = I(1)$. In order to obtain the requested result we should show that $I(0) \leq 0$ and that $I'(t) < 0$ for $t \in [0, 1]$.

Let us first show that $I(0) \leq 0$. Therefore is enough to show that $A + AB(0)A$ is positive. A and $AB(0)A$ are respectively positive definite and positive. It is then enough to show, according to Theorem 7.7.3 in (ref Matrix analysis) that $\rho(-B(0)A) \leq 1$.

$$\text{One has } -B(0)A = \begin{pmatrix} A_1(\sum_{j=1}^n A_j)^{-1} & \cdots & A_1(\sum_{j=1}^n A_j)^{-1} \\ \vdots & & \vdots \\ A_n(\sum_{j=1}^n A_j)^{-1} & \cdots & A_n(\sum_{j=1}^n A_j)^{-1} \end{pmatrix}$$

Therefore λ is an eigenvalue of $-B(0)A$ if and only if there exist a non-zero

$$X = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}, \text{ such that for all } i, A_i(\sum_{j=1}^n A_j)^{-1} \sum_{j=1}^n X_j = \lambda X_i.$$

Then, $\sum_{j=1}^n A_j(\sum_{j=1}^n A_j)^{-1} \sum_{j=1}^n X_j = \lambda \sum_{j=1}^n X_j$ That is to say $\lambda = 1$. This implies according to the preceding $I(0) \leq 0$.

One has

$$I'(t) = \sum_{j=1}^n \delta_j A_j (t\Delta - \sum_{j=1}^n A_j)^{-1} \Delta (t\Delta - \sum_{j=1}^n A_j)^{-1} \sum_{j=1}^n A_j \delta_j$$

According to the lemma Δ is symmetric and therefore

$$I'(t) = (t\Delta - \sum_{j=1}^n A_j)^{-1} \sum_{j=1}^n \delta_j A_j \Delta (t\Delta - \sum_{j=1}^n A_j)^{-1} \sum_{j=1}^n A_j \delta_j$$

The result is then a direct consequence of the negativity of Δ .

Provided the f_j^l s are convex, the very same proof can be used in the neighborhood of $q > 0$ as long as q verifies the assumptions R^q , BLS^q , LPP^q , and DZ^q . We can therefore state:

Proposition 6 *If the production sets are strictly convex, the pollution functions are convex and if there exist a representative utility maximizing consumer then $F'(q)$ is negative for all $q > 0$ such that R^q , BLS^q , LPP^q , and DZ^q hold and the demand for permits decrease with the price in the neighborhood of those q .*

As a straightforward corollary, using the inverse function theorem, we obtain a parametrization of the equilibria of the economy with permits via the total endowment in permits. Let P_0 denotes the initial level of emission.

Corollary 1 *Under the assumptions of proposition 5 there exist $\gamma > 0$ and a differentiable function Γ defined on $[P_0 - \gamma, P_0]$ such that Γ associates to a level of emission an equilibrium of the economy with permits*

4.3.1 Revenue effects

In the particular case of a private ownership economy, the emission allowance market functioning can have a redistributive effect. Consider for example the

case of two consumers, the first one holding much shares of the most polluting firms, and the second one holding much shares of the least polluting firms. The introduction of the allowance market will then proportionally increase the income of the second consumer (respectively proportionally decrease the income of the first one). This feature could interfere with the emission limitation and reduction objectives if the second consumer was very likely to consume commodities which production process entails a high level of CO₂ emissions. However, with a large number of consumers, whose preferences are very much alike, this should not have heavy consequences on the objectives of the emission allowance market.

4.3.2 Convexity of the functions f_j

The preceding result, proposition doesn't hold in case there exist partial increasing returns due to a decreasing marginal pollution. We give here an example of an economy where the marginal pollution is decreasing, and where the demand for emission allowances is increasing with respect to the allowance price when it is sufficiently high.

We consider an initial two goods economy. There is one consumer, whose preferences can be represented by the utility function u , defined on \mathbb{R}_+^2 by $u(x^1, x^2) = x^1 x^2$. The demand of this consumer, given the price vector $p = (p^1, p^2)$ in the simplex of \mathbb{R}^2 and the wealth $w \in \mathbb{R}$, is then $D(p, w) = \left(\frac{w}{2p^1}, \frac{w}{2p^2}\right)$. We denote by $\omega = (\omega^1, \omega^2) \in \mathbb{R}_{++}^2$ the initial endowments of the consumer. There is one producer with a constant return production technology represented by the production set $Y = \{(y^1, y^2) \in \mathbb{R}^2 \mid y^1 \leq 0, y^2 \leq -y^1\}$, and having a marginal pricing behavior. The pollution function f is supposed to be an increasing concave function of commodity 2, not depending on commodity 1.

Note that, in this example, there is no redistributive effect due to the emission allowance market since there is only one consumer, and that the profit of the firm is always zero.

One can easily check that a (non-zero) equilibrium in the perturbed economy E^q must be of the form:

$$\left(\left(\frac{1}{2}, \frac{1}{2} + qf'(y^2)\right), \left(\frac{\omega^1 + (1+2qf'(y^2))\omega^2}{2}, \frac{\omega^1 + (1+2qf'(y^2))\omega^2}{2(1+2qf'(y^2))}\right), (-y^2, y^2)\right),$$

with y^2 satisfying:

$$\frac{\omega^1 + (1+2qf'(y^2))\omega^2}{2} + y^2 = \omega^1.$$

When $1 + qf''(y^2)\omega^2 < 0$, i.e. when $f''(y^2) < -\frac{1}{q\omega^2}$, one can immediately check via the Implicit Function Theorem that y^2 (hence the CO₂ emissions $f(y^2)$) increases with the allowance price in the neighborhood of q . If the pollution function f is concave in the neighborhood of y^2 with a sufficiently big curvature

(depending here of ω^2 and q), then we can expect an augmentation of the CO₂ emissions as the permit price goes up.

Appendix 1: The case of marginal pricing behavior

Following Hotelling (1938), economic efficiency could be achieved only if every commodity is sold at marginal cost, i.e. if the firms minimize their costs and set their selling prices equal to marginal cost. As in Cornet (1989), we shall generalize this marginal cost pricing behavior to the case where the production sets are not supposed to have a smooth boundary, and set the firms to follow the marginal pricing rule, i.e. to fulfill the first order necessary condition for their profit maximization, in the mathematical sense formalized by Clarke's normal cone. Note that, in the particular case where the production sets are convex, the marginal pricing behavior coincides with the profit maximization.

Let us recall the definition of the Clarke's tangent and normal cones [see Clarke (1983)]. If Y is a nonempty subset of \mathbb{R}^ℓ and y is an element in $\text{cl}Y$, then Clarke's tangent cone to Y at y is:

$$T_Y(y) = \left\{ v \in \mathbb{R}^\ell \left| \begin{array}{l} \forall \{y^\nu\}_\nu \subset Y, \forall \{t^\nu\}_\nu \subset \mathbb{R}_+, t^\nu \rightarrow 0, \exists \{v^\nu\}_\nu \subset \mathbb{R}^\ell : \\ v^\nu \rightarrow v \text{ and } y^\nu + t^\nu v^\nu \in Y \text{ for } \nu \text{ large enough} \end{array} \right. \right\}.$$

Clarke's normal cone to Y at y , denoted by $N_Y(y)$, is then the negative polar cone of $T_Y(y)$. Note that, when Y is closed and convex, Clarke's tangent and normal cones to Y at y reduce to the classical tangent and normal cones of convex analysis.

From now on, we shall suppose that the firms follow the marginal pricing rule. Formally, this amounts to suppose that, for every j , the pricing rule φ_j is such that, for every $y_j \in \partial Y_j$,

$$\varphi_j(y_j) = N_{Y_j}(y_j) \cap \mathcal{H}.$$

In the case where all the firms have a marginal pricing behavior, the perturbations δ_j induced by the emission allowance market on the pricing rules can be precisely determined thanks to the properties of Clarke's normal cone.

Proposition 7 *Under Assumptions (P) and (PR*), for every j , firm j has a marginal pricing behavior in the economy with emission allowance $\mathcal{E}^*(b)$ if Y_j is regular in the sense of Clarke and if, for every $y_j \in \partial Y_j$, $\delta_j(y_j) = \nabla(f_j \circ \Lambda_j)(\text{proj}_{e^\perp}(y_j))$, where Λ_j is the homeomorphism from e^\perp to ∂Y_j .*

Proof: From Corollary 2 of Theorem 2.9.8 in Clarke (1983), recalling that

$$Y_j^* = \left\{ (y_j, \lambda) \in Y_j \times \mathbb{R} \mid \lambda \leq -f_j(y_j) \right\} = (Y_j \times \mathbb{R}) \cap \text{hypo}(-f_j),$$

for every $(y_j, -f_j(y_j)) \in \partial Y_j^*$, we have:

$$\begin{aligned} N_{Y_j^*}(y_j, -f_j(y_j)) &= N_{Y_j \times \mathbb{R}}(y_j, -f_j(y_j)) + N_{\text{hypo}(-f_j)}(y_j, -f_j(y_j)), \\ &= N_{Y_j}(y_j) \times \{0\} + (\nabla f_j(y_j), 1)\mathbb{R}_+ \\ &= \left\{ (p_j + q\nabla f_j(y_j), q) \mid p_j \in N_{Y_j}(y_j) \text{ and } q \in \mathbb{R}_+ \right\}. \end{aligned}$$

Furthermore, note that, for every $y_j \in \partial Y_j$, every $p_j \in N_{Y_j}(y_j)$ and every $q \in \mathbb{R}_+$, $p_j + q\nabla f_j(y_j) \in S$ if and only if:

$$\begin{aligned} p_j + q\nabla f_j(y_j) &= (1 - q\nabla f_j(y_j) \cdot e) \frac{p_j}{p_j \cdot e} + (q\nabla f_j(y_j) \cdot e) \frac{e}{\|e\|^2} + \text{proj}_{e^\perp}(q\nabla f_j(y_j)) \\ &= \frac{p_j}{p_j \cdot e} + q \left((\nabla f_j(y_j) \cdot e) \left(\frac{e}{\|e\|^2} - \frac{p_j}{p_j \cdot e} \right) + \text{proj}_{e^\perp}(\nabla f_j(y_j)) \right). \end{aligned}$$

Consequently, from Bonnisseau and Cornet (1990), we deduce that:

$$\varphi_j^*(y_j, -f_j(y_j)) \subset N_{Y_j^*}(y_j, -f_j(y_j)) \cap S \times \mathbb{R}_+,$$

if $\delta_j(y_j) = \nabla(f_j \circ \Lambda_j)(\text{proj}_{e^\perp}(y_j))$. □

Recalling that the production sets are implicitly supposed endowed with the \mathcal{C}^1 manifold structure of dimension $\ell - 1$ induced by the homeomorphisms Λ_j , we shall sum up Proposition 7 by assuming that $\delta_j = \nabla f_j$ for every j . The derivative of the total allowance demand function F at $q = 0$ is then given by:

$$F'(0) = -{}^t \nabla f(\hat{y})(ABA + A) \nabla f(\hat{y}).$$

Appendix 2: Lemmata

Proof of Lemma 1: For every $q \in \mathbb{R}$ and every j , the correspondence φ_j^q is clearly upper hemi-continuous with nonempty, convex compact values from the Assumptions (PR) and (Δ) .

Furthermore, if the set A_0 of attainable production allocations is compact, then the continuity of the applications δ_j induces the compactness of the following subset of e^\perp :

$$\Delta = \left\{ \delta_j(y_j) \mid j \in \{1, \dots, n\} \text{ and } (y_j) \in A_0 \right\}.$$

Consequently, if we let S^q be the convex hull of the compact sets $S := \mathcal{H} \cap \mathbb{R}_+^\ell$ and $S + q\Delta$, then S^q is clearly a compact subset of \mathcal{H} containing S satisfying $\prod_{j=1}^n \varphi_j^q(y_j) \subset (S^q)^n$ for every $(y_j) \in A_0$. \square

Proof of Lemma 2: For every j , let ϕ_j be the correspondence from $\mathbb{R} \times \partial Y_j$ into \mathcal{H} defined by $\phi_j(q, y_j) = \varphi_j^q(y_j)$. From Lemma 1, ϕ_j clearly has compact values in \mathcal{H} .

Furthermore, if we let $\{q^\nu\}_\nu$ and $\{y_j^\nu\}_\nu$ be to sequences, with values in \mathbb{R} and ∂Y_j respectively and converging to some $q \in \mathbb{R}$ and $y_j \in \partial Y_j$ respectively, and $\{p^\nu\}_\nu$ be a sequence with values in \mathcal{H} satisfying $p^\nu \in \phi_j(q^\nu, y_j^\nu)$ for every ν , then there exists a sequence $\{p_j^\nu\}_\nu$ with values in H such that $p_j^\nu \in \varphi_j(y_j^\nu)$ and $p^\nu = p_j^\nu + q^\nu \delta_j(y_j^\nu)$ for every ν . From the upper hemi-continuity of the pricing rule φ_j , we can suppose without any loss of generality that the sequence $\{p_j^\nu\}_\nu$ converges to some $p_j \in \varphi_j(y_j)$. The continuity of the mapping δ_j then implies that the sequence $\{p^\nu\}_\nu$ converges to $p_j + q\delta_j(y_j) \in \phi_j(q, y_j)$, which shows that the correspondence ϕ_j is upper hemi-continuous on $\mathbb{R} \times \partial Y_j$. \square

Proof of Lemma 3: Let Π be the correspondence from $\mathbb{R} \times \mathbb{R}_+$ into $\mathcal{H} \times \prod \partial Y_j$ defined by

$$\Pi(q, t) = \left\{ (p, (y_j)) \in PE^q \mid (y_j) \in A_t \right\}.$$

From Assumption (BLS^{q0}), the set $A_{\Theta(t_0)}$ is bounded, hence A_t is compact for every $t \in [0, \Theta(t_0)]$. Since, from Lemma 2, the correspondence ϕ_j is upper hemi-continuous, we deduce that $\Pi(q, t)$ is compact for every $(q, t) \in \mathbb{R} \times [0, \Theta(t_0)]$. Let $\{q^\nu\}_\nu$ and $\{t^\nu\}_\nu$ be to sequences, with values in \mathbb{R} and $[0, \Theta(t_0)]$ respectively and converging to some $q \in \mathbb{R}$ and $t \in [0, \Theta(t_0)]$ respectively, and $\{p^\nu\}_\nu$ and $\{(y_j^\nu)\}_\nu$ be to sequences with values in \mathcal{H} and $\prod_{j=1}^n \partial Y_j$ respectively, such that $(p^\nu, (y_j^\nu)) \in \Pi(q^\nu, t^\nu)$ for every ν . Since $A_{\Theta(t_0)}$ is compact, we can suppose without any loss of generality that the sequence $\{(y_j^\nu)\}_\nu$ converges to some $(y_j) \in A_{\Theta(t_0)}$, and we clearly have $(y_j) \in A_t$. From Lemma 2, we can then suppose without any loss of generality that the sequence $\{p^\nu\}_\nu$ converges to some $p \in \cap_{j=1}^n \varphi_j^q(y_j)$, which finally shows that the correspondence Π is upper hemi-continuous on $\mathbb{R} \times [0, \Theta(t_0)]$. \square

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