A PREDICTIVE SWITCHING STRATEGY FOR A CLASS OF HYBRID SYSTEMS: WAVE SUPPRESSION IN FLUID DYNAMIC SYSTEMS

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Abstract: In this contribution, predictive control of switched nonlinear systems is considered. A simple open loop switching parametrization is proposed to efficiently deal with the combinatorics associated with the discrete part of the dynamics. As an application, the problem of actuator switching for wave suppression in the Kuramoto-Sivashinsky PDE is considered. Numerical experiments are reported for some pathological cases to testify the tractability of the scheme.

1. INTRODUCTION

Model Predictive Control MPC or *Receding Horizon Control* RHC is becoming an almost standard control technique in the process industry. This success is mainly due to its ability to handle in a natural way constraints on both the inputs and outputs. With the advent of powerful dedicated processors and efficient optimization techniques, MPC is moving towards nonconventional applications where continuous and discrete dynamics are interacting, i.e. *hybrid systems.* A rather mature theory including modelling and control frameworks exist for such systems and some computational optimal open loop control schemes for specific classes have also been formulated see e.g. (Xu and Antsaklis, 2003) and the references therein.

In optimal control for general hybrid systems (Branicky *et al.*, 1998). One has to deal not only with the infinite dimensional optimization problems related to the continuous dynamics, but also with a potential combinatoric explosion related to the discrete part. In this context, one can distinguish three major approaches to tackle the problem. The first consists in approximating the continuous nonlinear dynamics by piecewise affine funcions and put the general model in the *Mixed Logical Dynamical* framework see, e.g. (Bemporad and Morari, 1999) where a mixed integer predictive controller is developed to stabilize MLD systems on desired reference trajectories. The resulting on line optimizations are solved through mixed integer quadratic programming tools.The second approach includes algorithms that are essentially based on the two stages optimization approach initiated independently in (Cassandras *et al.*, 2001) and (Xu and Antsaklis, 2003). At the first stage the aim is to find an optimal continuous control and the switching instants while, the second allows the variation of the sequences of active locations and the number of switches. This also includes some approaches based on new versions of the Maximum principle, see (Shaikh and Caines, 2003). The third approach consists in using effective *open loop parametrization* i.e. reduction of the set of admissible controls and switching paths. In this context embedding and pruning techniques have shown promising results for an abstracted class of hybrid systems i.e. *switched* systems, see e.g. respectively (Alamir and Attia, 2004),(Attia *et al.*, 2005),(Bengea and DeCarlo, 2005) and (Lincoln and Bernhardsson, 2002).

In this paper we propose a closed loop predictive control strategy for switched nonlinear systems. The peculiarity of the approach lies in the fact that a *local* feedback control policy is used in each location. The problem is then to find a switching strategy that minimizes a problem related cost functional. A combinatoric free open loop switching parametrization is then proposed to extract such a switching feedback. As an application, actuator switching policy for wave suppression in fluid dynamics systems is investigated. Stability and performance enhancement by switching are then reported for this case study. The application driven conclusions are those reported for some finite dimensional systems (McClamroch and Kolmanovsky, 2000).

The paper organization is as follows : section 2 presents basic definitions of switched systems. Section 3 is devoted to the predictive control formulation. In section 4, based on standard modelling assumptions see e.g. (El-Farra *et al.*, 2003), an application is fully treated and some validating numerical scenarios are reported. Finally, some conclusions and future work orientations are given in section 5.

2. SWITCHED NONLINEAR SYSTEMS

Definition 1. A controlled switched system is a tuple $\mathcal{S} = (\mathcal{D}, \mathcal{F}, \mathcal{K})$ where

- $\mathcal{D} = (\mathcal{Q}, \mathcal{E})$ is a directed graph representing the discrete structure of the system. The node or location set $\mathcal{Q} = \{1, 2, \ldots, Q\}$ is the set of indices for the configurations. The directed edge set $\mathcal{E}$ is a subset of the cartesian product $\mathcal{Q} \times \mathcal{Q}$ which contains all valid controlled transitions represented by the elements of the type $(q_1,\ q_2)$ meaning that a switching from location q_1 to location q_2 is allowed.
- $\mathcal{F} = \{f_q : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n, q \in \mathcal{Q}\}$ is a set of vector fields, where with each location $q \in \mathcal{Q}$ is associated a vector field $f_q(x, u)$, $x \in \mathbb{R}^n$ is the state vector and $u_q \in \mathcal{U}_q \subset \mathbb{R}^m$ is the control input, $\mathcal{U}_q$ is some compact set.
- $\mathcal{K} = \{K_q : \mathbb{R}^n \to \mathbb{R}^m, q \in \mathcal{Q}\}$ is a set of local feedback control strategies. The system in location q is then described by the following dynamics

$$\dot{x} = f_q(x, u_q) \tag{1}$$

$$u_q = K_q(x) \tag{2}$$

Locality of the feedback controllers K_p's is to be understood w.r.t. physical location and can be seen as a strategy to reduce the complexity of designing controllers for hybrid systems.

For the controlled switched system $\mathcal{S}$, the control input consists in a global switching strategy defined below

Definition 2. For a controlled switched system $\mathcal{S}$, an admissible switching strategy or profile $q(\cdot)$ is a piecewise constant function defined for all $t \in [t_0^s,\ \infty)$ as

$$q(t) = \begin{cases} q_0 & t_0^s \leq t < t_1^s \\ q_1 & t_1^s \leq t < t_2^s \\ \vdots & \vdots \\ q_K & t_K^s \leq t < t_{K+1}^s \\ \vdots & \vdots \end{cases} \tag{3}$$

where $\{t_k^s\}_{k \in \mathbb{N}}$ constitutes a strictly increasing sequence of switching instants, and $(q_k,\ q_{k+1}) \in \mathcal{E}$ for all $k \in \mathbb{N}$ meaning that $q(\cdot)$ follows the path as described in the discrete structure. Define also Σ as the set of all admissible switching strategies, $\Sigma \triangleq \{q(\cdot)$ defined on $[t_0,\ \infty)\}$.

The formulation allows the system to dwell in a location for a certain minimum time, thus excluding pathological phenomena like *Zeno* or *Fuller*, i.e., accumulation of location switching at finite time.

3. THE PREDICTIVE CONTROL STRATEGY

Definition 3. A constant open loop switching profile is a switching strategy that belongs to the following set

$$\Sigma_{\mathcal{Q}} = \{q(\cdot) \in \Sigma \mid \forall t \in [t_0,\ \infty), q(t) = q^*,\ q^* \in \mathcal{Q}\} \tag{4}$$

subset of Σ.

Let us denote a finite time switching signal as $q_{t_1,t_2} : [t_1,\ t_2] \to \mathcal{Q}$. The signal q_{t_1,t_2} represents a portion starting at t_1 and ending at t_2 of a switching signal.
By denoting the $i-th$ sampling period as $t_i = iT_s$ where i is a nonnegative integer and T_s the sampling period. The goal is to determine a sampled switching feedback law of the type

$$q(t) = s(x(t_i)), \quad t \in [t_i,\ t_{i+1}) \tag{5}$$

which asymptotically stabilizes the origin or at least enhance the performance of the switched system in some sense. By defining one step ahead reachable locations from p as

$$\mathcal{R}_p = \{s \in \mathcal{Q} \mid (p,\ s) \in \mathcal{E}\} \tag{6}$$

the discrete dynamics are captured and the following finite horizon optimal control problem can be formulated.

Problem 1. Given a prediction horizon N_{pr}, at every sampling instant t_i minimize with respect to $q_{t_i,t_i+N_{pr}T_s} \in \Sigma_{\mathcal{Q}}$ the following cost

$$J(x(t_i), q(\cdot)) = \psi_f(x(t_i + N_{pr}T_s)) + \int_{t_i}^{t_i+N_{pr}T_s} L(x(\tau), q(\tau))d\tau \tag{7}$$

subject to the following mixed dynamics

$$\dot{x} = f_q(x, K_q(x)) \tag{8}$$

$$q(t_i) \in \mathcal{R}_{q(t_{i-1})} \tag{9}$$

with $x(t_0) = x_0$, $q(t_0) = q_0$, $L : \mathbb{R}^n \times \mathcal{Q} \to \mathbb{R}_{\geq 0}$ and $\psi_f : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is a terminal penalty.

From the computational view point, solution to problem 1 can be found by an exhaustive search which consists in comparing the cost (7) for at most $card(\mathcal{Q})$ trajectories, meaning that the complexity is linear in the number of locations (this follows from the cardinality of the set $\Sigma_{\mathcal{Q}}$). This combinatoric free parametrization can thus be suitable for fast systems where hard real time contraints are to be met.

According to the receding horizon approach, the state feedback Model Predictive Controller (MPC) law is then derived by solving the open loop finite horizon problem 1 at every sampling time instant t_i

$$q^o_{t_i,t_i+N_{pr}T_s}(x(t_i), q(t_{i-1})) = \arg\min_{q \in \Sigma_{\mathcal{Q}}} J(x(t_i), q(\cdot)) \tag{10}$$

under (8)-(9). Applying

$$q(t) = q^o_{t_i,t_{i+1}} = s^{RH}(x(t_i), q(t_{i-1})), \quad t \in [t_i,\ t_{i+1}) \tag{11}$$

where $q^o_{t_i,t_{i+1}}$ is the first part of the optimal switching signal $q^o_{t_i,t_i+N_{pr}T_s}$ clearly defines a state feedback switching controller s^{RH}.

4. NUMERICAL EXPERIMENTS : STABILIZATION OF THE KURAMOTO-SIVASHINSKY EQUATION

The Kuramoto-Sivashinsky Equation KSE is a nonlinear dissipative *Partial Differential Equation* PDE that describes a variety of physical phenomena including flame fronts propagation fluid particles mixtures and falling liquid films (Chen and Chang, 1986). The equation is of the following form

$$\frac{\partial h}{\partial t} = -\nu\frac{\partial^4 h}{\partial z^4} - \frac{\partial^2 h}{\partial z^2} - h\frac{\partial h}{\partial z} \tag{12}$$

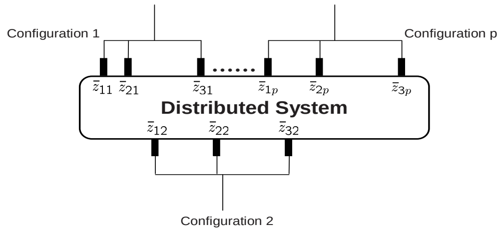

Fig. 1. Multi actuator multi configuration control architecture

where h is the internal state, the film height in falling liquids, and ν the *instability parameter* that depends on the fluid characteristics.

The control architecture of interest in this work is the one depicted in figure 1. It reflects the case where different configurations are available for control purposes or equivalently the case of moving, within a pre specified positions, actuators.

For the case of Q configurations each equipped with l distributed actuators with z as the spatial coordinate and $[-\pi,\ \pi]$ as the normalized evolution domain, the KSE while in configuration q is described as follows

$$\frac{\partial h}{\partial t} = -\nu\frac{\partial^4 h}{\partial z^4} - \frac{\partial^2 h}{\partial z^2} - h\frac{\partial h}{\partial z} + \sum_{i=1}^{l} b_{iq}(z)u_{iq}(t) \tag{13}$$

with the following periodic boundary conditions

$$\frac{\partial^j h}{\partial z^j}(-\pi, t) = \frac{\partial^j h}{\partial z^j}(\pi, t) \qquad j = 0, \dots 3 \tag{14}$$

$$h(z, 0) = h_0(z) \tag{15}$$

where $b_{iq}(\cdot)$ represents the spatial distribution of the $i-th$ actuator in the $q-th$ configuration. For the case of pointwise actuation and by denoting $\bar{z}_{iq}$ as the spatial coordinate of actuator i located in configuration q, b_{iq} can be written as

$$b_{iq}(z) = \frac{1}{\epsilon}\Big[\Gamma(\bar{z}_{iq} - \frac{\epsilon}{2}) - \Gamma(\bar{z}_{iq} + \frac{\epsilon}{2})\Big] \tag{16}$$

where $\Gamma(\cdot)$ is the spatial step function and ϵ a small positive real number representing the actuator spread.

Using standard arguments see e.g., (Chen and Chang, 1986; El-Farra *et al.*, 2003) the solution $y(z, t)$ to equation (12) can be expanded in terms of the following orthonormal eigenfunctions

$$y(z, t) = \frac{1}{\sqrt{\pi}}\sum_{k=1}^{\infty} x_k(t) \quad \sin(kz) \tag{17}$$

where x_k is the $k-th$ eigenmode's amplitude. After some basic manipulations and truncation of order r (for more details see e.g., (Chen and

Chang, 1986)) one obtains the following system of ODE written in matrix notations

$$\dot{x} = Fx + B_q u_q + f(x) = \bar{f}(x) + B_q u_q \quad (18)$$

where F is an $r \times r$ diagonal matrix containing the r eigenvalues, f is a nonlinear vector function and B_q is an $r \times l$ location dependent input matrix.

The state vector can be partitioned into x_s and x_f vectors $x = x_s \oplus x_f$ ($\oplus$ is the concatenation operator) where x_s represents the first (unstable for $\nu < 1$) r_s modes, and x_f the fastest r_f ($r = r_s + r_f$) modes. Based on this partition, the feedback controller associated to each configuration, while retaining the r_s slow modes, is based on the construction of quadratic location dependent Lyapunov function candidates V_q (El-Farra *et al.*, 2003). Using standard notations the controller writes

$$u_q = K_q(x_s) = \begin{cases} k_q(x_s)\ (L_{B_q} V_q)^T & L_{B_q} V_q \neq \mathbf{0}_{1\times l} \\ \mathbf{0}_{l\times 1} & L_{B_q} V_q = \mathbf{0}_{1\times l} \end{cases} \quad (19)$$

with $k_q(x_s) =$

$$-\frac{L_{\bar{f}} V_q + \sqrt{\left[L_{\bar{f}} V_q\right]^2 + (u_q^{max}(L_{B_q} V_q) u_q^{max}(L_{B_q} V)^T)^2}}{(L_{B_q} V_q)(L_{B_q} V_q)^T \left[1 + \sqrt{1 + (u_q^{max}(L_{B_q} V_q) u_q^{max}(L_{B_q} V_q)^T)}\right]}$$

where u_q^{max} denotes the actuator saturation level. By combining equations (18) and (19) the switched system can be written in the standard form (1)-(2). It is shown in (El-Farra *et al.*, 2003) that under the feedback control (19), local asymptotic stability of the KSE is achieved provided that the number of retained modes is large enough. The appropriate number is usually tested via closed loop simulation since no precise systematic methodology exists. The switching problem under interest reduces to the case where one has to activate at each instant a locally asymptotically stable vector field in such way that the closed loop switched system remains stable, while minimizing the following quadratic performance measure

$$J = x_s(t_0 + N_{pr} T_s)^T P_f x_s(t_0 + N_{pr} T_s) + \int_{t_0}^{t_0 + N_{pr} T_s} x_s(\tau)^T P x_s(\tau) d\tau \quad (20)$$

where P and P_f are respectively positive definite and semi definite matrices of appropriate dimensions. For simulation purposes a nonlinear model of order $r = 30$ is chosen, the model used in the controller (19) only retains the first two unstable modes ($r_s = 2$) with eigenvalues λ_1 and λ_2 of multiplicity 2, $\lambda_1 = \lambda_2 = 0.8$ for $\nu = 0.2$. The model used for prediction is based on the reduced

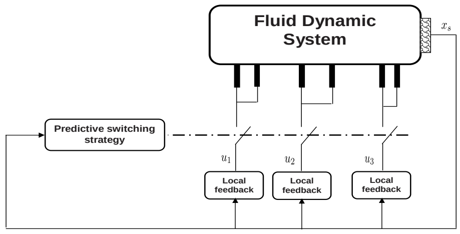

Fig. 2. The implemented control architecture

order model (r_s). The sampling period T_s is taken equal to 0.2 *sec* and the initial condition $h_0(z)$ as

$$h_0(z) = a_1 \sin(z) + a_2 \sin(2z) + \sin(\frac{z}{4}) + \sin(\frac{z}{16}) \quad (21)$$

with $a_1 = \frac{0.2}{\sqrt{\pi}}$ and $a_2 = \frac{1.5}{\sqrt{\pi}}$. In figure 3 is illustrated the open loop spatiotemporal evolution of h. As shown, the oscillations are sustained. The primary objective is to suppress such oscillations by actuator switching. In order to illustrate the control approach a set of actuators placement are taken into account. Three configurations with two actuators per configuration is taken. Indeed this is the least number of actuators per configuration that guarantees stabilizability of the reduced order model (since the eigenvalues are of multiplicity two). The actuators placement is reported in table 1. The control saturation levels are taken as $u_q^{max} = 2,\ q = 1, 2, 3$.

Configuration	$\bar{\mathbf{z}}_\mathbf{q}$
$q = 1$	$(-0.4\pi \quad -0.2\pi)$
$q = 2$	$(0.6\pi \quad 0.8\pi)$
$q = 3$	$(0.3\pi \quad 0.9\pi)$

Table 1. The different configurations and their corresponding actuators placement

The overall control architecture is reported in figure 2. The fluid dynamic system block is used to schematize the process described by a 30th order Galerkin approximation of the KS equation. The predictive switching strategy block depicted in figure 2 is based on the reduced order model (retaining only the first unstable modes) and the receding horizon strategy (11) with the quadratic cost functional (20). The local feedback laws are based on the expression (19).
The numerical experiments are conducted using the Matlab software and Mex compiled Fortran subroutines. The worst case execution times although not systematically reported are well under the allowed time slot of $T_s = 0.2$ *sec* (beyond $N_{pr} = 5$ the execution times exceeds the sampling period). In figure 4 is reported the spatiotemporal evolution of h, under the initial profile (21), when the first pair of actuators (first configuration) is active. As it can be seen, stability is not guaranteed since the initial conditions are far from the equilibria's region of attraction (the same holds when respectively only the second or third set

of actuators is active). The situation is better understood in the light of figure 5 where a phase portrait (retaining the two first modes) together with estimates of the regions of attraction are depicted. In figure 6 is shown the spatiotemporal profile of h under the predictive control strategy for the prediction horizon $N_{pr} = 1$. Stability is recovered and the wavy behaviour suppressed. Allowing actuator switching has thus enlarged the basin of attraction of the equilibrium. The phase portrait is the one depicted in figure 7. The corresponding switching profile is reported in figure 8. The region of attraction of the switched system can be numerically estimated by initializing the system at different initial conditions. In figure 9 is plotted the phase portrait of the switched system for initial conditions taken outside the regions of attraction. In figure 10 is depicted a scenario where the initial conditions is within the region of attraction. The predictive approach improves the behaviour even for these conditions. Indeed, the performance index as measured by (20) over the simulation horizon, has been almost quartered when actuator switching is allowed.

5. CONCLUSIONS

In this paper, a predictive control strategy for switched systems is presented. The approach is based on an efficient and simple open loop parametrization. An important example arising in fluid dynamics systems is investigated and some pathological interesting scenarios are shown to exist.

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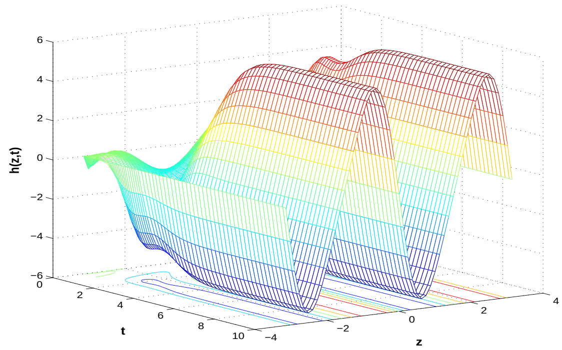

Fig. 3. Open loop spatiotemporal profile 30^{th} order Galerkin model. This corresponds to the case where no actuation is present in the system.

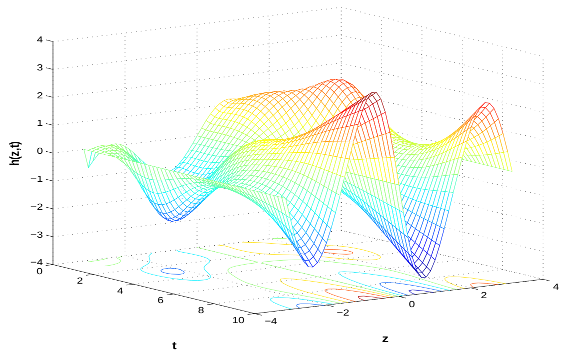

Fig. 4. Spatiotemporal profile when the first pair of actuators is in force. Oscillations are sustained despite the fact that the system is under control (first configuration), see table 1 for actuators placement

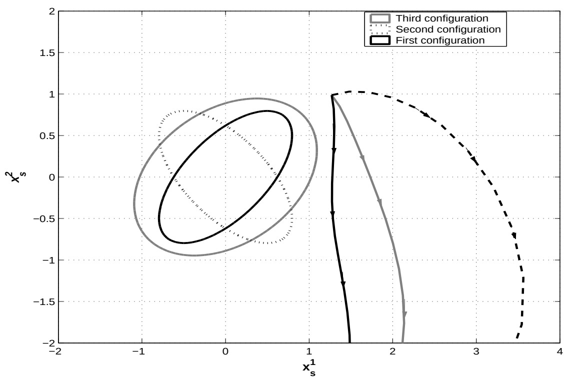

Fig. 5. Phase portrait together with the estimated regions of attraction without the predictive control approach. The initial conditions are outside the regions of attraction.

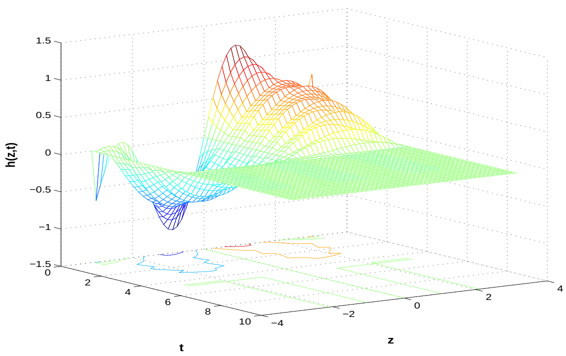

Fig. 6. Spatiotemporal profile for the switched system with a prediction horizon $N_{pr} = 1$. This figure is to be compared with figure 4. By allowing actuators switching the wavy behaviour is suppressed, despite the fact that the initial conditions are outside the regions of attraction, see figure 5

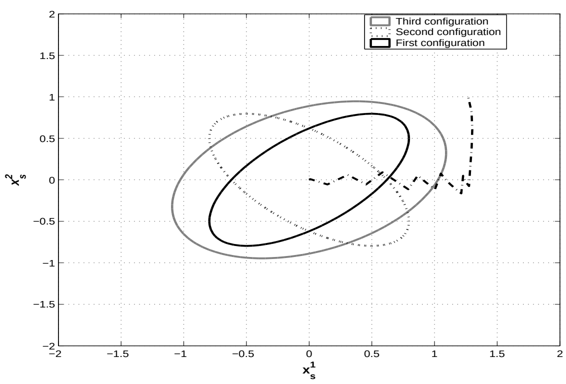

Fig. 7. Phase portrait together with the estimated region of attraction under the predictive control approach $N_{pr} = 1$. The initial conditions are those taken in figure 5, see also figure 6 for the spatiotemporal behaviour

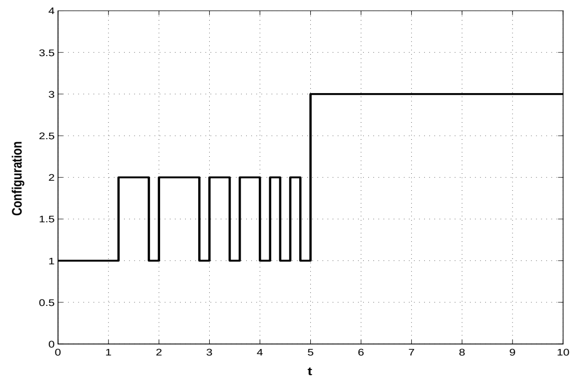

Fig. 8. Switching profile under the predictive control strategy for $N_{pr} = 1$, see the corresponding spatiotemporal behaviour, figure 6

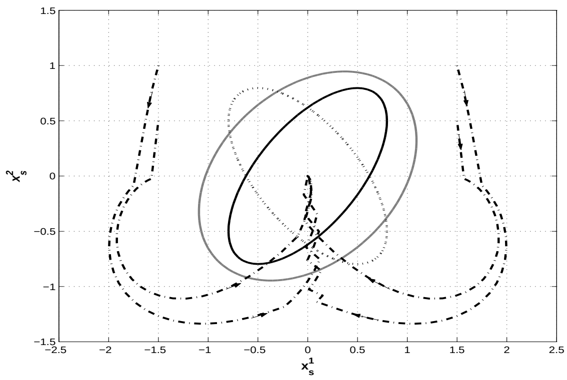

Fig. 9. Phase portrait with the predictive control approach for different initial conditions taken outside the regions of attraction. The external dahed-dot lines represent a numerical estimates of the region of attraction of the system under the switching policy. Notice the enlargement of the basin of attraction.

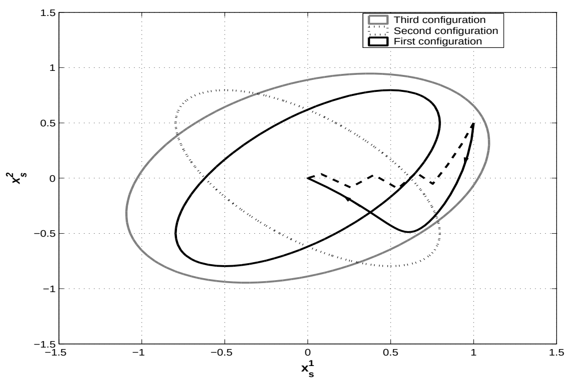

Fig. 10. Phase portrait with and without the predictive control approach, for the case where the initial conditions are within the region of attraction. One can remark that even for this case, switching brings benefits in the performance measure. The path traced under switching is shorter than the one without