

The Effect of Clock Frequency Offset on OFDM-CDMA systems

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I. Introduction

Recently, Orthogonal Frequency and Code Division Multiplexing (OFCDM) access technology has been investigated for the next generation of mobile communication systems [1]. A known drawback of these multi carrier systems is their sensitivity to synchronization errors [2] Particularly when the transmitter and receiver sampling frequencies are not perfectly synchronized, Inter Carrier Interference (ICI) is generated, which degrades the SINR at the receiving side. The contribution of this article is twofold. First, a generalized framework is proposed for modelling the effect of clock frequency offset on 2 dimensional spreading OFDM-CDMA systems [1]. Then, exploiting some results from the random matrix theory, an analytic expression of the Signal to Interference and Noise Ratio (SINR), modelling the impact of clock frequency offset, is derived. This formula is independent from the actual values of the spreading codes while taking into account their orthogonality. This is the novelty of this article. This model works for frequency selective channels and any single user detector.

II. System description

Figure 1 shows a 2 dimensional (2D) spreading OFDM-CDMA [6] transmitter/receiver chain for a downlink communication with N_u users.

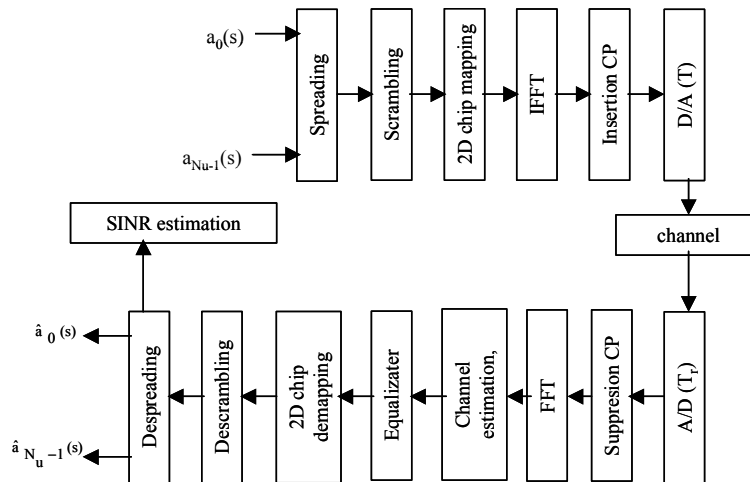


Figure 1: OFDM-CDMA transmitter.

Each symbol $a_m(s)$ of the m^{th} user is first spread by a Walsh-Hadamard sequence of N_c chips, and scrambled by a cell specific long pseudo random sequence. The resulting samples are then allocated on the time/frequency grid as shown on Figure 2 [1].

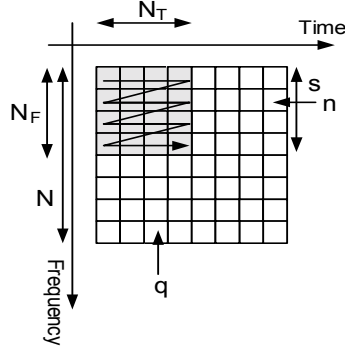


Figure 2: 2D spreading scheme.

The spreading factor is thus $N_c = N_F \times N_T$ where N_F and N_T are respectively the frequency and time domain spreading factors. Assuming a FFT of N points, each user transmits $S = N/N_F$ data symbols in an OFDM-CDMA block. Eventually, the signal at the IFFT input is:

$$b_q(sN_F + n) = \sum_{m=0}^{N_u-1} \sqrt{P_m} a_m(s) c_{m,s}[nN_T + q] \quad (1)$$

$$q = 0, \dots, N_T - 1; s = 0, \dots, S - 1, n = 0, \dots, N_F - 1$$

s is the index referring to the sub-band of N_F adjacent sub-carriers used for the transmission of the symbol $a_{i,m}(s)$. P_m is the power of the m^{th} user which is supposed constant for all sub-bands and $c_{m,s}$ represents its spreading sequence composed from the chip by chip multiplication of the user assigned Walsh-Hadamard sequence and the cell specific scrambling code. At the output of the IFFT and after the cyclic prefix insertion (v samples), the signal is passed through a multipath channel with delay spread Δ and corrupted by a complex value Additive White Gaussian Noise (AWGN) with variance σ^2 .

Without loss of generality, we assume at the receiving side, that one is interested by the symbols of user 0. In order to write the received signal with matrix-vector notation, the following matrices are defined: $P = \text{diag}(P_0, \dots, P_{N_u-1})$ is the $N_u \times N_u$ diagonal matrix which entries are the powers allocated to each code, $Q = \text{diag}(P_1, \dots, P_{N_u-1})$ is the $(N_u-1) \times (N_u-1)$ diagonal matrix containing the powers of the interfering codes, $C[s] = (C_0[s], C_1[s], \dots, C_{N_u-1}[s])$ is the $N_c \times N_u$ matrix containing all the spreading codes used in the s^{th} sub-band and $U[s] = (C_1[s], \dots, C_{N_u-1}[s])$ is the $N_c \times (N_u-1)$ matrix containing the interfering codes used in the s^{th} sub-band. $C[s]$ and $U[s]$ are isometric matrices, and depend on sub-band index s because of the long scrambling code.

We also define the vectors $a[s] = (a_0(s), \dots, a_{N_u-1}(s))^T$ and $\tilde{a}[s] = (a_1(s), \dots, a_{N_u-1}(s))^T$ corresponding respectively to the symbols of all users and interfering users transmitted in the s^{th} sub-band.

At the receiver side, after FFT operation, the signal on each carrier is multiplied by the one-tap equalizer coefficient $z_q(wN_F + p)$ which compensates for channel attenuation and signal rotation due to timing error. The output is then descrambled and despread. The estimated symbol on the sub-band w is [6]:

$$\begin{aligned} \hat{a}_0(w) &= I_0 + I_1 + I_2 + I_3 \\ I_0 &= \sqrt{P_0} C_0^H[w] Z(w) B(w, w) C_0[w] a_0(w) \\ I_1 &= C_0^H[w] Z(w) B(w, w) U[w] \sqrt{Q} \tilde{a}[w] \\ I_2 &= \sum_{\substack{s=0 \\ s \neq w}}^{S-1} C_0^H[w] Z(w) B(w, s) C[s] \sqrt{P} a[s] \\ I_3 &= C_0^H[w] Z(w) N(w) \end{aligned} \quad (2)$$

I_0 represents the useful signal, I_1 the Multiple Access Interference (MAI) generated in the same sub band w , I_2 the interference generated by all users from other sub-bands and I_3 the filtered noise. $H(w,s)$ is a $N_c \times N_c$ matrix modeling the combined effect of channel attenuation and clock frequency offset. It is defined by:

$$H(w,s) = \begin{pmatrix} A_{w,s}(0,0) & \cdots & \cdots & A_{w,s}(0,N_F-1) \\ A_{w,s}(1,0) & \ddots & & A_{w,s}(1,N_F-1) \\ \vdots & \vdots & \ddots & \vdots \\ A_{w,s}(N_F-1,0) & \cdots & \cdots & A_{w,s}(N_F-1,N_F-1) \end{pmatrix}$$

$$A_{w,s}(p,n) = \begin{pmatrix} \phi(w,s,p,n,0) & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \phi(w,s,p,n,N_F-1) \end{pmatrix}$$

$\phi(w,s,p,n,q)$ is the equivalent channel transfer function including the effect of the clock frequency offset [4]:

$$\begin{aligned} \phi(w,s,p,n,q) &= h(e(sN_F+n))\psi_N \left(\frac{e(sN_F+n)\Delta T}{N} + \frac{(w-s)N_F+(p-n)}{N} \right) \\ &\times \exp \left(j2\pi q(N+v) \frac{e(sN_F+n)\Delta T}{N} \right) \\ &\times \exp \left(j\pi(N-1) \left(\frac{e(sN_F+n)\Delta T}{N} + \frac{(w-s)N_F+(p-n)}{N} \right) \right) \end{aligned} \quad (3)$$

$h(l)$ is the l^{th} output of the FFT of the channel impulse response, $\Psi_N(x)$ is the function defined by $\psi_N(x) = \frac{\sin(\pi Nx)}{N \sin(\pi x)}$ and $e(n)$ is the function defined by $e(n)=n$ if $n \leq N/2$ and $e(n)=n-N$ if $n \geq N/2$. $Z(w)$ is a $N_c \times N_c$ diagonal matrix which components are the equalizer's coefficients

III. Asymptotic Performance

The symbols $a_m(s)$ are assumed i.i.d. zero mean variance random variables with unit variance. The SINR is deduced from (2) by calculating the expectations of I_0 , I_1 , I_2 and I_3 :

$$SINR = \frac{E|I_0|^2}{E|I_1|^2 + E|I_2|^2 + E|I_3|^2} \quad (4)$$

with

$$\begin{aligned} E|I_0|^2 &= P_0 |C_0^H[w]Z(w)H(w,w)C_0[w]|^2 \\ E|I_1|^2 &= C_0^H[w]Z(w)H(w,w)U[w]QU^H[w]H(w,w)^H Z(w)^H C_0^H[w] \\ E|I_2|^2 &= \sum_{\substack{s=0 \\ s \neq w}}^{S-1} C_0^H[w]Z(w)H(w,s)C[s]PC^H[s]H(w,s)^H Z(w)^H C_0^H[w] \\ E|I_3|^2 &= \frac{\sigma^2}{N_c} \text{tr}(Z(w)Z(w)^H) \end{aligned} \quad (5)$$

These expressions show that the SINR depends of actual value of the spreading codes. Hence (5) cannot be used practically due to its complexity and its sensitivity to the code allocation. In order to evaluate the different terms of (5) independently on the spreading codes, we use three properties of the random matrix and free probability theories.

A- If $C_m = \frac{1}{\sqrt{N_c}}(c_m(0), \dots, c_m(N_c - 1))$ where $c_m(l)$'s are iid complex random variances with zero mean, unit variance and finite eighth order moment and A is a $N_c \times N_c$ uniformly bounded deterministic matrix (independent of C_m) then [7]:

$$C_m^H A C_m \xrightarrow{N_c \rightarrow \infty} \frac{1}{N_c} \text{tr}(A) \quad (6)$$

B- Let C a Haar distributed unitary matrix of size $N_c \times N_c$. C can be decomposed into a vector C_0 of size N_c and another matrix U of size $N_c \times N_u - 1$ as $C = (C_0, U)$, thus [8]:

$$U Q U^H \xrightarrow{N_c \rightarrow \infty} \alpha \bar{P} (I - C_0 C_0^H) \quad (7)$$

C- If C is generated from a $N_c \times N_c$ Haar unitary random matrix then matrices $C P C^H$ and $Z^H H^H H Z$ are asymptotically free almost everywhere [9], one concludes

$$\frac{1}{N_c} \text{tr}(Z H C P C^H H^H Z^H) \xrightarrow{N_c \rightarrow \infty} \frac{1}{N_c} \text{tr}(Z H H^H Z^H) \times \frac{1}{N_c} \text{tr}(C P C^H) \quad (8)$$

As was already explained in [5], the Haar distributed assumption is only technical. The final results obtained with real spreading matrices match with the model prediction. Invoking the properties (6), (7) and (8), the expectation terms of (5) simplifies as follows:

$$\begin{aligned} E|I_0|^2 &= P_0 \left| \frac{1}{N_c} \text{tr}(Z(w)H(w,w)) \right|^2 \\ E|I_1|^2 &= \alpha \bar{P}_s \left(\frac{1}{N_c} \text{tr}(Z(w)H(w,w)H(w,w)^H Z(w)^H) - \left| \frac{1}{N_c} \text{tr}(Z(w)H(w,w)) \right|^2 \right) \\ E|I_2|^2 &= \frac{\alpha \bar{P}_s}{N_c} \sum_{\substack{s=0 \\ s \neq s'}}^{S-1} \text{tr}(Z(w)H(w,s)H(w,s)^H Z(w)^H) \\ E|I_3|^2 &= \frac{\sigma^2}{N_c} \text{tr}(Z(w)Z(w)^H) \end{aligned} \quad (9)$$

$\alpha = N_u / N_c$ is the system load and $\bar{P} = \frac{1}{N_u - 1} \sum_{m=1}^{N_u - 1} P_m$ is the average power of the interfering users.

In order to validate the asymptotic SINR evaluated with (9), we first compare its results with Monte Carlo simulations. The common simulations assumptions are: FFT size: $N=64$, the scrambling sequence is the concatenation of 19 Gold sequences of 128 chips, constellation: QPSK.

Figure 3 illustrates the comparison between theoretical and simulated SINRs of the fifth band ($s=5$) for an OFDM-CDMA system ($N_F \times N_T = 8 \times 4$), a BRAN A channel model and a MMSE equalizer ($z_q(wN_F + p) = \frac{\phi^*(w, w, p, p, q)}{|\phi(w, w, p, p, q)|^2 + \gamma}$, $\gamma = \frac{\alpha \bar{P}}{\sigma^2}$). It shows that our theoretical model matches perfectly with simulations, even for a relatively small spreading factor ($N_c=32$).

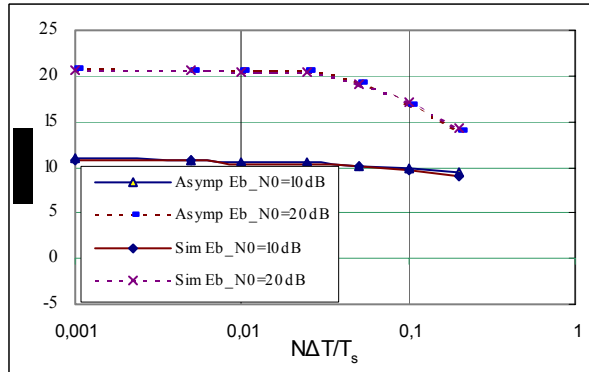


Figure 3: validation of theoretical model by SINR comparison (BRAN A channel).

Since, the asymptotical model has been validated, we will exploit some induced results to give more insights about the comparison of the different spreading schemes. We define then the SINR degradation of the system defined in dB by $Deg_{(dB)} = 10 * \log_{10} \left(\frac{SINR_{max}}{SINR} \right)$ where SINRmax is the SINR obtained without clock frequency offset. Figure 4 shows that the MC-CDMA scheme ($N_F \times N_T = 32 \times 1$) is slightly less sensitive than OFDM-CDMA ($N_F \times N_T = 8 \times 4$) and MC-DS-CDMA ($N_F \times N_T = 1 \times 32$) systems to clock frequency offset. Moreover, it shows that for a given signal bandwidth, this sensitivity increases with the system load and number of carriers.

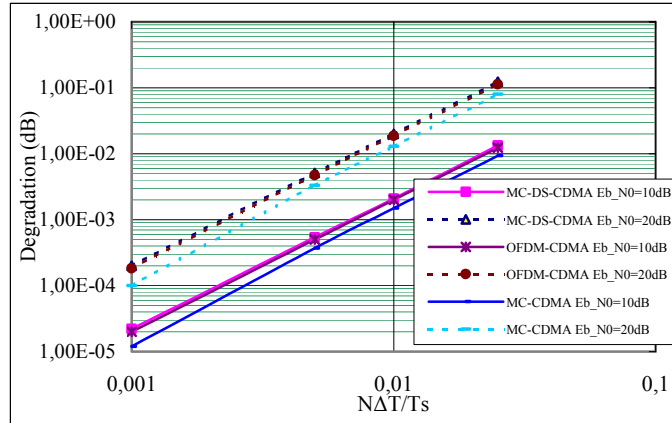


Figure 4: Degradation of spreading schemes (Bran A channel $E_b/N_0=20\text{dB}$).

Let us give some more insights about the comparison between MC-CDMA ($N_F=N, N_T=1, S=1$) and MC-DS-CDMA ($N_F=1, N_T=N, S=N$) schemes. In order to compare the different spreading schemes, we define the parameter: $I(w) = E|I_1|^2 + E|I_2|^2 + E|I_3|^2$. Let us note $I_{MC-DS-CDMA}(w)$ the interference power of the w^{th} sub-carrier of the MC-DS-CDMA scheme, and $I_{MC-CDMA}$ the total interference for a pure MC-CDMA system. For a zero-forcing (ZF) equalizer ($\gamma=0$), we show that:

$$I_{MC-CDMA} = \frac{1}{N} \sum_{w=0}^{N-1} I_{MC-DS-CDMA}(w) \quad (10)$$

With a ZF equalizer, the total interference power of a MC-CDMA system is the average value of the interference experienced by each sub-carrier in a MC-DS-CDMA system.

Figure 5-a and Figure 5-b show the validity of equation (10) in gaussian and BRAN A channels ($E_b/N_0 = 20 \text{ dB}$). The legend 'mean MC-DS-CDMA' refers to the interference computed with equation (10). Figure 6 shows that (10) is not valid for a MMSE equalizer in a frequency selective channel.

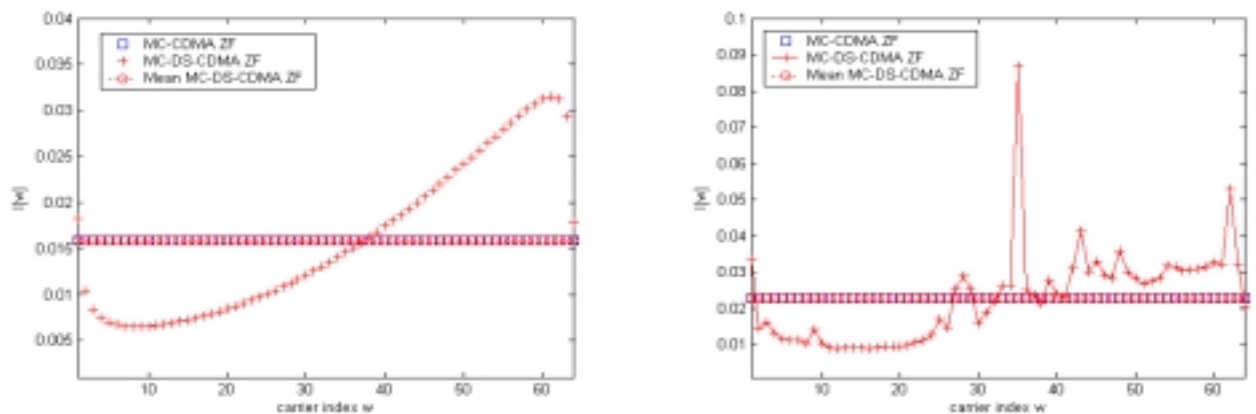


Figure 5: Interference power (a- ZF and Gaussian channel, b-ZF and BRAN A channel) $\Delta T/T = 1560 \text{ ppm}$.

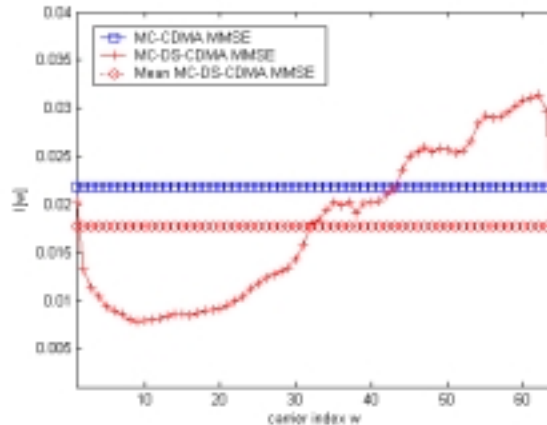


Figure 6: Interference power (MMSE and BRAN A channel, $\Delta T/T = 1560$ ppm).

IV. Conclusion

In this article, we have investigated the effect of clock frequency offset on the performance of 2 dimensional OFDM-CDMA spreading schemes. A new analytical expression of the SINR has been derived. It is independent of the actual value of the spreading codes but takes their orthogonality into account. It is valid for different single user detector (MMSE, ZF) and for any frequency selective channel. Exploiting this model, we found that, for a ZF equalizer, the total interference power of a MC-CDMA system is the average of the interference experienced by each sub-carrier in a MC-DS-CDMA system whatever the channel model. The latter does not hold for a MMSE detector.

V. References

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