AN ANALYTICAL METHOD FOR POWER QUALITY ASSESMENT APPLIED TO GRID-CONNECTED POWER ELECTRONIC CONVERTERS

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INTRODUCTION

The wide spread use of the power electronic devices in the power networks are due to their multiple functions: compensation, protection and interface for generators. Adapting and transforming the electric energy, they make possible the insertion in the power network of independent generators and renewable sources of energy. However, because of their switching components, the power electronic converters generate current and voltage harmonics which may cause measurements, stability and control problems.
In order to avoid the harmonic disturbances, a good knowledge on the harmonic generation and propagation is necessary. The understanding of the harmonic transfer mechanisms could make the harmonic attenuation more efficient, optimising the filters and improving the power electronics control.

STATE OF ART

The harmonic analysis is usually effectuated in the time or in the frequency domain. In the time domain currents and voltages spectra can be obtained by application of the Fourier transform after the resolution of the system differential equations. This method can not give an analytical expression for the system harmonics and for that reason is rarely used. Moreover, it may induce errors in the harmonics calculation when the choice of the step time is not accurate, or the transient process is still persistent.
In the frequency domain several methods for power network harmonic analysis are used [1]. The simplest one consists to model the network by representing the power electronic devices by known sources of harmonic currents. Another technique models the converters by their Norton equivalent. Both methods are often used in the network harmonic analysis thanks to their simplicity. However, they are not accurate enough, because they do not take into account the dynamics of the switching components.
More detailed models especially designed for the power electronic devices exist. Such a model is the transfer function model, which links the converter state variables by matrix equations. Another method proposed in [2] describes the converter by a set of nonlinear equations solved by Newton's method. These models have a good accuracy, but because of their complexity, they can not be applied to systems containing multiple converters.
For an accurate network harmonic analysis a simple and efficient method, taking into account the harmonics caused by the switching process is required.
The method proposed in this paper use the periodicity of the converter variables in steady state in order to put them in a matrix form in the frequency domain. Previous researches in this area have been already made. In [3] the models of power electronics structures are built using harmonic transfer matrices and implemented in Matlab/Simulink. The presented method is especially used for stability analysis and for that reason data are simplified and the high frequencies are neglected. In [4] a method using the periodicity of the variables is presented, but it gives only a numerical solution and it is not applied in the case of switching circuits and network analysis. Both methods do not give analytical expressions of the harmonics.
In this paper the presented method describes the considered system by differential equations, which are then converted in the frequency domain. Being periodic signals, currents and voltages are described by terms of Fourier series and then by vectors of harmonics. The passive elements and the switching functions are described by matrices. The resolution of the matrix equations gives time and frequency expressions of the converter voltages and currents.
The presentation of the proposed method begins with the construction of the harmonic transfer matrices for the converter components: the switching elements and the passive components (resistors, inductors and capacitors). Then, the method is applied to a closed-loop three phase PWM AC/DC/AC converter. The results obtained by the method are verified by measurements and simulation.

BASIC ANALYSIS AND FUNDAMENTALS

Power electronic systems can be considered as combination of switching and passive components. In this section the harmonic propagation through these elements is analysed and the necessity of their matrix representation is demonstrated.
When building the harmonic transfer matrices, some assumptions are made: the switching and the passive components are supposed ideal, the considered system is supposed to be in steady state and periodically time-variant.

Harmonic transfer matrix for switching elements

For the switching process presented in fig.1, the relation between the ac and the dc currents is given by:

$$i_{dc}(t) = u(t) i_{ac}(t) \,. \quad (1)$$

$i_{ac}(t)$ is supposed T_i-periodic (periodic with period of T_i seconds).
$u(t)$ is T_u-periodic with $T_i = NT_u$, where N is an integer, so

that $u(t)$ can be considered as T_i -periodic.

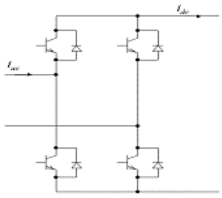

Figure 1 A switching process

These two signals can be decomposed in Fourier series as a function of the same fundamental frequency $\frac{1}{T_i}$. Therefore, equation (1) become:

$$i_{dc}(t)=\left(\sum_{n=-\infty}^{\infty}<i_{ac}>_n e^{jn\omega_i t}\right)\left(\sum_{m=-\infty}^{\infty}<u>_m e^{jm\omega_i t}\right)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}<i_{ac}>_n<u>_m e^{j(n+m)\omega_i t} \quad (2)$$

$$i_{dc}(t)== \sum_{k=-\infty}^{\infty}\left(\sum_{n=-\infty}^{\infty}<i_{ac}>_n<u>_{k-n}\right)e^{jk\omega_i t}$$, where

$\omega_i=\frac{2\pi}{T_i}$ and $<x>_n=\frac{1}{T_i}\int_{T_i}x(t)e^{-jn\omega_i t}dt$ is the n^{th} harmonic component of the T_i -periodic signal $x(t)$

Equation (1) shows that $i_{dc}(t)$ can be viewed as a T_i periodic signal, with

$$<i_{dc}>_k=\sum_{n=-\infty}^{\infty}<u>_{k-n}<i_{ac}>_n \quad (3)$$

Equation (3) can be written in a matrix form as follows:

$$\begin{bmatrix} . \\ . \\ <i_{dc}>_{k-1} \\ <i_{dc}>_k \\ <i_{dc}>_{k+1} \\ . \\ . \end{bmatrix}=\begin{bmatrix} . & . & . & & & & \\ . & . & . & . & & & \\ . & . & <u>_0 & <u>_{-1} & <u>_{-2} & & \\ & . & <u>_1 & <u>_0 & <u>_{-1} & . & \\ & & <u>_2 & <u>_1 & <u>_0 & . & . \\ & & & . & . & . & . \\ & & & & . & . & . \end{bmatrix}\begin{bmatrix} . \\ . \\ <i_{ac}>_{k-1} \\ <i_{ac}>_k \\ <i_{ac}>_{k+1} \\ . \\ . \end{bmatrix} \quad (4)$$

or with a shorter notation:

$$[I_{dc}]=[U][I_{ac}] \quad (5)$$

Harmonic transfer matrix for passive elements

For the passive elements, for example a capacitor, the relation between current and voltage harmonics is given by the formulae:

$$i=C\frac{dv}{dt} \Rightarrow <i>_k=C\left\langle\frac{dv}{dt}\right\rangle_k=C\frac{d<v>_k}{dt}+jk\omega C<v>_k \quad (6)$$

As the converter is considered in its steady state, the harmonics do not vary with time:

$$<v>_k=const \Rightarrow \frac{d<v>_k}{dt}=0 \Rightarrow <i>_k=jk\omega C<v>_k \quad (7)$$

and the relation between the voltage and current harmonics is presented in the following matrix form:

$$\begin{bmatrix} . \\ . \\ <i>_{k-1} \\ <i>_k \\ <i>_{k+1} \\ . \\ . \end{bmatrix}=\begin{bmatrix} . & 0 & 0 & 0 & 0 & 0 & 0 \\ 0. & . & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & j(k-1)\omega C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & jk\omega C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j(k+1)\omega C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & . \end{bmatrix}\begin{bmatrix} . \\ . \\ <v>_{k-1} \\ <v>_k \\ <v>_{k+1} \\ . \\ . \end{bmatrix} \quad (8)$$

The matrix representation of the harmonic transfer matrix via the passive components is not obligatory. However, it is used in order to describe the whole considered system by matrix equations.

Analogically, the harmonic transfers via an inductor and a resistor can be expressed by matrix equations (6,7), all matrices having a diagonal structure.

$$\begin{bmatrix} . \\ . \\ <v>_{k-1} \\ <v>_k \\ <v>_{k+1} \\ . \\ . \end{bmatrix}=\begin{bmatrix} . & 0 & 0 & 0 & 0 & 0 & 0 \\ 0. & . & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & j(k-1)\omega L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & jk\omega L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j(k+1)\omega L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & . \end{bmatrix}\begin{bmatrix} . \\ . \\ <i>_{k-1} \\ <i>_k \\ <i>_{k+1} \\ . \\ . \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} . \\ . \\ <v>_{k-1} \\ <v>_k \\ <v>_{k+1} \\ . \\ . \end{bmatrix}=\begin{bmatrix} . & 0 & 0 & 0 & 0 & 0 & 0 \\ 0. & . & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & . \end{bmatrix}\begin{bmatrix} . \\ . \\ <i>_{k-1} \\ <i>_k \\ <i>_{k+1} \\ . \\ . \end{bmatrix} \quad (10)$$

APPLICATION TO AN AC/DC/AC CONVERTER

In order to illustrate the capacities of the method, the model of a closed-loop AC/DC/AC PWM converter is described in this section (fig. 2). The considered structure is chosen because of its complexity and its wide spread use as power interface.

The presented structure is considered as composed of two converters having the same structure, so the application of the method is presented only for the AC/DC converter. The method is analogically applied to the whole converter structure, adding the equations for the second converter.

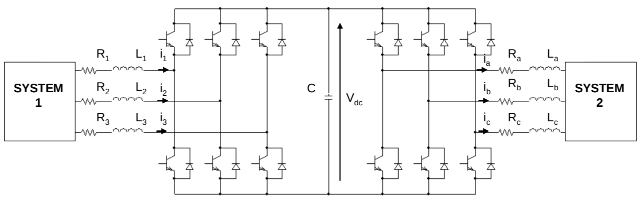

Figure 2 AC/DC/AC three phase converter used as power interface

Application to an AC/DC PWM converter

The method is applied to the AC/DC converter (fig.3), the DC/AC converter from fig.2 is replaced by a resistor. By supposing that the switching components, the passive elements and the network voltage are ideal, the converter can be described by a set of the differential equations (11):

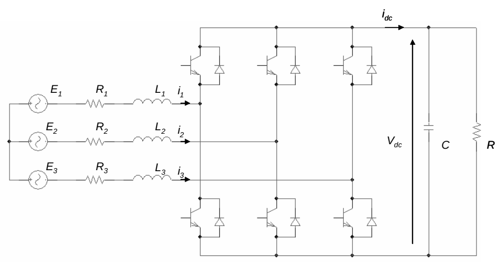

Figure 3 AC/DC converter

$$\begin{cases} L_1\dfrac{di_1(t)}{dt}=V_1(t)-R_1i_1(t)-\left(2u_1(t)-u_2(t)-u_3(t)\right)\dfrac{V_{dc}(t)}{6} \\ L_2\dfrac{di_2(t)}{dt}=V_2(t)-R_2i_2(t)-\left(2u_2(t)-u_1(t)-u_3(t)\right)\dfrac{V_{dc}(t)}{6} \\ L_3\dfrac{di_3(t)}{dt}=V_3(t)-R_3i_3(t)-\left(2u_3(t)-u_1(t)-u_2(t)\right)\dfrac{V_{dc}(t)}{6} \\ C\dfrac{dV_{dc}(t)}{dt}=\dfrac{1}{2}\left(u_1(t)i_1(t)+u_2(t)i_2(t)+u_3(t)i_3(t)\right)-\dfrac{V_{dc}(t)}{R} \end{cases}, \qquad (11)$$

where $u_i(t)$ is the switching function of the i^{th} leg:

$$u_i(t)=\begin{cases}1\\-1\end{cases} \qquad (12)$$

In steady state these equations can be converted in the frequency-domain and presented in a matrix form (13):

$$\begin{cases} [L_1][I_1]=[V_1]-[R_1][I_1]-\frac{1}{6}(2[U_1]-[U_2]-[U_3])[V_{dc}] \\ [L_2][I_2]=[V_2]-[R_2][I_2]-\frac{1}{6}(2[U_2]-[U_1]-[U_3])[V_{dc}] \\ [L_3][I_3]=[V_3]-[R_3][I_3]-\frac{1}{6}(2[U_3]-[U_1]-[U_2])[V_{dc}] \\ [C][V_{dc}]=\frac{1}{2}([U_1][I_1]+[U_2][I_2]+[U_3][I_3])-[R]^{-1}[V_{dc}] \end{cases}, \qquad (13)$$

The state variables are represented by vectors of harmonics and the system parameters by matrices (14), as described in the previous section:

$$\begin{aligned} &[I_k]=\left[\;.\;\;.\;\;.\;\;\langle I_k\rangle_{-2}\;\;\langle I_k\rangle_{-1}\;\;\langle I_k\rangle_{0}\;\;\langle I_k\rangle_{1}\;\;\langle I_k\rangle_{2}\;\;.\;\;.\;\;.\;\right]^T \quad k=1,2,3 \\ &[V_{dc}]=\left[\;.\;\;.\;\;.\;\;\langle V_{dc}\rangle_{-2}\;\;\langle V_{dc}\rangle_{-1}\;\;\langle V_{dc}\rangle_{0}\;\;\langle V_{dc}\rangle_{1}\;\;\langle V_{dc}\rangle_{2}\;\;.\;\;.\;\;.\;\right]^T \\ &[L_k]=diag(j\omega HL_k) \quad k=1,2,3 \\ &[R_k]=diag(j\omega HR_k) \quad k=1,2,3, \\ &[C]=diag(j\omega HC) \\ &[R]=diag(R) \end{aligned} \qquad (14)$$

where $[H]$ is a vector containing the harmonics ranks:

$$[H]=[\;.\;\;.\;\;.\;\;-2\;\;-1\;\;0\;\;1\;\;2\;\;.\;\;.\;\;.\;] \qquad (15)$$

The matrices $[U_1]$, $[U_2]$ and $[U_3]$ contain the Fourier coefficients of the switching functions:

$$\begin{aligned} &[U_1]=toeplitz\left[\;.\;\;.\;\;.\;\;\langle u_1\rangle_{-2}\;\;\langle u_1\rangle_{-1}\;\;\langle u_1\rangle_{0}\;\;\langle u_1\rangle_{1}\;\;\langle u_1\rangle_{2}\;\;.\;\;.\;\;.\;\right] \\ &[U_2]=toeplitz\left[\;.\;\;.\;\;.\;\;\langle u_2\rangle_{-2}\;\;\langle u_2\rangle_{-1}\;\;\langle u_2\rangle_{0}\;\;\langle u_2\rangle_{1}\;\;\langle u_2\rangle_{2}\;\;.\;\;.\;\;.\;\right] \\ &[U_3]=toeplitz\left[\;.\;\;.\;\;.\;\;\langle u_3\rangle_{-2}\;\;\langle u_3\rangle_{-1}\;\;\langle u_3\rangle_{0}\;\;\langle u_3\rangle_{1}\;\;\langle u_3\rangle_{2}\;\;.\;\;.\;\;.\;\right] \end{aligned} \qquad (16)$$

which are obtained through a double Fourier series decomposition. For naturally sampled PWM, the Fourier series decomposition of the switching function [5] is given by:

$$\begin{aligned} &u_i(t)=M\cos(\omega_0 t+\theta_0+(i-1)\frac{2\pi}{3})+ \\ &\frac{4}{\pi}\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\frac{1}{m}\left\{\sin\left([m+n]\frac{\pi}{2}\right)J_n\left(m\frac{\pi}{2}M\right)\cos\left(m[\omega_c t+\theta_c]+n\left[\omega_0 t+\theta_0(i-1)\frac{2\pi}{3}\right]\right)\right\} \end{aligned} \qquad (17)$$

where

M fundamental magnitude or modulation depth

ω_c, θ_c Carrier pulsation and phase

ω_0, θ_0 Fundamental pulsation and phase

$J_n(\;)$ Bessel function from order n

Control modeling

In open loop the fundamental magnitude M and phase θ_0 used for the calculation of the Fourier coefficients of the switching function are constant. In closed loop these two parameters are not fixed, because they are used to control the magnitude of some converter state variables and for that reason are dependant on their reference values. In order to take into account the control system, the converter equations are solved in the time domain for the fundamental component before the application of the developed method.

When expressing the converter equations only for the fundamental component, the three phase state variables and switching functions can be represented by vector phasors:

$$\begin{cases} i_1=i \\ i_2=ie^{-j\frac{2\pi}{3}} \\ i_3=ie^{j\frac{2\pi}{3}} \end{cases} \quad \begin{cases} V_1=V \\ V_2=Ve^{-j\frac{2\pi}{3}} \\ V_3=Ve^{j\frac{2\pi}{3}} \end{cases} \quad \begin{cases} u_1=u \\ u_2=ue^{-j\frac{2\pi}{3}} \\ u_3=ue^{j\frac{2\pi}{3}} \end{cases} \qquad (18)$$

By supposing that the passive elements have the same values in the three phases, equations (11) become:

$$\begin{cases} L_1\dfrac{di}{dt}=V_1-u\dfrac{V_{dc}}{6}-R_1 i \\ C\dfrac{dV_{dc}}{dt}=\dfrac{3}{2}ui-\dfrac{V_{dc}}{R} \end{cases} \qquad (19)$$

Equations (19) are transformed in the dq0 frame in order to make appear the state variables as constant:

$$\begin{cases} L_1\dfrac{di_d}{dt}=\omega L_{res}i_q-R_1i_d+V_1-\dfrac{1}{2}u_dV_c \\ L_1\dfrac{di_q}{dt}=-\omega L_{res}i_d-R_1i_q-\dfrac{1}{2}u_qV_c \\ C\dfrac{dV_{dc}}{dt}=\dfrac{3}{2}\left(u_di_d+u_qi_q\right)-\dfrac{V_{dc}}{R} \end{cases} \qquad (20)$$

The magnitudes of the state variables are constant in the dq0 frame (21). By supposing the PI controllers ideal, the d and the q components of the switching functions are found (22). From the obtained values, the fundamental magnitude and phase of the switching function are calculated (23) :

$$\frac{di_d}{dt}=0 \qquad \frac{di_q}{dt}=0 \qquad \frac{dV_{dc}}{dt}=0 \qquad (21)$$

$$\begin{cases} V_{dc}=\sqrt{\dfrac{3}{2}R\left(i_d\left(\omega L_1i_q-R_1i_d-V_1\right)+i_q\left(-\omega L_1i_d-R_1i_q\right)\right)} \\ u_d=\dfrac{2}{V_{dc}}\left(\omega L_1i_q-R_1i_d-V_1\right) \\ u_q=\dfrac{2}{V_{dc}}\left(-\omega L_1i_d-R_1i_q\right) \end{cases} \qquad (22)$$

$$\begin{cases} M=\sqrt{u_d^2+u_q^2} \\ \theta_0=-\arctan\left(\dfrac{u_q}{u_d}\right) \end{cases} \qquad (23)$$

Application to the whole converter structure

The method is applied to the whole system. Analogical equations are used to describe the DC/AC converter. The converter structure is connected to the grid, a resistor is used as load. The PWM frequency is 2kHz. The obtained results are compared with those obtained by measurements and simulation and are presented in the following section.

SIMULATION AND PRACTICAL RESULTS

The results obtained thanks to the previous theoretical method, the Matlab/simulink simulation and the experimental bench are compared in this section.

Theoretical method

The matrix equations describing the converter and its control system are implemented. The switching functions and the known state variables as the input voltage are decomposed in Fourier series and the corresponding harmonic transfer matrices and vectors are built. The resolution of the matrix system equations gives the time and frequency expression of the converter state variables. The calculation time depends on the number of the considered harmonics.

Matlab/Simulink simulation

A model of the converter based on its differential equations is realized under Matlab/Simulink. The obtained results are in the time domain and a Fourier transformation is used to obtain the currents and the voltages spectra.

Experimental bench

The experimental bench and its structure are presented in fig.4. The network voltage is adapted through autotransformers.

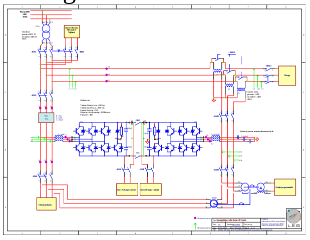

Figure 4 Experimental bench

Results

The method results are compared to those obtained by measurements and Matlab/Simulink simulations. In fig.5 the spectrum of the ac current from the network side is shown between 1500 and 6500 Hz (for the PWM harmonics). In the three cases, the harmonics are situated at the same frequencies and have almost the same magnitudes. The small differences are due to the assumptions used in our method, the simulation errors, and the disturbances in the real system (non-ideal components, noises, etc.). The results obtained for the dc voltage and the ac current from the load side are quite similar.

CONCLUSION

The presented method is designed for power systems with periodically switching components. It does not need a long calculation time and gives a theoretical solution of the considered system. One of its main advantages is the analytical expression of the currents and voltages harmonics. It allows to determine the influence of the system parameters (control strategy, passive elements, etc.) on the harmonic contents of the converter state variables. It can be successfully applied for power quality assessment, harmonic filters optimization and converter control design.

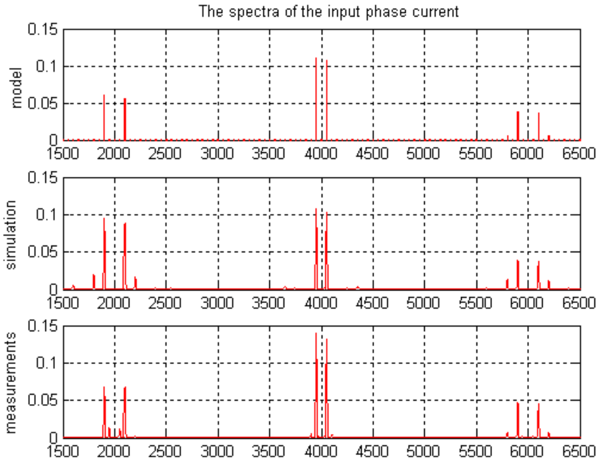

Figure 5 The spectrum of the ac current from the network side theoretical, simulation and experimental results

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