

Sensitivity of OFDM-CDMA Systems to Carrier Frequency Offset

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Abstract— This paper presents the impact of a carrier frequency offset on the performance of 2 dimensional spreading OFDM-CDMA systems. This is measured by the degradation of the Signal to Interference plus Noise Ratio (SINR) obtained after despreading and equalization. Using some properties of random matrix and free probability theories, a new expression of the SINR is derived. It is independent of the actual value of the spreading codes while still accounting for the orthogonality between codes. This model is validated by means of Monte-Carlo simulations. . It is also exploited to compare the sensitivities of MC-CDMA and MC-DS-CDMA systems to carrier offset in a frequency selective channel. This work is carried out for zero forcing (ZF) and minimum mean square error (MMSE) equalizers.

Keywords: MC-CDMA, MC-DS-CDMA, OFDM-CDMA, Carrier offset, SINR.

I. INTRODUCTION

Recently, Orthogonal Frequency and Code Division Multiplexing (OFCDM) access technology has been investigated for the next generation of mobile communication systems [1][2]. It is a combination of Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA). To achieve high spectrum efficiency, these systems will implement a large number of sub-carriers, and, as a consequence, will be highly sensitive to synchronization errors. More specifically, when the transmitter and receiver carrier frequencies are not synchronized, inter-carrier interference is generated at the receiver, which degrades the Signal to Interference and Noise Ratio (SINR). This effect has been intensively studied for OFDM systems [3][4]. Concerning MC-CDMA and MC-DS-CDMA schemes, their sensitivity to carrier frequency offset has been evaluated in [5] and [6]. To derive an analytic expression of the SINR, these articles considered the particular case of a gaussian channel and a zero-forcing equalizer. Moreover, they assume that the chips of the spreading sequences are independent and identically distributed (i.i.d.) binary random variables. This is the classical random spreading assumption. Unfortunately, this model is not accurate for the downlink of actual CDMA or OFDM-CDMA systems, since it does not take into account the orthogonality between codes. When using an isometric spreading matrix, the codes are no longer independent.

The contribution of this article is thus twofold. First, a generalized framework is proposed for modelling the effect of carrier frequency offset on 2 dimensional spreading OFDM-CDMA systems. This encompasses the particular cases of

MC-CDMA and MC-DS-CDMA. Then, exploiting some results from the random matrix theory, an analytic expression of the SINR modeling the impact of carrier frequency offset is derived. This model works for frequency selective channels and any single user detector. To derive the SINR formula, the mathematical background developed in [11], [12] and [14] has been reused. This formula is independent from the actual values of the spreading codes while taking into account their orthogonality. This is the main novelty of this article. To confirm the validity of the proposed model, the mean theoretical SINR is compared to the mean SINR measured via Monte Carlo simulations for a BRAN A channel model. It is then used to compare the sensitivities of MC-CDMA and MC-DS-CDMA systems to carrier offset in a frequency selective channel, with a zero forcing (ZF) or a minimum mean square error (MMSE) equalizer.

This article is structured as follows. Section 2 describes the system model for OFDM-CDMA with carrier frequency offset. Section 3 derives an asymptotic expression of the SINR using some properties of random matrix theory. Section 4 first validates the proposed model by mean of simulations and then the SINR formula is exploited to compare the sensitivities of MC-CDMA and MC-DS-CDMA systems.

II. SYSTEM MODEL

A. System Description

In this section, the generalized framework describing a OFDM-CDMA system is presented. Figure 1 shows a 2 dimensional (2D) spreading OFDM-CDMA transmitter/receiver chain for a downlink communication with N_u users [1].

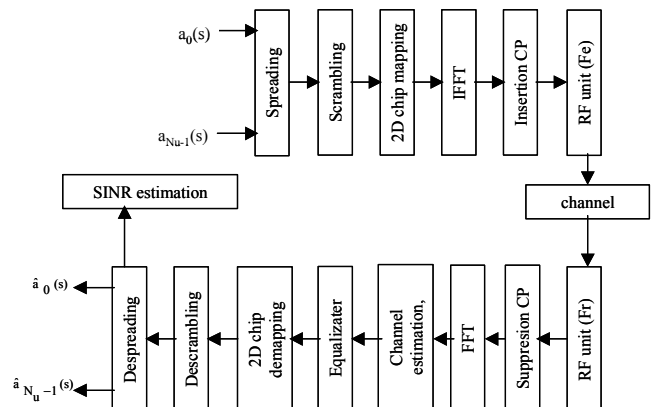


Figure 1- OFDM-CDMA Transceiver.

Each symbol is first spread by a Walsh-Hadamard sequence of N_c chips, and scrambled by a cell specific long pseudo random sequence. This scrambling code is used to minimize the multi-cell interference. The resulting samples are then allocated on the time/frequency grid as shown on Figure 2.

The first N_T samples are allocated in the time direction. The next blocks of N_T chips are allocated identically on adjacent sub-carriers. Each bin with coordinate (n,q) represents the signal transmitted on the n^{th} sub-carrier of the q^{th} OFDM symbol.

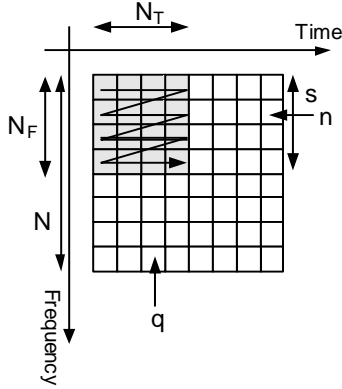


Figure 2- Time Frequency grid

The spreading factor is thus $N_c = N_F \times N_T$ where N_F and N_T are respectively the frequency and time domain spreading factors. Assuming a FFT of N points, each user transmits $S = N/N_F$ data symbols in an OFDM-CDMA block. Particularly, for $N_T = 1$, this resumes to a MC-CDMA scheme [7] and for $N_F = 1$, to MC-DS-CDMA [8].

Given the above notations, the signal at the IFFT input is:

$$b_q(sN_F + n) = \sum_{m=0}^{N_u-1} \sqrt{P_m} a_m(s) C_{m,s} [nN_T + q] \quad (1)$$

$$q = 0, \dots, N_T - 1; s = 0, \dots, S - 1, n = 0, \dots, N_F - 1$$

s is the index referring to the sub band used for the transmission of the symbol $a_m(s)$ of the m^{th} user. P_m is its transmit power which is identical in all sub-bands, $C_{m,s}$ represents its spreading sequence (chip by chip multiplication of the user assigned Walsh-Hadamard sequence and the cell specific scrambling code).

At the output of the IFFT, a cyclic prefix of v samples is inserted to discard the inter symbol interference:

$$x_q(k) = \sum_{s=0}^{S-1} \sum_{n=0}^{N_F-1} b_q(sN_F + n) \exp\left(j2\pi \frac{k(sN_F + n)}{N}\right) \quad (2)$$

$$k = -v, \dots, N - 1$$

After D/A conversion, transposition to the carrier frequency F_c by the RF unit, and transmission through the channel, the signal at the receiver side (Figure 1) is transposed to base band with the receiver carrier frequency F_r and sampled. The base-band time-discrete received signal, after guard time removal is then:

$$r_q(u) = \sum_{k=0}^{N-1} x_q(k) g_q(u-k) \exp(j2\pi\Delta F T_s u + \beta_q) + n_q(u) \quad (3)$$

with $u = 0, \dots, N-1$ and $\beta_q = j2\pi\Delta F T_s q(N+v)$

$g_q(k)$ ($k = 0 \dots W-1$) are the complex coefficients of the discrete equivalent low-pass channel, with delay spread W samples ($v < W$). T_s is the sampling clock period assumed identical at both sides. Notice that we use a discrete representation of the transmission with sampling-frequency $F_s = 1/T_s$ equal to the bandwidth of the real band pass transmit signal. Thus, the coefficients $g_q(k)$, of the discrete channel can be expressed as a function of the real parameters of the analog channel $g_q(k) = \sum_{l=0}^{L-1} \alpha_q^{(l)} \cdot \text{sinc}\left\{ \pi F_s \left(\frac{k}{F_s} - \tau_q^{(l)} \right) \right\}$.

$\alpha_q^{(l)}$ and $\tau_q^{(l)}$ are respectively the complex amplitudes and the delays of the l^{th} multi-path, and $\text{sinc}(x) = \sin(x)/x$. $\Delta F = F_r - F_c$ is the residual carrier frequency offset. The initial phase β_q for the q -th OFDM symbols means that the receiver is perfectly synchronized at the beginning of each 2D symbol. $n_q(u)$ is a complex additive white Gaussian noise with variance σ^2 .

Assuming a perfect synchronization of the FFT window, the receiver selects N consecutive samples and transposes them to the frequency domain thanks to the FFT.

The value of $(wN_F + p)^{\text{th}}$ sub-carrier of the q^{th} OFDM symbol is given by :

$$R_q(wN_F + p) = \sum_{s=0}^{S-1} \sum_{n=0}^{N_F-1} b_q(sN_F + n) \phi(w, s, p, n, q) + N_q(wN_F + p) \quad (4)$$

w is the desired sub-band index ($w = 0, \dots, S-1$) and p is the index of a sub-carrier in the w^{th} sub-band ($p = 0, \dots, N_F-1$). $\phi(w, s, p, n, q)$ is the equivalent channel transfer function including the effect of the carrier frequency offset :

$$\phi(w, s, p, n, q) = h(sN_F + n) \Psi_N \left(\Delta F T_s + \frac{(w-s)N_F + (p-n)}{N} \right) \times \exp(j2\pi\Delta F T_s q(N+v)) \times \exp\left(j\pi(N-1) \left(\Delta F T_s + \frac{(w-s)N_F + (p-n)}{N} \right) \right) \quad (5)$$

$h(sN_F + n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{W-1} g(k) e^{-j2\pi \frac{k(sN_F + n)}{N}}$ is the FFT of the channel impulse response and $\Psi_N(x)$ is the function defined by $\Psi_N(x) = \frac{\sin(\pi N x)}{N \sin(\pi x)}$.

Without loss of generality, we assume that one is interested by the symbols of user 0. In order to write the received signal with matrix-vector notation, the following matrices are defined: $P = \text{diag}(P_0, \dots, P_{N_u-1})$ is the $N_u \times N_u$ diagonal matrix which entries are the power allocated to each user, $Q = \text{diag}(P_1, \dots, P_{N_u-1})$ is the $(N_u-1) \times (N_u-1)$ diagonal matrix containing the power of the interfering users, $C[s] = (C_0[s], C_1[s], \dots, C_{N_u-1}[s])$ is the $N_c \times N_u$ matrix containing all the spreading codes used in the s^{th} sub-band and

$U[s] = (C_1[s], \dots, C_{N_u-1}[s])$ is the $N_c \times (N_u-1)$ matrix containing the codes of the interfering users in the s^{th} sub-band. Both matrices $C[s]$ and $U[s]$ depend on sub-band index s because of the long scrambling code. We also define the vectors $a[s] = (a_0[s], \dots, a_{N_u-1}[s])^T$ and $\tilde{a}[s] = (a_1[s], \dots, a_{N_u-1}[s])^T$ corresponding to the symbols of all and interfering users respectively transmitted in the s^{th} sub-band.

The signal at the FFT output is multiplied by the one-tap channel equalization coefficients, descrambled and then it is multiplied by the spreading sequence of the reference user and summed over the chips to obtain the sample at the input of the decision device. The estimated symbol of the reference user on the sub-band w is then:

$$\begin{aligned} \hat{a}_0(w) &= I_0 + I_1 + I_2 + I_3 \\ I_0 &= \sqrt{P_0} C_0^H[w] Z(w) B(w, w) C_0[w] a_0(w) \\ I_1 &= C_0^H[w] Z(w) B(w, w) U[w] \sqrt{Q} \tilde{a}[w] \\ I_2 &= \sum_{\substack{s=0 \\ s \neq w}}^{S-1} C_0^H[w] Z(w) B(w, s) C[s] \sqrt{P} a[s] \\ I_3 &= C_0^H[w] Z(w) N(w) \end{aligned} \quad (6)$$

(Notation: A^H is the transpose-conjugate of matrix A).

I_0 represents the useful signal, I_1 the Multiple Access Interference (MAI) generated in the same sub band w , I_2 the interference generated by all users from other sub-bands and I_3 the filtered noise. $B(w, s)$ is a $N_c \times N_c$ matrix modeling the combined effect of channel attenuation and carrier frequency offset. It is defined by:

$$B(w, s) = \begin{pmatrix} A_{w,s}(0,0) & \dots & \dots & A_{w,s}(0, N_F-1) \\ A_{w,s}(1,0) & \ddots & & A_{w,s}(1, N_F-1) \\ \vdots & \vdots & \ddots & \vdots \\ A_{w,s}(N_F-1,0) & \dots & \dots & A_{w,s}(N_F-1, N_F-1) \end{pmatrix}$$

$$A_{w,s}(p, n) = \begin{pmatrix} \phi(w, s, p, n, 0) & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \phi(w, s, p, n, N_T-1) \end{pmatrix}$$

If there is no carrier offset, $B(w, s)$ is the null matrix for $w \neq s$ and is diagonal for $w = s$. In this case, I_2 is equal to zero, there is no inter carrier interference.

$Z(w)$ is a $N_c \times N_c$ diagonal matrix which components are the equalizer's coefficients.

B. SINR evaluation

The symbols $a_m(s)$ are assumed i.i.d. zero mean and unit variance random variables. The SINR for every sub band for one channel realisation is deduced from (6) by calculating the expectations of I_0 , I_1 , I_2 and I_3 :

$$SINR = \frac{E|I_0|^2}{E|I_1|^2 + E|I_2|^2 + E|I_3|^2} \quad (7)$$

The expectations in (7) are given by :

$$\begin{aligned} E|I_0|^2 &= P_0 |C_0^H[w] Z(w) B(w, w) C_0[w]|^2 \\ E|I_1|^2 &= C_0^H[w] Z(w) B(w, w) U[w] Q U^H[w] B(w, w)^H Z(w)^H C_0^H[w] \\ E|I_2|^2 &= \sum_{\substack{s=0 \\ s \neq w}}^{S-1} C_0^H[w] Z(w) B(w, s) C[s] P C^H[s] B(w, s)^H Z(w)^H C_0^H[w] \\ E|I_3|^2 &= \frac{\sigma^2}{N_c} \text{tr}(Z(w) Z(w)^H) \end{aligned} \quad (8)$$

These expressions show that the SINR depends of actual value of the spreading codes. Hence (8) cannot be used practically due to its complexity and its sensitivity to the code allocation. In the sequel, we derive a new SINR formula that will be independent from the spreading codes while taking into account their orthogonality. This work relies on results of random matrix and free probability theories.

III. ASYMPTOTIC PERFORMANCE

Tse and Hanly [9] were the pioneer in the application of random matrix theory to CDMA systems analysis. They studied the asymptotic performance of the multi-user MMSE receiver for a CDMA system, with random spreading and synchronous reception. They found that the dependence of the SINR on the spreading codes was vanishing in the asymptotic regime (N_c and $N_u \rightarrow \infty$ while the ratio $\alpha = N_u/N_c$ is kept constant). The performances only depend on the system load α , noise variance and the power distribution. This work was then extended to a multipath fading channel in [10].

Unfortunately, the model with random spreading is not accurate for the downlink of actual CDMA or OFDM-CDMA systems, since it does not take into account the orthogonality between codes.

To solve this issue the authors of [11] proposed a trick. They assume that the spreading matrix $C[s]$ is extracted from a Haar distributed unitary matrix. Such a matrix is random and isometric, which capture the orthogonality of conventional spreading matrices. This assumption allows applying very powerful results from the free probability theory. In [11] and [12], it was found that the dependence of the SINR on the spreading codes was also vanishing in the asymptotic regime. However, as opposed to [10], the orthogonality between codes was accounted for. In addition, it was found that the aforementioned assumption is only technical. The simulation results obtained in [11][12][13] with the combinaison of Walsh-Hadamard sequences and a scrambling code match very well with the theoretical model. This is achieved even for relatively small spreading factors ($N_c \geq 32$).

In order to evaluate the different terms of (8) and get ride off the dependence on the spreading codes, we apply three properties from the random matrix and free probability theories. The details of the computations are given in the Appendix. The final results are the following:

$$\begin{aligned}
E|I_0|^2 &= P_0 \left| \frac{1}{N_c} \text{tr}(Z(w)B(w;w)) \right|^2 \\
E|I_1|^2 &= \alpha \bar{P} \left(\frac{1}{N_c} \text{tr}(Z(w)B(w;w)B(w;w)^H Z(w)^H) - \left| \frac{1}{N_c} \text{tr}(Z(w)B(w;w)) \right|^2 \right) \\
E|I_2|^2 &= \frac{\alpha \bar{P}}{N_c} \sum_{s=0}^{S-1} \sum_{s' \neq s} \text{tr}(Z(w)B(w;s)B(w;s')^H Z(w)^H) \\
E|I_3|^2 &= \frac{\sigma^2}{N_c} \text{tr}(Z(w)Z(w)^H)
\end{aligned} \quad (9)$$

$\alpha = N_u/N_c$ is the system load and $\bar{P} = \frac{1}{N_u - 1} \sum_{m=1}^{N_u-1} P_m$ is the average power of the interfering users.

IV. PERFORMANCE EVALUATION

We will now exploit (9) to compare MC-CDMA, MC-DS-CDMA and OFDM-CDMA spreading schemes. First, we assume that the receiver uses a MMSE equalizer :

$$z_q(wN_F + p) = \frac{\phi^*(w, w, p, p, q)}{|\phi(w, w, p, p, q)|^2 + \gamma}$$

γ is the inverse of the SNR per sub-carrier : $\gamma = \frac{\alpha \bar{P}}{\sigma^2}$.

This implicitly assumes a perfect estimation of the rotated and attenuated channel frequency response on sub-carrier ($wN_F + p$):

$$\phi(w, w, p, p, q) = h(wN_F + p)\psi_N(\Delta F T_s) \exp(j\pi \Delta F T_s (2q(N+v) + N - 1))$$

In order to compare the different spreading scheme, we define the parameter: $I(w) = E|I_1|^2 + E|I_2|^2 + E|I_3|^2$. With these assumptions the different terms of (9) become:

$$\begin{aligned}
E|I_0|^2 &= \frac{P_0}{N_F^2} \left| \sum_{p=0}^{N_F-1} \frac{h(wN_F + p)\psi_N(\Delta F T_s)}{|\phi(w, w, p, p, q)|^2 + \gamma} \right|^2 \\
I(w) &= \frac{\alpha \bar{P}}{N_F} \sum_{s=0}^{S-1} \sum_{p=0}^{N_F-1} \sum_{n=0}^{N_F-1} \frac{h^*(wN_F + p)h(sN_F + n)\psi_N(\Delta F T_s) \psi_N(\Delta F T_s + \frac{(w-s)N_F + p - n}{N})}{|\phi(w, w, p, p, q)|^2 + \gamma} \\
&\quad - \alpha \bar{P} \left[\frac{1}{N_F} \sum_{p=0}^{N_F-1} \frac{|h(wN_F + p)\psi_N(\Delta F T_s)|^2}{|\phi(w, w, p, p, q)|^2 + \gamma} + \frac{\sigma^2}{N_F} \sum_{p=0}^{N_F-1} \frac{|h(wN_F + p)\psi_N(\Delta F T_s)|^2}{|\phi(w, w, p, p, q)|^2 + \gamma} \right] \quad (10)
\end{aligned}$$

In order to validate the asymptotic formula, we first compare the result computed with (7) and (10) with the SINR measured via Monte Carlo simulations. The simulations assumptions are the followings:

- FFT size: $N=64$, Spreading factor: $N_c=32$ chips, QPSK modulation.
- Scrambling code: concatenation of 19 Gold codes of 128 chips each.
- Spreading schemes: MC-CDMA: ($N_F=32$, $N_T=1$), OFDM-CDMA: ($N_F=8$, $N_T=4$), MC-DS-CDMA: ($N_F=1$, $N_T=32$).

Figure 3 illustrates the comparison between theoretical and simulated SINRs for BRAN A channel model [17]. The SINRs have been measured in the first sub-band ($w=0$). Due to the variations of the channel caused by the mobile speed (3 km/h), we compare the means SINRs. The simulations performances are realized for a mean ratio \bar{E}_b/N_0 of 20 dB. It is given by $\frac{\bar{E}_b}{N_0} = \frac{N+v}{N} \frac{\sum_{l=0}^{L-1} E(|\alpha^{(l)}|^2)}{2\sigma^2} P_0$ where $|\alpha^{(l)}|^2$ is the power of each multipath of the channel. Figure 3 shows that our theoretical model matches perfectly with simulations, even for a relatively small spreading factor ($N_c=32$). As for conventional multicarrier systems, the degradation becomes noticeable for $N\Delta FT > 1\%$. The sensitivity of the 3 spreading schemes to carrier offset is comparable.

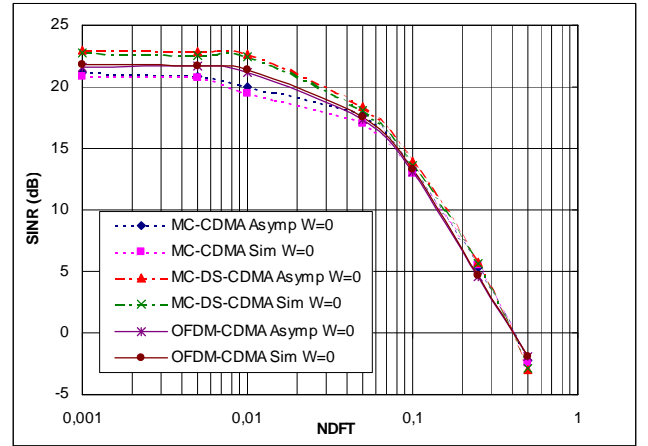


Figure 3: validation of theoretical model (BRAN A).

Figure 4 presents the sensitivity of the MC-DS-CDMA scheme to system load for a gaussian channel. As predicted by equation (10), the degradation increases with the load. This result is in contradiction with the conclusion of [5] which stated that the MC-DS-CDMA scheme was not sensitive to system load since [5] supposes that in the latter that the MAI is eliminated by the equalizer.

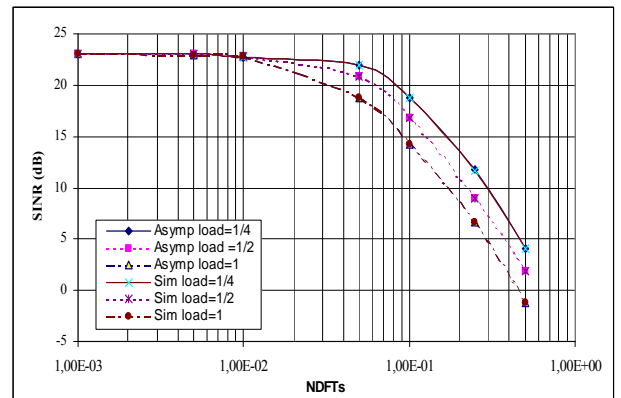


Figure 4: sensitivity of MC-DS-CDMA to system load (Gaussian channel).

On other side, it was demonstrated in [18] that the MAI at the output of a linear receiver is asymptotically gaussian, as $N_c \rightarrow \infty$. This property allows using the SINR as a criterion to determine the system performance. For a QPSK modulation, the Bit Error Ratio (BER) is related to the SINR by:

$$BER = E \left[Q \left(\sqrt{SINR} \right) \right] \quad (11)$$

$Q(x)$ is the gaussian tail function, and the expectation is taken over the channel realizations. This is validated in Figure 5, which shows that the BER obtained with (11) and (10) matches perfectly with Monte Carlo simulations. However, it is known that the channel coding scheme has a huge impact on the comparison between OFDM-CDMA and COFDM schemes [19]. As MC-DS-CDMA is very similar to a COFDM system, the channel coding scheme must be considered for the comparison between spreading schemes.

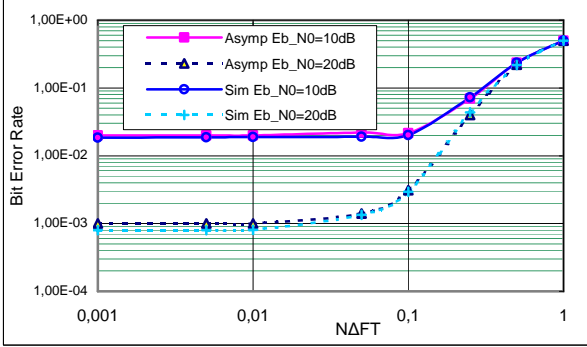


Figure 5: BER comparison between asymptotic model and simulation (Bran A channel OFDM-CDMA).

Since the theoretical model has been validated, we will exploit equation (10) to give more insight on the sensitivities of MC-CDMA and MC-DS-CDMA to carrier frequency offset.

If the receiver implements a zero-forcing (ZF) equalizer ($\gamma=0$), equation (10) becomes:

$$E|I_0|^2 = P_0$$

$$I(w) = \frac{\alpha \bar{P}}{N_F} \sum_{s=0}^{S-1} \sum_{p=0}^{N_F-1} \sum_{n=0}^{N_F-1} \left| \frac{h(sN_F + n) \psi_N \left(\Delta FT_s + \frac{(w-s)N_F + p - n}{N} \right)}{h(wN_F + p) \psi_N(\Delta FT_s)} \right|^2$$

$$- \alpha \bar{P} + \frac{\sigma^2}{N_F} \sum_{p=0}^{N_F-1} \frac{1}{|h(wN_F + p) \psi_N(\Delta FT_s)|^2}$$

Let us note $I_{MC-DS-CDMA}(w)$ the interference power of the w^{th} sub-carrier of a MC-DS-CDMA scheme ($N_F=1, N_T=N, S=N$), and $I_{MC-CDMA}$ the total interference for a pure MC-CDMA system ($N_F=N, N_T=1, S=1$). Based on the above formula, we observe that:

$$I_{MC-CDMA} = \frac{1}{N} \sum_{w=0}^{N-1} I_{MC-DS-CDMA}(w) \quad (12)$$

With a ZF equalizer, the total interference power of a MC-CDMA system is the average of the interference experienced by each sub-carrier in a MC-DS-CDMA system. For a Gaussian channel, it is easily proven that $I(w+k) = I(w)$. This is due to the periodicity of function $\psi_N(x)$ ($\psi_N(x+1) = \psi_N(x)$). Hence, as shown in Figure 6, the interference power is independent of the carrier index. The legend ‘mean MC-DS-CDMA’ refers to the interference computed according to equation (12). The accuracy of equation (12) for a ZF equalizer in a BRAN A channel is demonstrated in Figure 7

by giving the mean over channel realizations of the total interference power. Figure 8 shows that it is not the case for a MMSE equalizer.

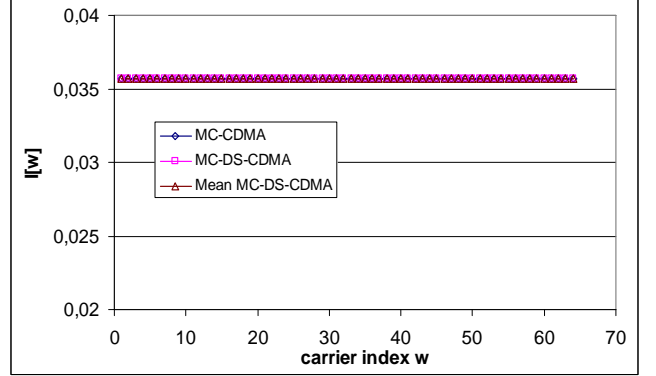


Figure 6: Interference power (ZF and Gaussian channel) $\Delta F=1560$ ppm.

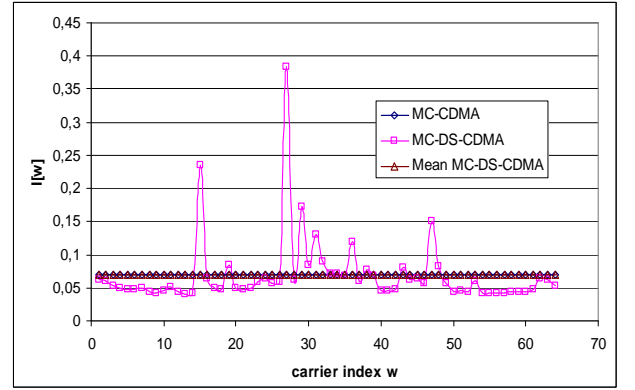


Figure 7: Mean Interference power (ZF and BRAN A channel) $\Delta F=1560$ ppm.

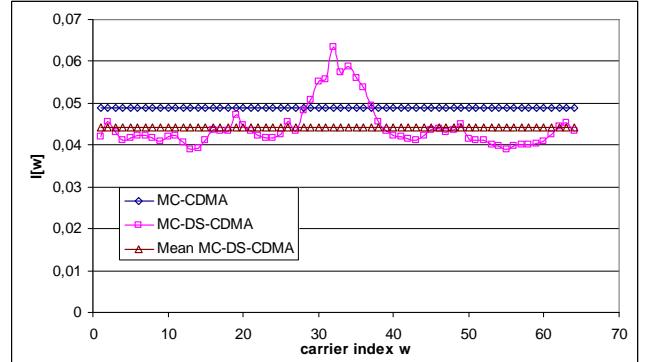


Figure 8: Mean Interference power (MMSE BRAN A channel) $\Delta F=1560$ ppm.

V. CONCLUSION

In this article, we have investigated the effect of carrier frequency offset on the performance of 2 dimensional OFDM-CDMA spreading schemes. A new analytical expression of the SINR has been derived. It is valid for various kind of single user equalizer (MMSE, ZF) and for any frequency selective channel. It is independent of the actual value of the spreading codes but takes their orthogonality into account. Exploiting this model, we found that, for a ZF equalizer, the total interference power of a MC-CDMA system is the average of the interference experienced by each sub-carrier in a MC-DS-CDMA system.

VI. APPENDIX

In this section, the 3 properties from the random matrix and free probability theories are first defined. Then their application for the computation of (9) is detailed.

Property 1: If A is a $N_c \times N_c$ uniformly bounded deterministic matrix and $C_m = \frac{1}{\sqrt{N_c}}(c_m(0), \dots, c_m(N_c - 1))$ where $c_m(l)$'s are iid complex random variables with zero mean, unit variance and finite eighth order moment, then [10]:

$$C_m^H A C_m \xrightarrow{N_c \rightarrow \infty} \frac{1}{N_c} \text{tr}(A) \quad (13)$$

C_0 is obtained by the multiplication of a Walsh-Hadamard sequence with a long scrambling code. Hence, the assumptions needed for (13) are easily satisfied. This property is used to evaluate $E|I_0|^2$, $E|I_1|^2$, $E|I_2|^2$ and $E|I_3|^2$.

Property 2: Let C be a Haar distributed unitary matrix of size $N_c \times N_u$ [11]. $C = (C_0 U)$ can be decomposed into a vector

C_0 of size N_c and a matrix U of size $N_c \times (N_u - 1)$. Given these assumptions, it is proven in [12] that:

$$U Q U^H \xrightarrow{N_c \rightarrow \infty} \alpha \bar{P} (I - C_0 C_0^H) \quad (14)$$

$\alpha = N_u / N_c$ is the system load and $\bar{P} = \frac{1}{N_u - 1} \sum_{m=1}^{N_u-1} P_m$ is the average power of the interfering users. This property is used to evaluate $E|I_1|^2$.

Property 3: If C is generated from a $N_c \times N_c$ Haar unitary random matrix then matrices $C[s] Q C^H[s]$ and $Z(w)^H B(w,s)^H B(w,s) Z(w)$ are asymptotically free almost everywhere [15]. In other words, verifying the above conditions, one can conclude:

$$\begin{aligned} & \frac{1}{N_c} \text{tr} \left(Z(w) B(w,s) C[s] P C^H[s] B(w,s)^H Z(w)^H \right) \xrightarrow{N_c \rightarrow \infty} \\ & \frac{1}{N_c} \text{tr} \left(Z(w) B(w,s) B(w,s)^H Z(w)^H \right) \times \frac{1}{N_c} \text{tr} \left(C[s] P C^H[s] \right) \end{aligned} \quad (15)$$

For definition of freeness, the reader may refer to [11] for more details.

Assuming that $C_0[w]$ is random (13) is used to evaluate $E|I_0|^2$. Since we use a long scrambling code, $C_0[w]$ and $C[s]$ are independent for $w \neq s$. Using (13), $E|I_2|^2$ becomes :

$$E|I_2|^2 = \sum_{\substack{s=0 \\ s \neq w}}^{S-1} \text{tr} \left(Z(w) B(w,s) C[s] P C^H[s] B(w,s)^H Z(w)^H \right) \quad (16)$$

This computation method has initially been applied for the analysis of multi-cell downlink CDMA systems in [16].

Applying (14) for the computations of $E|I_1|^2$ and (15) for the computations of $E|I_2|^2$ in (16), (9) is obtained.

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