Towards the Optimal Amplify-and-Forward Cooperative Diversity Scheme

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Abstract

How to find a cooperative diversity scheme that achieves the transmit diversity bound is still an open problem. In fact, all previously proposed amplify-and-forward (AF) and decode-and-forward (DF) schemes do not improve with the number of relays in terms of the diversity-multiplexing tradeoff (DMT) for multiplexing gains r higher than 0.5. In this work, we study the class of slotted amplify-and-forward (SAF) schemes. We first establish an upper-bound on the DMT for any half-duplex SAF scheme with an arbitrary number of relays N and number of slots M. Then, we propose a naive SAF scheme that can exploit the potential diversity gain in the high multiplexing gain regime. More precisely, in certain conditions, the naive SAF scheme achieves the proposed DMT upper-bound which tends to the transmit diversity bound when M goes to infinity. In particular, for the two-relay case, the three-slot naive SAF scheme achieves the proposed upper-bound and outperforms the NAF scheme of Azarian *et al.* for multiplexing gains $r \leq 2/3$. Numerical results reveal a significant gain of our scheme over the previously proposed AF schemes, especially in high spectral efficiency and large network size regime.

Index Terms

Cooperative diversity, relay, diversity-multiplexing tradeoff (DMT), slotted amplify-and-forward (SAF), relay scheduling.

I. Introduction and Problem Description

As a new way to exploit spatial diversity in a wireless network, cooperative diversity techniques have recently drawn more and more attention. Since the work of Sendonaris *et al.* [1], [2], a flood

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of works has appeared on this subject and many cooperative protocols have been proposed (see, for example, [3]–[8]). A fundamental performance measure to evaluate different cooperative schemes is the diversity-multiplexing tradeoff (DMT) which was introduced by Zheng and Tse [9] for the MIMO Rayleigh channel. It is well known that the DMT of any N-relay cooperative diversity scheme is upper-bounded (referred to as the *transmit diversity bound* in [4]) by the DMT of a MISO system with $N+1$ antennas,

$$d(r) = (N+1)(1-r)^{+}. \tag{1}$$

This bound is actually proved achievable by the cooperative multiple access (CMA) scheme [6], using a Gaussian code with an infinite cooperation frame length.

However, how to achieve (1) in a single-user setting (*i.e.*, half-duplex relay channel) in the general case is still an open problem, even with an infinite cooperation frame length. In the single-relay case, the best known cooperative scheme, in the class of amplify-and-forward strategies, is the Non-orthogonal Amplify-and-Forward (NAF) scheme and the Dynamic Decode-and-Forward (DDF) scheme in the class of decode-and-forward strategies. The NAF scheme was proposed by Nabar *et al.* [5] and has been proved to be the optimal amplify-and-forward scheme for a single-relay channel by Azarian *et al.* [6]. It is therefore impossible to achieve (1) by only amplifying-and-forwarding and one relay. The DDF scheme was proposed independently in [6], [10], [11] in different contexts. In [6], it is shown that the DDF scheme does achieve (1) in the low multiplexing gain regime ($r < 0.5$) but it fails in the high multiplexing gain regime, which is due to the *causality* of the decode-and-forward scheme. Intuitively, to achieve the MISO bound with a multiplexing gain r, the source and the relay need to cooperate during at least r-portion of the time. However, before this might possibly happen, the relay also needs at least r-portion of the time to decode the source signal (even with a Gaussian source-relay link). Therefore, it is impossible for the DF schemes to achieve the MISO bound for $2r > 1$.

Being optimal in the single-relay case, the generalization of the NAF and the DDF schemes proposed in [6], also the best known in each class, fails to exploit the potential spatial diversity gain in the high multiplexing gain regime ($r > 0.5$) with the growth of the network size. The suboptimality of these two schemes becomes very significant for a large number of relays, as shown in Fig. 1. Our goal is therefore to find a practical scheme that can possibly fill the gap between the two schemes and the MISO bound. In this work, we focus on the class of slotted

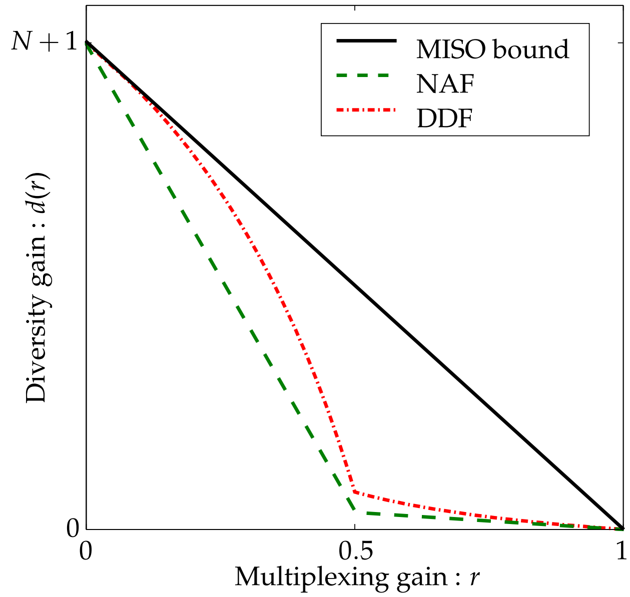


Fig. 1. Diversity-multiplexing tradeoff of an N-relay channel : NAF, DDF *vs.* MISO bound.

amplify-and-forward (SAF) schemes because of the following attractive properties :

1) Low relaying complexity. The relays only need to scale the received signal and retransmit it.
2) Existence of optimal codes with finite framelength. We will show that any SAF scheme is equivalent to a linear fading channel, whose DMT is achieved by perfect [12] $M \times M$ codes. The code length for an M-slot SAF scheme is therefore at most M^2.
3) Flexibility. The source does not have to know the number of relays or the relaying procedure. The coding scheme only depends on the number of slots M and is always optimal in terms of DMT.

A natural question is raised : *Is it possible for a SAF scheme to achieve the MISO bound (1)? And how to achieve it if it is possible?* This question is partially answered in this work. The main contributions of this work are as follows :

- For a general N-relay M-slot SAF scheme, we establish a new upper-bound :
$$d^*(r) = (1-r)^+ + N\left(1 - \frac{M}{M-1}r\right)^+, \tag{2}$$
from which we conclude that it is impossible to achieve the MISO bound with a finite length. This bound is however tending to the MISO bound when M goes to infinity. Then,

we argue that the suboptimality of the N-relay NAF scheme is due to the fact that only half of the source signal is *protected* by the relays.

- To approach the upper-bound (2), we propose a naive SAF scheme. The basic idea is to let as many slots as possible (*i.e.*, $M-1$) be forwarded by the relays. By introducing two relay scheduling strategies, we show that the naive SAF achieves the DMT upper-bound for arbitrary (N, M) when all relays are isolated from each other, *i.e.*, there is no physical link between the relays. For $M = 2$ and an arbitrary N, the proposed scheme corresponds to the single-relay NAF scheme combined with the relay selection scheme [7] and the DMT upper-bound is achieved without the relay isolation assumption.
- In particular, we show explicitly that the two-relay three-slot naive SAF scheme dominates the two-relay NAF scheme for multiplexing gains $r \leq 2/3$. It is therefore the best known two-relay amplify-and-forward scheme.

In this paper, we use boldface lower case letters $\boldsymbol{v}$ to denote vectors, boldface capital letters $\boldsymbol{M}$ to denote matrices. $\mathcal{CN}$ represents the complex Gaussian random variable. $[\cdot]^{\mathsf{T}}, [\cdot]^{\dagger}$ respectively denote the matrix transposition and conjugated transposition operations. $\|\cdot\|$ is the vector norm and $\|\cdot\|_{\mathrm{F}}$ is the Frobenius matrix norm. $(x)^{+}$ means $\max(0, x)$. The dot equal operator $\doteq$ denotes asymptotic equality in the high SNR regime, *i.e.*,

$$p_1 \doteq p_2 \quad \text{means} \quad \lim_{\mathsf{SNR}\to\infty} \frac{\log p_1}{\log \mathsf{SNR}} = \lim_{\mathsf{SNR}\to\infty} \frac{\log p_2}{\log \mathsf{SNR}},$$

and $\dot{\leq}$, $\dot{\geq}$ are similarly defined.

The rest of the paper is organized as follows. Section II introduces the system model and the class of SAF schemes. In Section III, we establish an upper-bound on the DMT of any SAF schemes, using a genie-aided model. Then, Section IV proposes a naive SAF scheme that achieves the previously provided DMT upper-bound with certain conditions, when using two relay scheduling schemes. To show the performance of the proposed scheme, numerical results with the naive SAF scheme are presented in Section V, compared to the NAF scheme and the non-cooperative scheme. Finally, we provide some concluding remarks in Section VI. For continuity of demonstration, most proofs are left in the Appendix.

II. System Model

The considered system model consists of one source s, one destination d and N relays (cooperative terminals) $\mathsf{r}_1, \ldots, \mathsf{r}_N$. The physical links between terminals are slowly faded and are modeled as independent quasi-static Rayleigh channels, *i.e.*, the channel gains do not change during the transmission of a cooperation frame, which is defined according to different schemes (protocols). The gain of the channel connecting s and d is denoted by g_0. Similarly, g_i and h_i respectively denote the channel gains between r_i and d and the ones between s and r_i. γ_{ij} is used to denote the channel gain between r_i and r_j. Channel quality between terminals is parameterized by the variance of the channel gains.

A. Slotted Amplify-and-Forward

In the paper, we study half-duplex slotted amplify-and-forward (SAF) cooperative schemes. For an N-relay M-slot scheme, the cooperation frame, composed of M slots of l symbols, is of length $M\,l$. During any slot i, $i = 1, \dots, M$, the source s transmits a sub-frame of l symbols, denoted by a vector $\boldsymbol{x}_i \in \mathbb{C}^l$ and the relay r_j, $j = 1, \dots, N$, can transmit $\boldsymbol{x}_{r_j,i} \in \mathbb{C}^l$, a linear combination of the vectors it received in previous slots. Under the half-duplex constraint, a relay does not receive while transmitting. For example, the NAF scheme [6] is an N-relay $(2N)$-slot scheme and the non-orthogonal relay selection scheme [7] is an N-relay two-slot scheme.

Obviously, the transmission of a cooperation frame with any SAF scheme is equivalent to l channel uses of the following vector (MIMO) channel

$$\boldsymbol{y} = \sqrt{\mathsf{SNR}}\,\boldsymbol{H}\boldsymbol{x} + \boldsymbol{z} \tag{3}$$

where $\boldsymbol{x}$ is the transmitted signal, $\boldsymbol{z} \sim \mathcal{CN}(0, \boldsymbol{\Sigma}_{\boldsymbol{z}})$ is the equivalent additive colored noise with covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{z}}$ and $\boldsymbol{H}$ is an $M \times M$ lower-triangular matrix representing the equivalent "space-time" channel between the source and the destination. Moreover, we have $H_{ii} = c_i\, g_0$ with c_i being a constant related to the transmission power.

B. Diversity-Multiplexing Tradeoff and Achievability

Let us recall the definition of the multiplexing and diversity gains.

Definition 1 (Multiplexing and diversity gain [9]): A coding scheme $\{\mathcal{C}(\mathsf{SNR})\}$ is said to achieve *multiplexing gain* r and *diversity gain* d if

$$\lim_{\mathsf{SNR}\to\infty} \frac{R(\mathsf{SNR})}{\log \mathsf{SNR}} = r \quad \text{and} \quad \lim_{\mathsf{SNR}\to\infty} \frac{\log P_{\mathrm{e}}(\mathsf{SNR})}{\log \mathsf{SNR}} = -d$$

where $R(\mathsf{SNR})$ is the data rate measured by bits per channel use (PCU) and $P_{\mathrm{e}}(\mathsf{SNR})$ is the average error probability using the maximum likelihood (ML) decoder.

Theorem 1: The DMT of any SAF scheme with equivalent channel model (3) is

$$d(r) = d_{\boldsymbol{H}}(Mr), \tag{4}$$

with $d_{\boldsymbol{H}}$ being the DMT of the linear channel (3). Furthermore, by vectorizing a full rate $M \times M$ space-time code with non-vanishing determinant (NVD), we get a code that achieves the tradeoff $d(r)$ for the SAF scheme.

Proof: The equality (4) is obvious, since M is the normalization factor of the channel use. The achievability is immediate from the results in [13], [14], stating that the DMT of a fading channel with any fading statistics can be achieved by a full rate NVD code. ■

C. Properties of the SAF Schemes

Theorem 1 implies that for any SAF scheme, the optimal code construction is available, using the NVD codes design (see, for example, [12], [15]) and the code length is at most[1] M^2. Since the optimal code construction is independent of the fading statistics of the channel, the only information that the source needs is the number of slots M. In practice, M can be decided by the destination, based on the channel coherence time, decoding complexity, etc. The relaying strategies are between the destination and the relays and can be completely ignored by the source. When no relay is helping, the equivalent channel can be the identity matrix (by setting c_i's identical). In this case, even if the source is not aware of the non-relay situation, the destination can decode the signal in linear complexity when "perfect" codes are used, since they are a rotated version of a vector of symbols from the original constellation ($\mathbb{Z}^{Ml}$) [12], [15]. All these properties make SAF schemes very flexible and suitable for wireless networks, especially for *ad hoc* networks where the network topology changes frequently.

III. Upper-bound of the SAF Schemes

The following theorem states the best DMT that we can have with SAF schemes.

Theorem 2: The DMT of any N-relay M-slot half-duplex SAF scheme is upper-bounded by

$$d^*(r) = (1-r)^+ + N\left(1 - \frac{M}{M-1}r\right)^+, \tag{5}$$

[1]In some particular cases, the code length can be shorter. For example, the NAF scheme has a block-diagonal equivalent channel. As shown in [14], we can have an optimal code of length $2M$.

for any $M > 1$.

In this theorem, we exclude the case $M = 1$ for the obvious reason that the single-slot SAF scheme corresponds to the non-cooperative case. For $N = 1, M = 2$, this bound is achieved by the NAF scheme, as shown in [6]. In fact, we can easily show that this bound is achievable for arbitrary N with $M = 2$ slots by combining the relay-selection scheme [7] with the single-relay NAF scheme. The rest of this section is dedicated to the proof of this theorem, by introducing a genie-aided model.

A. The Genie-Aided Model

To get an upper-bound of the DMT, we consider the following genie-aided SAF model. We assume that the source-relay links are noiseless broadcast channels, which means that before the transmission of the i th slot, the relays know exactly the coded signal $\boldsymbol{x}_j$ for any $j < i$. However, the relays are not allowed to decode the message embedded in the signal, according to the AF constraint. The half-duplex constraint is also removed, *i.e.*, in the i th slot, the relays can transmit any linear combinations of the vectors $\boldsymbol{x}_j$ for $j < i$. With the above assumptions, the equivalent noise $\boldsymbol{z}$ in (3) is spatially white and the equivalent channel matrix $\boldsymbol{H}$ is lower-triangular with off-diagonal components being any linear functions of the channel gains $g_0, \ldots, g_N$.

B. Upper-bound on the DMT

Lemma 1: For the genie-aided model, let us define $|g_{\max}|^2 \triangleq \max_{i=0\ldots N} |g_i|^2$, then we have

$$\det\left(\mathbf{I} + \mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^\dagger\right) \dot{\leq} \left(1 + \mathsf{SNR}\,|g_0|^2\right)^M + \left(1 + \mathsf{SNR}\,|g_{\max}|^2\right)^{M-1}. \tag{6}$$

Proof: See Appendix B. ■

Now, define $\boldsymbol{\alpha} \triangleq [\alpha_{g_0} \ldots \alpha_{g_N}]$, where α_{g_i} is such that $|g_i|^2 \doteq \mathsf{SNR}^{-\alpha_{g_i}}$. By applying Lemma 3 on the right hand side (RHS) of (6), we get an upper-bound on the DMT

$$\bar{d}_{\boldsymbol{H}}(r) = \inf_{\mathcal{O}(\boldsymbol{\alpha},r)} \sum_{i=0}^{N} \alpha_{g_i}$$

with

$$\mathcal{O}(\boldsymbol{\alpha}, r) = \left\{ \begin{array}{c} M(1-\alpha_{g_0})^+ < r; \\ (M-1)(1-\alpha_{g_i})^+ < r, \quad \text{for } i = 1, \ldots, N \end{array} \right\}.$$

Due to the symmetry of α_{g_i} for $i = 1, \ldots, N$, we can solve the linear programming problem by adding the constraint $\alpha_{g_1} = \ldots = \alpha_{g_N}$. Applying Theorem 1, we can get the closed-form DMT (5).

C. Discussion

From the upper-bound (5), two observations can be made : 1) SAF schemes can never achieve the MISO bound with a finite number of slots, even without the half-duplex constraint, and 2) SAF schemes can never beat the non-cooperative scheme for $r > \frac{M-1}{M}$. In fact, the first observation can be seen as a necessary condition of the second one, and it applies to all AF schemes.

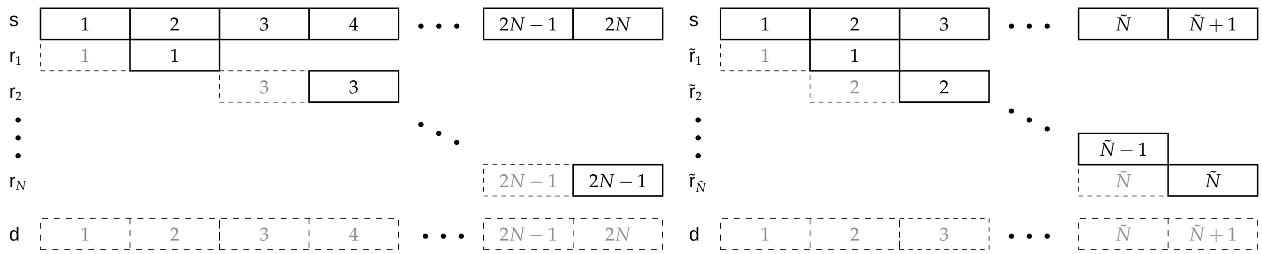
Intuitively, even in the genie-aided model, the last slot is not protected by any relay. This is due to the causality of the relay channel, not to the half-duplex constraint. Therefore, at most $M-1$ slots out of M slots can be protected, which explains the suboptimality for $r > \frac{M-1}{M}$. In the same way, since only N slots out of $2N$ slots are protected by one relay in the NAF scheme, the NAF scheme is not better than the non-cooperative scheme for $r > 0.5$.

As stated in [6], an important guideline for cooperative diversity is to let the source keep transmitting all the time so that the maximum multiplexing gain is achieved. Here, we provide another guideline : *let most of the source signal be protected by extra paths*. Based on this guideline, we propose, in next section, a naive SAF scheme and we show that this scheme actually achieves the upper-bound (5) for some particular cases.

IV. The Naive SAF Scheme

As previously stated, the NAF scheme is optimal in the single-relay case, due to the half-duplex constraint. We consider the multiple-relay case in the rest of the paper.

Let us consider the following naive SAF scheme. First of all, the source must transmit during all the M slots. Then, from the beginning of the second slot, in each slot, there is one and only one relay forwarding a scaled version of what it received in the previous slot. In such a way, $M-1$ slots out of M slots of the source signal are forwarded by at least one relay. Here, we can see that this is only possible when we have more than one relay, where different relays can alternatively help the source. Thus, we have $\tilde{N} \triangleq M-1$ effective relays $\tilde{\mathsf{r}}_1, \ldots, \tilde{\mathsf{r}}_{\tilde{N}}$ during the transmission of a specific source. The mapping between the real relays and the effective relays



(a) NAF

(b) Naive SAF

Fig. 2. Frame structure and relaying procedure of NAF and naive SAF, solid box for transmitted signal and dashed box for received signal.

is accomplished by relays scheduling that will be discussed later on. The frame structure and the relaying procedure are illustrated in Fig. 2, compared to the NAF scheme.

A. Equivalent Linear Fading Channel

In SAF schemes, there's no difference in data processing for different symbols within the same slot. Thus, we can consider one symbol from a slot, without loss of generality. With the previous description of the naive SAF scheme, we have the following signal model :

$$\begin{cases} y_{d,i} = \sqrt{\pi_i \,\mathsf{SNR}}\, g_0\, x_i + \sqrt{\bar{\pi}_i \,\mathsf{SNR}}\, \tilde{g}_{i-1}\, \tilde{b}_{i-1}\, y_{r,i-1} + u_i \\ y_{r,i} = \sqrt{\pi_i \,\mathsf{SNR}}\, \tilde{h}_i\, x_i + \sqrt{\bar{\pi}_i \,\mathsf{SNR}}\, \tilde{\gamma}_{i-1,i}\, \tilde{b}_{i-1}\, y_{r,i-1} + v_i \end{cases} \tag{7}$$

where x_i is the transmitted symbol from the source in the i th slot; $y_{r,i}$ and $y_{d,i}$ are the received symbols at the i th effective relay and at the destination, respectively, in the i th slot; u_i's and v_i's are independent AWGN with unit variance; $\tilde{h}_i$ and $\tilde{g}_i$, $i = 1, \ldots, \tilde{N}$, are the channel gains from the source to the ith effective relay and from the ith effective relay to the destination, respectively; $\tilde{\gamma}_{i-1,i}$ is the channel gain between the $i-1$th and the ith effective relay; $\tilde{b}_i$ is the processing gain at the ith effective relay subject to the power constraint $\mathbb{E}_{\boldsymbol{u},\boldsymbol{v}} \left|\tilde{b}_i\, y_{r,i}\right|^2 \leq 1$. The power allocation factors π_i, $\bar{\pi}_i$, $i = 1, \ldots, M$, are independent of the channel coefficients and satisfy $\sum_{i=1}^{M} (\pi_i + \bar{\pi}_i) = M$. Finally, by definition, we have $\bar{\pi}_1 = 0$ and $\tilde{b}_0 = 0$.

By carefully treating the signal part and the noise part, we can express the signal model of M slots in the vectorized form

$$\boldsymbol{y}_d = \sqrt{\mathsf{SNR}}\, \boldsymbol{H} \boldsymbol{x} + \boldsymbol{z}.$$

The equivalent channel matrix and the noise are in the following form :

$$\boldsymbol{H} = \operatorname{diag}\left(\sqrt{\pi_i}\, g_0\right) + \boldsymbol{T} \operatorname{diag}\left(\sqrt{\pi_i}\, h_i\right) \tag{8}$$

$$\boldsymbol{z} = \boldsymbol{u} + \boldsymbol{T}\, \boldsymbol{v}, \tag{9}$$

where[2] $\boldsymbol{T} \triangleq \boldsymbol{U_c}(\mathbf{I} - \boldsymbol{U_d})^{-1}$, and $\boldsymbol{U_c}$, $\boldsymbol{U_d}$ are $M \times M$ matrices defined as

$$\boldsymbol{U_c} \triangleq \begin{bmatrix} \mathbf{0}^{\mathsf{T}} & 0 \\ \operatorname{diag}(c_i) & \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{U_d} \triangleq \begin{bmatrix} \mathbf{0}^{\mathsf{T}} & 0 \\ \operatorname{diag}(d_i) & \mathbf{0} \end{bmatrix}$$

with $c_i \triangleq \sqrt{\pi_i \mathsf{SNR}}\, \tilde{g}_i\, \tilde{b}_i$ and $d_i \triangleq \sqrt{\pi_i \mathsf{SNR}}\, \tilde{\gamma}_{i,i+1}\, \tilde{b}_i$ for $i = 1, \ldots, \tilde{N}$. Both $\boldsymbol{U_c}$ and $\boldsymbol{U_d}$ are forward-shift like matrices. From (9), the covariance matrix of the noise is $\boldsymbol{\Sigma_z} = \mathbf{I} + \boldsymbol{T}\boldsymbol{T}^{\dagger}$. We can show that the largest and smallest eigenvalues of $\boldsymbol{\Sigma_z}$ satisfy $\lambda_{\max}(\boldsymbol{\Sigma_z}) \doteq \lambda_{\min}(\boldsymbol{\Sigma_z}) \doteq \mathsf{SNR}^0$, which implies that the DMT of the proposed scheme depends only on $\boldsymbol{H}$ and not on $\boldsymbol{\Sigma_z}$. Unfortunately, the complex form of the equivalent channel matrix $\boldsymbol{H}$ (8) prevents us from obtaining the closed-form DMT in the general case. Nevertheless, in some particular cases, we can have the closed-form DMT and furthermore, it achieves the upper-bound (5).

B. *Isolated Relays*

Let us consider a special scenario where the relays have weak interconnections. In this case, we can assume that the relays are isolated from each other. Then, the DMT of the naive SAF scheme can be obtained explicitly.

Proposition 1: When the relays are isolated from each other, *i.e.*, $\tilde{\gamma}_{i,i+1} = 0,\ \forall i$, the DMT (5) is achievable with the naive SAF scheme.

We prove this proposition in the following paragraphs. With the assumption of relay isolation, we have $\boldsymbol{T} = \boldsymbol{U_c}$ and $\boldsymbol{H}$ is therefore a bidiagonal matrix. The special form of $\boldsymbol{H}$ allows us get the following lemma that is crucial to the proof.

[2] We can get the signal part recursively by the matrix $\boldsymbol{U_d}$, where we use the identity $\mathbf{I} + \boldsymbol{U_d} + \boldsymbol{U_d}^2 + \cdots = (\mathbf{I} - \boldsymbol{U_d})^{-1}$.

Lemma 2:

$$\det\left(\mathbf{I}+\mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^{\dagger}\right)\,\dot{\geq}\,\left(1+\mathsf{SNR}\left|g_0\right|^2\right)^M + \prod_{i=1}^{\tilde{N}}\left(1+\mathsf{SNR}\left|\tilde{g}_i\tilde{h}_i\right|^2\right) \tag{10}$$

Proof: The matrix $\mathbf{I}+\mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^{\dagger}$ being tridiagonal, we can use the recursive determinant formula to get the RHS of (10) in the high SNR regime. See Appendix C for details. ■

Inspired by the RHS of (10), we propose the following two scheduling strategies :

- **Dump scheduling**: For $\tilde{N}=kN$ with k being any integer, the relays help the source in a round-robin manner, *i.e.*, $\tilde{\mathsf{r}}_i=\mathsf{r}_{(i-1)_N+1}$. For $\tilde{N}=kN+m$ with $m\in[1,N-1]$, we first order the relays $\mathsf{r}_1,\ldots,\mathsf{r}_N$ in such a way that

$$\min\{|g_1\,h_1|\,,\ldots,|g_m\,h_m|\}\geq\max\{|g_{m+1}\,h_{m+1}|\,,\ldots,|g_N\,h_N|\}.$$

  Then, we apply the round-robin scheduling.
- **Smart scheduling**: First, select the two "best" relays in the sense that they have largest $|g_i\,h_i|$. Then, we apply the dump scheduling on these two relays, as if we were in the two-relay M-slot case.

These two scheduling strategies maximize *statistically* the RHS of (10) in the high SNR regime, so that upper-bound (5) is achieved. The detailed proof is provided in Appendix D. Even though both schemes achieve DMT (5) under the relay isolation assumption, the smart scheme outperforms the dump scheme in a general case, without relay isolation. The basic idea of the smart scheduling is to avoid using the "bad" relays, where the noise level is higher than the other relays in average. Therefore, in M slots, noise amplification is less significant with the smart scheduling than with the dump scheduling. The impact is investigated in the next section, with the simulation results.

As an example, Fig. 3 shows the DMT of different cooperative schemes for a three-relay channel, with relay isolation assumption. For $M=2$, the DMT of the proposed scheme coincides with that of the NAF scheme. With increasing M, the proposed scheme is approaching the MISO bound, which makes it asymptotically optimal.

C. Two-Slot with Arbitrary Number of Relays

Note that for the particular cases $M=2$, i.e., *$k=0$ and $m=1$, the above analysis is valid whether the relays are isolated from each other or not.* This is because a 2×2 triangular matrix

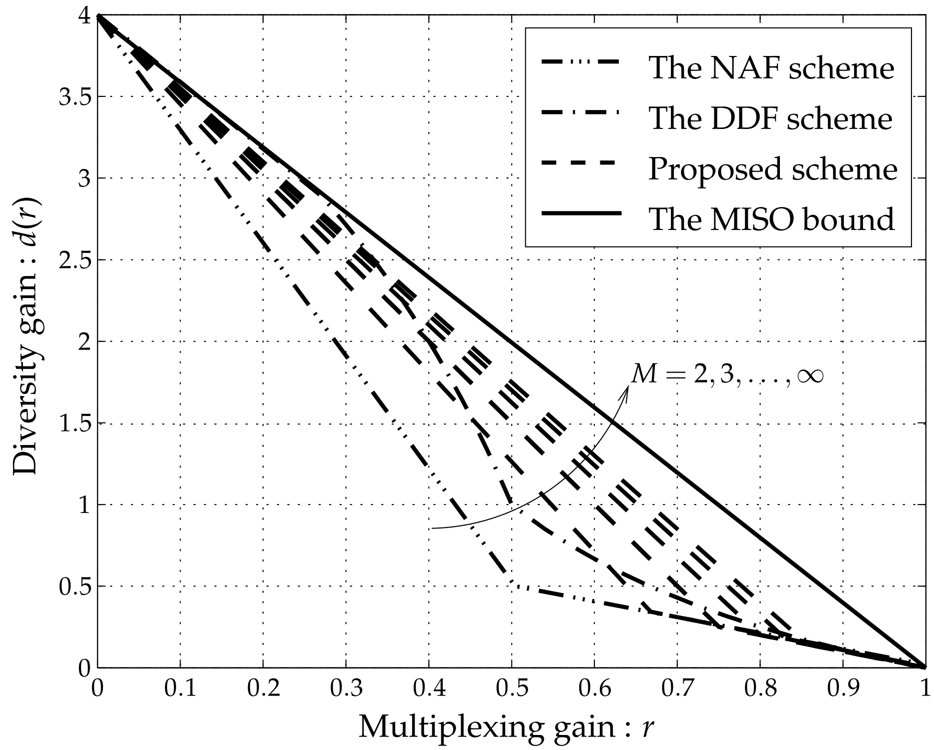


Fig. 3. D-M tradeoff of different three-relay schemes with isolated relays.

is also a bidiagonal matrix. Therefore, the achievability of (5) for $M = 2$ and arbitrary N is proved. And the proposed scheme is actually the single-relay NAF scheme combined with the relay selection scheme [7].

D. Two-Relay and Three-Slot

Proposition 2: The two-relay three-slot naive SAF scheme achieves the DMTs of Fig. 4, where the relay ordering is such that $|h_2|^2 \geq |h_1|^2$, *i.e.*, the relay with worse source-relay link transmits first.

Proof: The DMTs are obtained with the same method as previously, by expressing explicitly the determinant $\det\left(\mathbf{I} + \mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^{\dagger}\right)$. See Appendix E for details. ■

As shown in Appendix E, even though we have the closed-form determinant expression, we can only have a lower-bound on the DMT because of the complex determinant form. Unfortunately, the lower-bound we get does not coincide with the upper-bound (5) for $r < 0.5$. By adding a relay ordering procedure ($|h_2|^2 \geq |h_1|^2$), we finally get a lower-bound equal to the upper-bound.

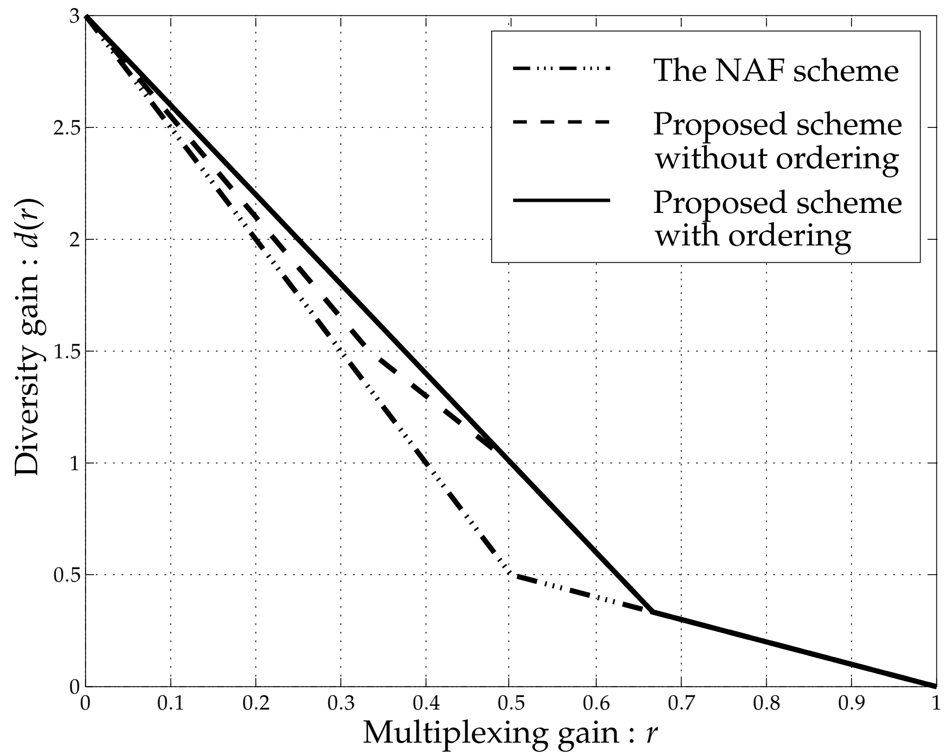


Fig. 4. Diversity-multiplexing tradeoff of the two-relay schemes.

However, this does not necessarily mean that the relay ordering improves the performance, as we will show in the next section.

As shown in Fig. 4, the naive SAF scheme (with or without relay ordering) outperforms the two-relay NAF scheme. Since with the three-slot structure we protect $\frac{2}{3}$ of the source signal, we can beat the non-cooperative scheme for $0 \leq r \leq \frac{2}{3}$. It is therefore the best AF scheme known for the two-relay case. To further improve the DMT, we should increase the number of slots.

E. Practical Considerations

To implement the naive SAF schemes, the relay ordering is essential for the smart scheduling and the $\tilde{N} \neq kN$ case of the dump scheduling. If we have the reciprocity for the forward and the backward relay-destination links, *i.e.*, the channel gains are the same (g_i) for the forward and backward links, an intelligent way to implement the relay ordering is similar to the RTS/CTS scheme proposed in [7]. First, the relays measure the source-relay channel quality $|h_i|$ by the reception of the *RTS* (Ready-to-Send) frame from the source. Then, the destination broadcasts a *relay-probing* frame, from which the relays can estimate the relay-destination channel $|g_i|$. Each relay calculates the product gain $|g_i h_i|$ and reacts by sending an *availability* frame after t_i time which is proportional to $|g_i h_i|$. Therefore, the relay with the strongest product gain is identified as relay 1, and so on. Finally, based on the order, the destination decides a scheduling strategy and broadcasts the parameters (*e.g.*, the relay ordering for the relays and number of slots M for the source, etc...) in the *CTS* (Clear-to-Send) frame. When there is no reciprocity for the relay-destination links, we modify the last three steps as follows. Each relay quantizes the source-relay gain and sends it in the *availability* frame to the destination using its own signature. Then, the destination can estimate the relay-destination links quality $|g_i|$ and also gets the estimates $|h_i|$ by decoding the signal. Finally, the destination decides the order based on the product gains and broadcasts the *CTS* frame.

Since we only consider slow fading channels, the ordering would not be so frequent and the signaling overhead is negligible in both cases (the overhead issue is mentioned in [7]). In the worst case where the above signaling is impossible, a cooperation order for the relays should be predefined and we apply the dump scheduling with a slot number M such that $M - 1 = kN$. In this case, the same DMT is achieved.

V. Numerical Results

In this section, we investigate the numerical results obtained by Monte-Carlo simulations. By default, we consider a symmetric network, where all the channel coefficients are i.i.d. Rayleigh distributed with unit variance. There is therefore no *a priori* advantage of the source-relay links over the source-destination link. The power allocation factors are $\pi_i = \bar{\pi}_i = 0.5$ for $i = 2, \ldots, M$ and $\pi_1 = 1$. Information rate is measured in bits per channel use (BPCU). We compare the proposed naive SAF scheme to the NAF scheme and the non-cooperative scheme in both small network scenarios (2 relays) and large network scenarios (12 relays).

A. Two-Relay Scenario

1) Three-Slot Case: Fig. 5 shows the performance of the proposed two-relay three-slot scheme for different spectral efficiencies. Note that with a low spectral efficiency (2 BPCU), the proposed schemes have almost the same performance as the NAF scheme. However, when increasing the spectral efficiency, the gain of our schemes compared to the NAF strategy increases. For

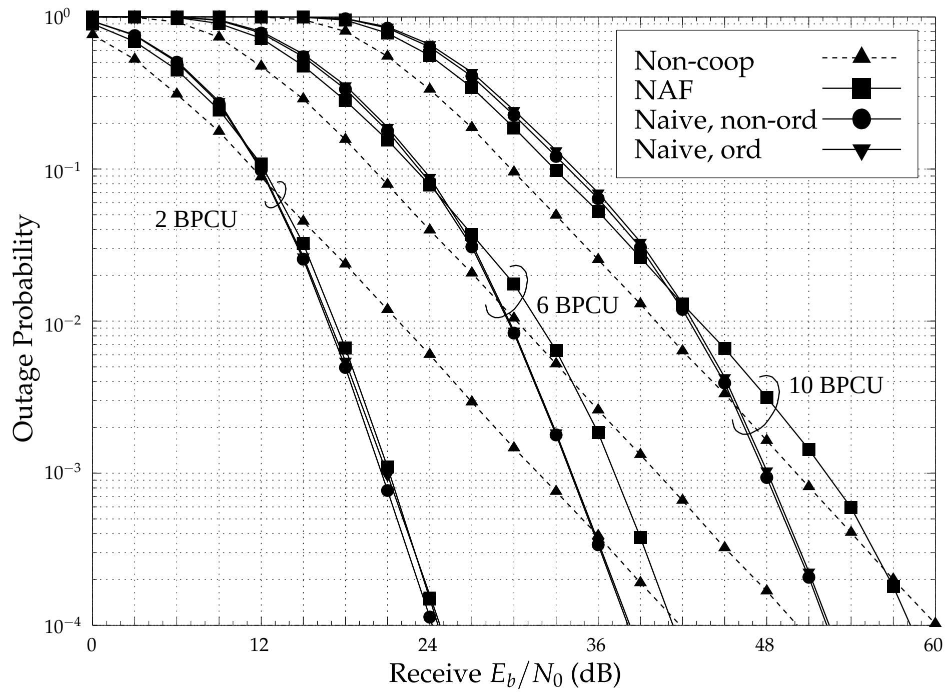
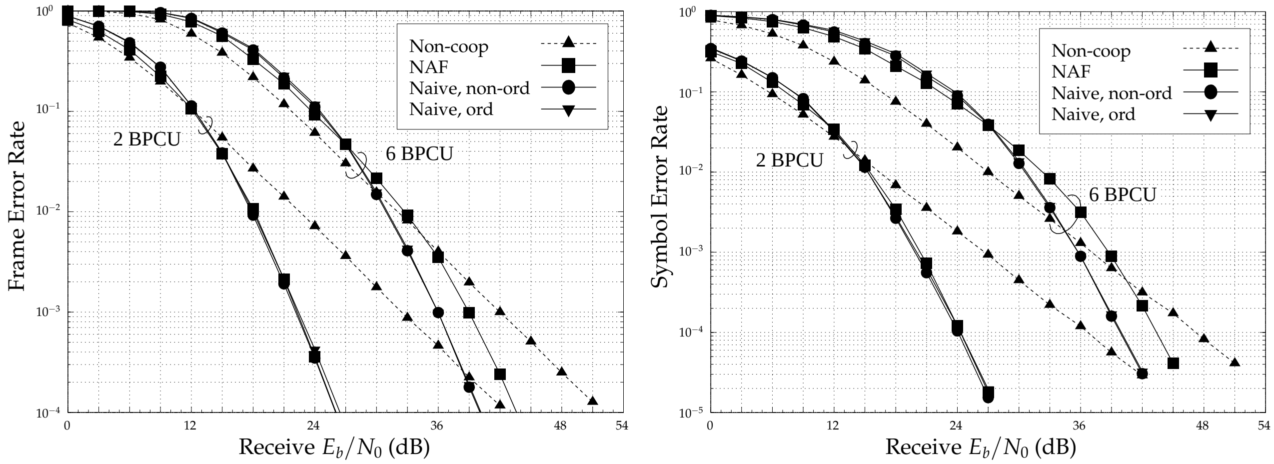


Fig. 5. Outage probabilities for the non-cooperative, NAF and naive SAF scheme with three slots. Two-relay symmetric network. Considered information rates: 2, 6 and 10 BPCU.

10 BPCU, the NAF scheme barely beats the non-cooperative scheme. Also note that in all cases, the scheme with relay ordering proposed in Sec. IV-D is not better than the one without relay ordering. Based on that observation, we conjecture that we can achieve the DMT (2) even without relay ordering in the two-relay three-slot case.

Then, we consider the error rate performance of NVD codes (*i.e.*, achieving the DMT) under ML decoding. For the two-relay NAF scheme, we use the optimal code $\mathcal{C}_{2,1}$ (QAM) proposed in [14]. For the naive SAF scheme, we use the perfect 3×3 code construction proposed in [15], based on QAM constellations, the best known 3×3 real rotation [16] and the "non-norm" element $\gamma = \frac{1+2i}{2+i}$. The vectorized code (frame) lengths are 8 and 9 QAM symbols for the NAF and the naive SAF, respectively. 4-QAM and 64-QAM uncoded constellations are used, corresponding to the 2 BPCU and 6 BPCU counterpart in the outage performance. The frame error rate (FER) is shown in Fig. 6(a). It is surprising to see such a similarity between code performance and outage performance: for a given probability (error or outage respectively), all SNR differences between the compared schemes are almost the same. We have a power gain of



(a) Frame Error Rate

(b) Symbol Error Rate

Fig. 6. Error rate performance: naive SAF *vs.* NAF scheme. Two-relay symmetric network, perfect 3×3 code for the three-slot SAF scheme and $\mathcal{C}_{2,1}$ for the NAF scheme for the NAF. 4- and 64-QAM for 2 and 6 BPCU, respectively.

more than 3 dB for FER lower than 10^{-3} with 64-QAM. For fairness of comparison between different frame length, we also show the symbol error rate performance in Fig. 6(b).

As stated in theorem 1, we can always construct optimal codes for a given SAF scheme. To focus on the cooperative scheme itself, we only consider the outage probability hereafter.

2) Impact of the Number of Slots: Fig. 7 shows the outage performance with different numbers of slots. For 2 BPCU, the difference is minor (within 1 dB). However, for 6 BPCU, the power gain compared to the three-slot scheme increases to 2 and 3 dB for 5 slots and 13 slots, respectively. The increasing SNR gain shows the superiority of the schemes with a larger number of slots in terms of DMT, even without the relay isolation assumption.

3) Inter-Relay Geometric Gain: In Fig. 8, we show the impact of the inter-relay geometric gain (defined as $\mathbb{E}\left|\gamma_{ij}\right|^2/\mathbb{E}\left|h_j\right|^2$) on the outage performance. In this scenario, all paths have the same average channel gain (0 dB), except for the inter-relay channels whose channel gains vary form -20 dB (bad interconnection) to 20 dB (good interconnection). The y-axis represents the power gain to the non-cooperative scheme with 6 BPCU and outage probability of 10^{-3}. The x-axis represents the inter-relay geometric gain. As shown in Fig. 8, the NAF scheme is independent of the geometric gain since there is no inter-relay communication at all in the NAF

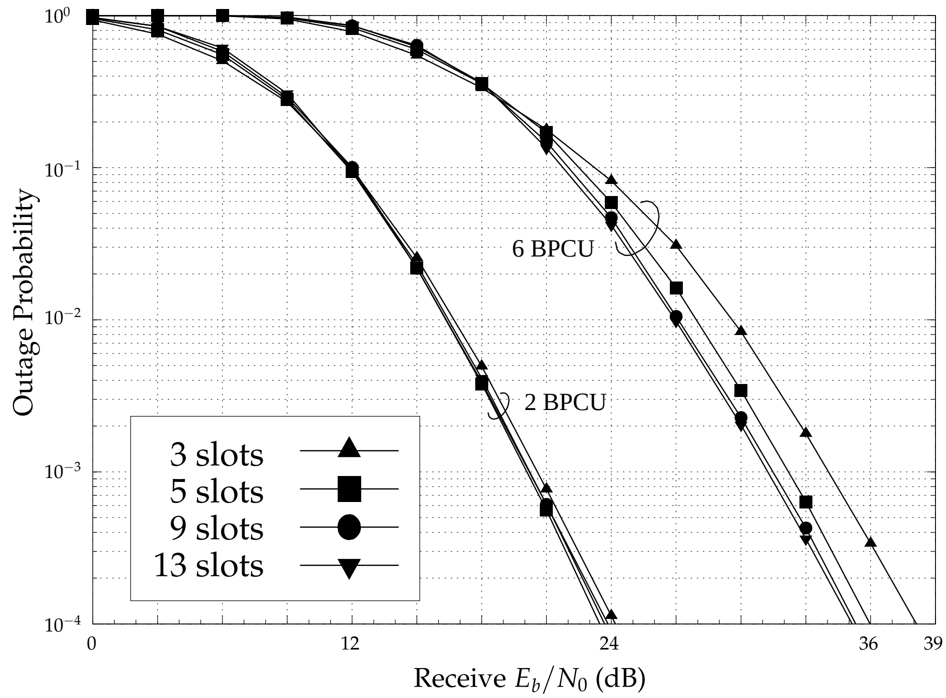


Fig. 7. Outage probability of the naive SAF scheme with 3, 5, 9 and 13 slots. Two-relay symmetric network.

scheme. In the bad interconnection regime (< 0 dB), the naive SAF scheme is not sensitive to the geometric gain and we always have a better performance by increasing the slot number. However, in the good interconnection regime (> 0 dB), the performance degrades dramatically with the increase of inter-relay gain and the increase of the number of slots. Intuitively, the task of the i th effective relay is to protect the source signal $\boldsymbol{x}_i$, transmitted in the i th slot. A strong interconnection between the $(i-1)$ th relay and the i th relay makes $\boldsymbol{x}_i$ drowned in the combined signal of $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{i-1}$ from the $(i-1)$ th relay.

B. Large Network : Dumb vs. *Smart Scheduling*

Now, we consider a large symmetric network with 12 available relays. We compare the proposed scheme to the NAF scheme. To ensure fairness, the considered NAF is combined with the relay selection scheme, *i.e.*, the source is only helped by the best relay (with largest $|g_i\, h_i|$). For the naive SAF scheme, both the dumb and the smart schedulings are considered. Obviously, with 3 slots, the dumb scheduling is the same as the smart scheduling. As shown in Fig. 9, the power gain increases with spectral efficiency, showing the superiority of our scheme

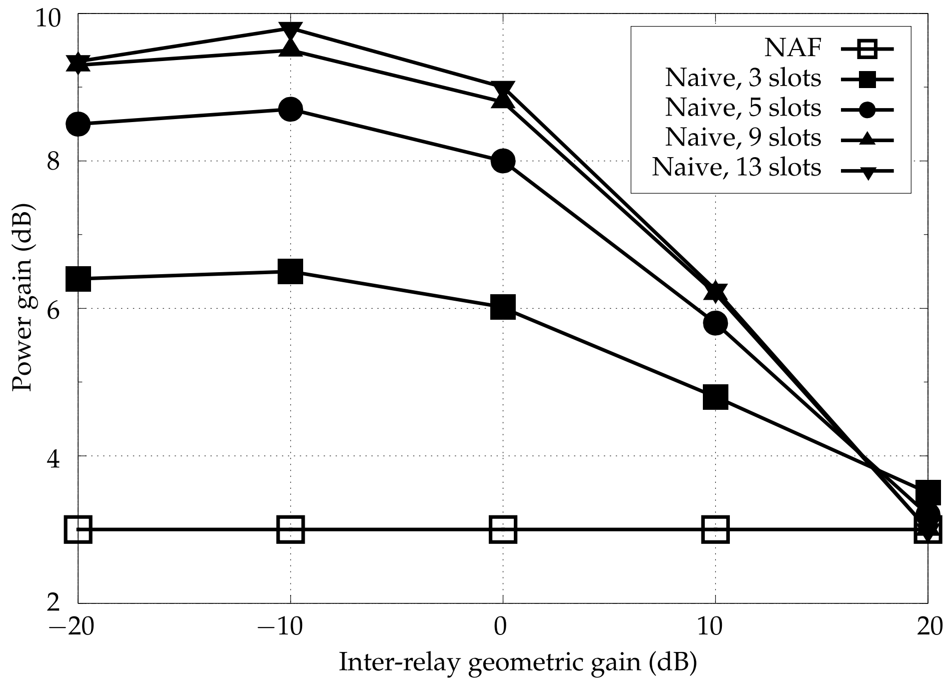


Fig. 8. Power gain to the non-cooperative scheme : impact of the inter-relay geometric gain. Two-relay network. Target information rate : 6 BPCU. Target outage probability : 10^{-3}.

in terms of DMT. The increase is more significant with a larger slot number. With the same slot number, the curve of the dump scheduling is parallel to that of the smart scheduling, meaning the same DMT for the same slot number. The power gain is up to 8 and 16 dB for 6 BPCU and 10 BPCU, respectively. For 2 BPCU, the 13-slot dump scheduling scheme is worse than the NAF, since the noise amplification is significant. As we see, the smart scheduling is always better than the dump scheduling. In the considered cases, the 5-slot smart scheduling outperforms the 13-slot dump scheduling. Since the optimal codes are respectively of length 5^2 and 13^2 for the 5 slot and the 13 slot cases, the use of smart scheduling can dramatically reduce the decoding complexity.

VI. Conclusion and Future Work

In this paper, we considered the class of slotted amplify-and-forward schemes. We first derived, for the SAF schemes, an upper-bound of the DMT which asymptotically (when the framelength grows to infinity) achieves the MISO bound. Then, we proposed and analyzed a naive SAF scheme for which the DMT upper-bound is achieved in some special cases. In particular, the two-

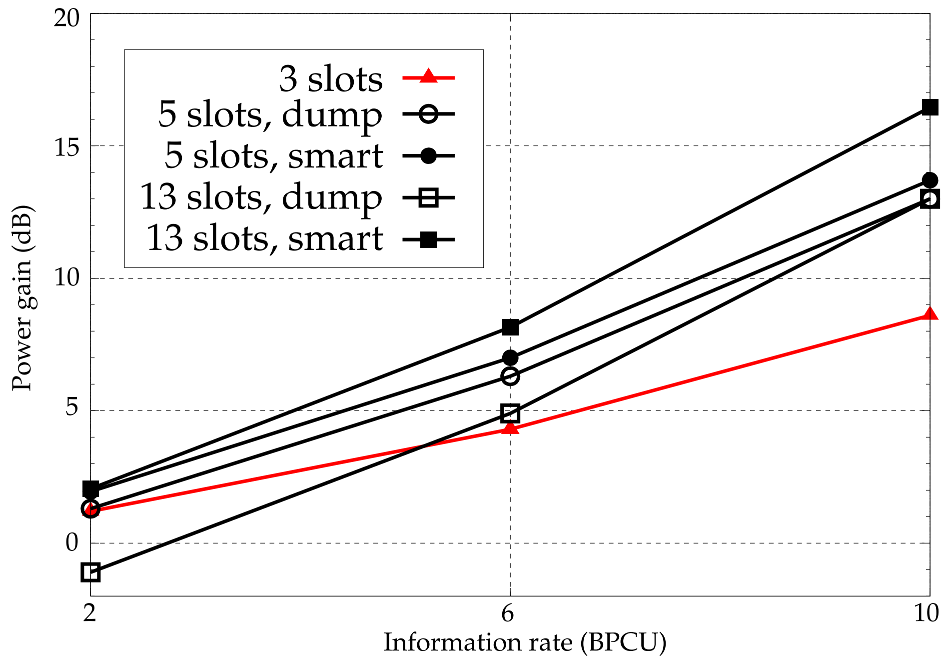


Fig. 9. Power gain to the NAF scheme with selection : dump *vs.* smart scheduling. Symmetric network with 12 relays. Target outage probability : 10^{-3}.

relay three-slot naive SAF is optimal within the $N = 2, M = 3$ class and therefore outperforms all previously proposed two-relay AF schemes.

The superiority of the naive SAF scheme over the previously proposed AF schemes lies in the fact that it exploits the potential diversity gain in the high multiplexing gain regime ($r > 0.5$), whereas all previously proposed AF schemes do not beat the non-cooperative scheme for $r > 0.5$. An important guideline for the design of AF schemes was then proposed : let most of the source signal be protected by extra paths. We also showed that, by using a smart relay scheduling, the complexity of decoding can be dramatically reduced. Numerical results on both the outage and error rate performance reveal a significant gain of our scheme compared to previously proposed AF schemes. Since we can always find optimal codes of finite length for any SAF scheme and the code construction is independent of the number of relays, the proposed scheme is a combination of efficiency and flexibility.

Even though we showed that the naive SAF scheme is asymptotically optimal in some particular cases, the DMT for the general case is unknown. It would also be interesting to find a new SAF scheme, more sophisticated than the naive one in order to improve the statistical properties of the equivalent channel matrix.

APPENDIX

A. Preliminaries

For any linear fading Gaussian channel

$$\boldsymbol{y} = \sqrt{\mathsf{SNR}}\,\boldsymbol{H}\,\boldsymbol{x} + \boldsymbol{z}$$

where $\boldsymbol{z}$ is an AWGN with $\mathbb{E}\{\boldsymbol{z}\boldsymbol{z}^\dagger\} = \mathbf{I}$ and $\boldsymbol{x}$ is subject to the input power constraint $\mathrm{Tr}\left\{\mathbb{E}\left[\boldsymbol{x}\boldsymbol{x}^\dagger\right]\right\} \le 1$, the DMT $d_{\boldsymbol{H}}(r)$ can be found as the exponent of the outage probability in the high SNR regime, *i.e.*,

$$\begin{aligned} P_{\text{out}}(r\log\mathsf{SNR}) &\doteq \text{Prob}\left\{\log\det\left(\mathbf{I} + \mathsf{SNR}\,\boldsymbol{H}\boldsymbol{H}^\dagger\right) \le r\log\mathsf{SNR}\right\} \\ &= \text{Prob}\left\{\det\left(\mathbf{I} + \mathsf{SNR}\,\boldsymbol{H}\boldsymbol{H}^\dagger\right) \le \mathsf{SNR}^r\right\} \\ &\doteq \mathsf{SNR}^{-d_{\boldsymbol{H}}(r)}. \end{aligned} \tag{11}$$

Lemma 3 (Calculation of diversity-multiplexing tradeoff): Consider a linear fading Gaussian channel defined by $\boldsymbol{H}$ for which $\det\left(\mathbf{I} + \mathsf{SNR}\,\boldsymbol{H}\boldsymbol{H}^\dagger\right))$ is a function of $\boldsymbol{\lambda}$, a vector of positive random variables. Then, the DMT $d_{\boldsymbol{H}}(r)$ of this channel can be calculated as

$$d_{\boldsymbol{H}}(r) = \inf_{\mathcal{O}(\boldsymbol{\alpha},r)} \varepsilon(\boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}_i \triangleq -\log v_i / \log\mathsf{SNR}$ is the exponent of v_i, $\mathcal{O}(\boldsymbol{\alpha}, r)$ is the outage event set in terms of $\boldsymbol{\alpha}$ and r in the high SNR regime, and $\varepsilon(\boldsymbol{\alpha})$ is the exponential order of the pdf $p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ of $\boldsymbol{\alpha}$, *i.e.*,

$$p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) \doteq \mathsf{SNR}^{-\varepsilon(\boldsymbol{\alpha})}.$$

Proof: This lemma can be justified by (11) using Laplace's method, as shown in [9]. ■

Lemma 4: Let X be a χ^2-distributed random variable with $2t$ degrees of freedom and Y be a uniformly distributed random variable in an interval including 0. Define $\xi \triangleq -\frac{\log X}{\log\mathsf{SNR}}$ and $\eta \triangleq -\frac{\log|Y|^2}{\log\mathsf{SNR}}$, then we have

$$p_\xi \doteq \begin{cases} \mathsf{SNR}^{-\infty} & \text{for } \xi < 0, \\ \mathsf{SNR}^{-t\xi} & \text{for } \xi \ge 0; \end{cases}$$

and

$$p_\eta \doteq \begin{cases} \mathsf{SNR}^{-\infty} & \text{for } \eta < 0, \\ \mathsf{SNR}^{-\eta/2} & \text{for } \eta \ge 0. \end{cases}$$

B. Proof of Lemma 1

Any $(n+1)\times(n+1)$ lower-triangular matrix, denoted $\boldsymbol{H}_{n+1}$ can be written as

$$\boldsymbol{H}_{n+1} = \begin{bmatrix} \boldsymbol{H}_n & \mathbf{0} \\ \boldsymbol{v}_n^\dagger & g \end{bmatrix}$$

Let us define $D_{n+1} \triangleq \det\left(\mathbf{I} + \mathsf{SNR}\boldsymbol{H}_{n+1}\boldsymbol{H}_{n+1}^\dagger\right)$ and $C \triangleq 1 + \mathsf{SNR}\,|g|^2$. Then, we have

$$\begin{aligned} D_{n+1} &= C\det\left(\mathbf{I} + \frac{\mathsf{SNR}}{C}\boldsymbol{v}_n\boldsymbol{v}_n^\dagger + \mathsf{SNR}\boldsymbol{H}_n^\dagger\boldsymbol{H}_n\right) \\ &\overset{(a)}{\leq} C\left(1 + \mathsf{SNR}\lambda_1 + \frac{\mathsf{SNR}}{C}\|\boldsymbol{v}_n\|^2\right)\prod_{i=2}^{n}(1+\mathsf{SNR}\lambda_i) \\ &= C\,D_n + \mathsf{SNR}\|\boldsymbol{v}_n\|^2\prod_{i=2}^{n}(1+\mathsf{SNR}\lambda_i) \\ &\overset{(b)}{\leq} C\,D_n + \mathsf{SNR}\|\boldsymbol{v}_n\|^2\left(\frac{1}{n-1}\sum_{i=2}^{n}(1+\mathsf{SNR}\lambda_i)\right)^{n-1} \\ &\leq C\,D_n + \left(1+\mathsf{SNR}\|\boldsymbol{v}_n\|^2\right)\left(1+\mathsf{SNR}\,\|\boldsymbol{H}_n\|_{\mathrm{F}}^2\right)^{n-1} \\ &\dot{\leq} C\,D_n + \left(1+\mathsf{SNR}\,|g_{\max}|^2\right)^n \end{aligned}$$

with λ_i the ith smallest eigenvalue of $\boldsymbol{H}_n\boldsymbol{H}_n^\dagger$. The inequality (a) comes from the fact that $\boldsymbol{v}_n\boldsymbol{v}_n^\dagger$ has only one nonzero eigenvalue and that for any nonnegative matrix $\boldsymbol{A}$ and $\boldsymbol{B}$, $\det(\boldsymbol{A}+\boldsymbol{B})$ is maximized when they are simultaneously diagonalizable and have eigenvalues in reverse order. (b) uses the arithmetic-geometric means inequality. And the last asymptotic inequality holds because $\|\boldsymbol{v}_n\| \;\dot{\leq}\; |g_{\max}|^2$ and $\|\boldsymbol{H}\|_{\mathrm{F}}^2 \;\dot{\leq}\; |g_{\max}|^2$. The above result leads directly to (6) in a recursive manner. ■

C. Proof of Lemma 2

For any $(k+1)\times(k+1)$ bidiagonal matrix $\boldsymbol{G}$ with $G_{ii} = x_0$ and $G_{i+1,i} = x_i$, the matrix $\boldsymbol{M}_{k+1} \triangleq \mathbf{I} + \boldsymbol{G}\boldsymbol{G}^\dagger$ is a tridiagonal matrix in the form

$$\begin{bmatrix} 1+|x_0|^2 & x_0x_1^* & \cdots & 0 \\ x_0^*x_1 & 1+|x_0|^2+|x_1|^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & x_0x_k^* \\ 0 & \cdots & x_0^*x_k & 1+|x_0|^2+|x_k|^2 \end{bmatrix}.$$

Define $X_i \triangleq |x_i|^2$ for $i = 0, \ldots, k$, $D_k \triangleq \det(\boldsymbol{M}_k)$ and use the formula for the calculation of the determinant of a tridiagonal matrix [17], we have

$$\begin{aligned} D_{k+1} &= (1 + X_0 + X_k) D_k - X_0 X_k D_{k-1} \\ &= (1 + X_0) D_k + X_k (D_k - X_0 D_{k-1}). \end{aligned} \tag{12}$$

Let us rewrite the last equation as

$$D_{k+1} - X_0 D_k = X_k (D_k - X_0 D_{k-1}) + D_k \tag{13}$$

and define $B_k \triangleq D_k - X_0 D_{k-1}$, from (12) and (13), we get

$$\begin{bmatrix} D_{k+1} \\ B_{k+1} \end{bmatrix} = \begin{bmatrix} 1 + X_0 & X_k \\ 1 & X_k \end{bmatrix} \begin{bmatrix} D_k \\ B_k \end{bmatrix}. \tag{14}$$

First, it is easy to show that $D_2 = X_0^2 + 2X_0 + (X_1 + 1)$ and $B_2 = X_0 + X_1 + 1$. Then, from (14), it is obvious that, as a polynomial of $(X_0, \ldots, X_k)$, D_{k+1} has nonnegative coefficients for any k. Finally, as a polynomial of X_0, D_{k+1}'s coefficients can be found recursively using (12) and we have

$$D_{k+1}(X_0) = X_0^{k+1} + \prod_{i=1}^{k} (1 + X_i) + P(X_0).$$

where $P(X_0) \geq 0$ is a polynomial of X_0 and is always nonnegative. Thus, we have

$$D_{k+1}(X_0) \geq X_0^{k+1} + \prod_{i=1}^{k} (1 + X_i)$$

which can be used to get (10). ■

D. Lower-bound on the DMT with Isolated Relays

1) Dump scheduling: In the $\tilde{N} = k\,N$ case with any integer k, a round-robin scheme is optimal since the $\tilde{N}$ slots are equally protected by all the relays. The RHS of (10) becomes

$$\left(1 + \mathsf{SNR}\,|g_0|^2\right)^M + \prod_{i=1}^{N} \left(1 + \mathsf{SNR}\,|g_i\,h_i|^2\right)^k. \tag{15}$$

We carry out the same calculations as in section III with some modifications. Define $\boldsymbol{\alpha} \triangleq [\alpha_{g_0} \ldots \alpha_{g_N}\;\; \alpha_{h_1} \ldots \alpha_{h_N}]$. By applying Lemma 3 on (15), we have

$$\underline{d}_{\boldsymbol{H}}(r) = \inf_{\mathcal{O}(\boldsymbol{\alpha}, r)} \left(\alpha_{g_0} + \sum_{i=1}^{N} (\alpha_{g_i} + \alpha_{h_i}) \right)$$

with

$$\mathcal{O}(\boldsymbol{\alpha}, r) = \left\{ \begin{array}{c} M(1-\alpha_{g_0})^+ < r; \\ k\sum_{i=1}^N (1-\alpha_{g_i}-\alpha_{h_i})^+ < r \end{array} \right\}.$$

Note that by using the variable changes $\alpha'_{g_i} \triangleq \alpha_{g_i} + \alpha_{h_i}$ for $i = 1, \ldots, N$, we get a linear programming problem with symmetry of $\alpha'_{g_1}, \ldots, \alpha'_{g_N}$. The optimum must satisfy $\alpha'_{g_1} = \ldots = \alpha'_{g_N} = \beta$, and the optimization problem reduces to

$$\underline{d}_{\boldsymbol{H}}(r) = \inf_{\mathcal{O}(\alpha_{g_0}, \beta, r)} (\alpha_{g_0} + N\beta) \tag{16}$$

with

$$\mathcal{O}(\alpha_{g_0}, \beta, r) = \left\{ \begin{array}{c} M(1-\alpha_{g_0})^+ < r; \\ (M-1)\beta < r \end{array} \right\}.$$

Solving this problem, we get exactly (5).

In the $\tilde{N} = kN + m$ case, the RHS of (10) is directly revised as

$$\begin{aligned} &\left(1 + \mathsf{SNR}\,|g_0|^2\right)^M \\ &+ \left(\prod_{n=1}^N \left(1 + \mathsf{SNR}\,|g_n\,h_n|^2\right)^k \right) \prod_{i=1}^m \left(1 + \mathsf{SNR}\,|g_i\,h_i|^2\right). \end{aligned} \tag{17}$$

Then, we have the same optimization problem (16) with different constraints, due to the relay ordering. Using the same variable changes, we have

$$\mathcal{O}(\boldsymbol{\alpha}, r) = \left\{ \begin{array}{c} M(1-\alpha_{g_0})^+ < r; \\ k\sum_{i=1}^N (1-\alpha'_{g_i})^+ + \sum_{i=1}^m (1-\alpha'_{g_i})^+ < r; \\ \max\{\alpha'_{g_1}, \ldots, \alpha'_{g_m}\} \le \min\{\alpha'_{g_{m+1}}, \ldots, \alpha'_{g_N}\} \end{array} \right\}.$$

The second and the third constraints together are equivalent to

$$\left\{ k\sum_{i=1}^N (1-\alpha'_{g_i})^+ + \sum_{i=1}^m (1-\alpha'_{g_{\mathcal{S}(i)}})^+ < r, \quad \forall\, \mathcal{S} \subseteq \{1, \ldots, N\} \text{ and } |\mathcal{S}| = m \right\}, \tag{18}$$

from which we get a symmetric problem for α'_{g_i}, $i = 1, \ldots, N$. We can then prove the same result as the previous case.

2) Smart scheduling: Using the two "best" relays, we can arrive at (18) with $N = 2$. Since our definition of "best" also corresponds to minimum value of α'_{g_i}, it is not difficult to verify that the outage region in this case is included in the region (18). Thus, the DMT is lower-bounded by that of the dump scheduling and the achievability is proved.

■

E. Proof of Proposition 2

Fact 1: Let $\boldsymbol{f} \triangleq \begin{bmatrix} f_1 & f_2 \end{bmatrix}^{\mathsf{T}}$, $\boldsymbol{U} \triangleq \begin{bmatrix} u_{11} & 0 \\ u_{21} & u_{22} \end{bmatrix}$ and $\boldsymbol{H}$ be a 3×3 upper-triangular matrix defined by

$$\boldsymbol{H} \triangleq \begin{bmatrix} \boldsymbol{U} & \boldsymbol{0} \\ \boldsymbol{f}^{\mathsf{T}} & g \end{bmatrix}$$

with g being a scalar. Then, we have

$$\begin{aligned} \det(\mathbf{I} + \mathsf{SNR}\,\boldsymbol{H}\boldsymbol{H}^{\dagger}) = (1 + \mathsf{SNR}\,|g|^2) \det\big(\mathbf{I} + \mathsf{SNR}\,\boldsymbol{U}\boldsymbol{U}^{\dagger}\big) \\ + \mathsf{SNR}\,\|\boldsymbol{f}\|^2 + \mathsf{SNR}^2\,|f_2 u_{11}|^2 \\ + \mathsf{SNR}^2\,|u_{22} f_1 - u_{21} f_2|^2 . \end{aligned} \tag{19}$$

Since non-zero multiplicative constants independent of SNR do not appear in the high SNR regime analysis, from (8), we consider the following matrix

$$\boldsymbol{H} = \begin{bmatrix} g_0 & 0 & 0 \\ g_1\, h_1 & g_0 & 0 \\ g_2\, \gamma_{12}\, h_1 & g_2\, h_2 & g_0 \end{bmatrix}, \tag{20}$$

where the coefficients $\sqrt{\mathsf{SNR}}\, b_1$ and $\sqrt{\mathsf{SNR}}\, b_2$ are neglected ($\mathsf{SNR}\,|b_i|^2 \doteq \mathsf{SNR}^0$). With (19), we can now obtain the outage event set, in terms of the entries of $\boldsymbol{H}$.

In order to apply lemma 3, however, we must get the outage event set in the high SNR regime, in terms of α. To this end, we must rewrite $|u_{22} f_1 - u_{21} f_2|^2$ in (19) in a more convenient form of positive variables. Let us use the notation $V = |v|^2$ for v being any variable. Then, from (19) and (20), we have

$$F_1 \doteq G_2 H_1 \Gamma_{12}; \qquad F_2 \doteq G_2 H_2;$$

$$U_{11} \doteq U_{22} \doteq G_0; \qquad U_{21} \doteq G_1 H_1.$$

Let us rewrite

$$\begin{aligned} |u_{22} f_1 - u_{21} f_2|^2 &= U_{22} F_1 + U_{21} F_2 - 2\sqrt{U_{21} U_{22} F_1 F_2}\cos\theta \\ &= (1 - \cos\theta)(U_{22} F_1 + U_{21} F_2) \\ &\quad + \cos\theta \left| \sqrt{U_{22} F_1} - \sqrt{U_{21} F_2} \right|^2 \end{aligned}$$

with θ uniformly distributed in $[0,\pi]$ and is independent of the other random variables. The outage probability conditioned on θ is maximized when θ is close to 0^+, where $1-\cos\theta \approx \frac{\theta^2}{2}$. In this region, we have

$$\begin{aligned} |u_{22}f_1 - u_{21}f_2|^2 \doteq & \frac{\theta^2}{2}(U_{22}F_1 + U_{21}F_2) \\ & + \left|\sqrt{U_{22}F_1} - \sqrt{U_{21}F_2}\right|^2 \end{aligned} \tag{21}$$

Then, from (19) and (21), we have the outage region $\mathcal{O}(\boldsymbol{H}, r)$

$$\left\{ \begin{array}{rcl} (1+\mathsf{SNR}G_0)\det(\mathbf{I}+\mathsf{SNR}\boldsymbol{U}\boldsymbol{U}^\dagger) & \dot{\leq} & \mathsf{SNR}^r \\ 1+\mathsf{SNR}(F_1+F_2) & \dot{\leq} & \mathsf{SNR}^r \\ 1+\mathsf{SNR}^2F_2U_{11} & \dot{\leq} & \mathsf{SNR}^r \\ 1+\mathsf{SNR}^2\theta^2(U_{22}F_1+U_{21}F_2) & \dot{\leq} & \mathsf{SNR}^r \\ 1+\mathsf{SNR}^2\left|\sqrt{U_{22}F_1}-\sqrt{U_{21}F_2}\right|^2 & \dot{\leq} & \mathsf{SNR}^r \end{array} \right\} \tag{22}$$

The last inequality in (22) implies

$$1+\mathsf{SNR}^2(U_{22}F_1+U_{21}F_2) \,\dot{\leq}\, \mathsf{SNR}^r + 2\mathsf{SNR}^2\sqrt{U_{21}U_{22}F_1F_2},$$

which means that, in the high SNR regime, the outage region $\mathcal{O}(\boldsymbol{H}, r)$ is included[3] in the region $\mathcal{O}(\boldsymbol{\alpha}, r)$ defined by

$$\left\{ \begin{array}{rcl} 3(1-\alpha_{g_0}) & \leq & r \\ (1-\alpha_{g_0})+(1-\alpha_{g_1}-\alpha_{h_1}) & \leq & r \\ 2-\alpha_{g_0}-\alpha_{g_2}-\alpha_{h_2} & \leq & r \\ 1-\alpha_{g_2}-\alpha_{\gamma_{12}}-\alpha_{h_1} & \leq & r \\ 2-\alpha_{g_0}-\alpha_{g_2}-\alpha_{\gamma_{12}}-\alpha_{h_1}-\alpha_\theta & \leq & r \\ 2-\alpha_{g_1}-\alpha_{g_2}-\alpha_{h_1}-\alpha_{h_2}-\alpha_\theta & \leq & r \\ 2-\alpha_{g_0}-\alpha_{g_2}-\alpha_{\gamma_{12}}-\alpha_{h_1} & \leq & \max\{r,\phi(\boldsymbol{\alpha})\} \\ 2-\alpha_{g_1}-\alpha_{g_2}-\alpha_{h_1}-\alpha_{h_2} & \leq & \max\{r,\phi(\boldsymbol{\alpha})\} \end{array} \right\}$$

[3]In this case, we have $\mathcal{O}(\boldsymbol{H}, r) \subseteq \mathcal{O}(\boldsymbol{\alpha}, r)$ but $\mathcal{O}(\boldsymbol{\alpha}, r) \nsubseteq \mathcal{O}(\boldsymbol{H}, r)$

with $\phi(\boldsymbol{\alpha}) \triangleq 2 - \frac{1}{2}(\alpha_{g_0} + \alpha_{g_1} + \alpha_{\gamma_{12}} + \alpha_{h_2}) - \alpha_{h_1} - \alpha_{g_2}$. Let us define

$$\begin{aligned} \mathcal{O}_{\mathcal{T}}(\boldsymbol{\alpha}, r) &\triangleq \mathcal{O}(\boldsymbol{\alpha}, r) \cap \mathcal{T}(\boldsymbol{\alpha}, r) \\ \mathcal{O}_{\overline{\mathcal{T}}}(\boldsymbol{\alpha}, r) &\triangleq \mathcal{O}(\boldsymbol{\alpha}, r) \cap \overline{\mathcal{T}}(\boldsymbol{\alpha}, r) \end{aligned}$$

with

$$\mathcal{T}(\boldsymbol{\alpha}, r) \triangleq \{\boldsymbol{\alpha} \ : \quad r \leq \phi(\boldsymbol{\alpha})\}.$$

Since $\mathcal{O}(\boldsymbol{\alpha}, r) = \mathcal{O}_{\mathcal{T}}(\boldsymbol{\alpha}, r) \cup \mathcal{O}_{\overline{\mathcal{T}}}(\boldsymbol{\alpha}, r)$, we have

$$\inf_{\mathcal{O}(\boldsymbol{\alpha},r)} \varepsilon(\boldsymbol{\alpha}) = \min\left\{\inf_{\mathcal{O}_{\mathcal{T}}(\boldsymbol{\alpha},r)} \varepsilon(\boldsymbol{\alpha}), \inf_{\mathcal{O}_{\overline{\mathcal{T}}}(\boldsymbol{\alpha},r)} \varepsilon(\boldsymbol{\alpha})\right\},$$

with $\varepsilon(\boldsymbol{\alpha}) = \alpha_{g_0} + \alpha_{g_1} + \alpha_{g_2} + \alpha_{h_1} + \alpha_{h_2} + \alpha_{\gamma_{12}} + \frac{1}{2}\alpha_{\theta}$ by lemma 4 and the independence between the random variables. Thus, the DMT can be obtained with two linear optimizations. This problem can be solved numerically using sophisticated linear programming algorithms or softwares. If the relay ordering is such that $|h_2| > |h_1|$, we add $\alpha_{h_1} > \alpha_{h_2}$ to the constraints and carry out the same optimization problem. We can finally get the DMTs of Fig. 4.

■

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