

## LINEAR TIME PERIODIC MODELLING OF POWER ELECTRONIC DEVICES FOR POWER SYSTEM HARMONIC ANALYSIS AND SIMULATION

*Vanya Ignatova\**, *Seddik Bacha\**, *Pierre Granjon\*\**, *Nicolas Retière\**

\*LEG: Laboratory of Electric Engineering of Grenoble

\*\*LIS: Laboratory of Images and Signals

### ABSTRACT

This paper presents an analytical frequency-domain method for harmonic estimation able to assess harmonics and interharmonics levels and to capture the coupling between frequencies. It is based on the linear time – periodic system theory as a result of the periodic behaviour of the state variables in steady state operation. It consists of describing the considered system with a set of differential equations, presenting them in a matrix form in the frequency domain by decomposition of the converter state variables into Fourier series and finally solving the matrix equations in order to calculate the current and the voltage harmonics. The method is applied to a single phase full wave rectifier and to an association rectifier- inverter, the results are verified by simulation.

### 1. INTRODUCTION

The variety and the wide spread use of power electronic devices in the power networks is due to their diverse and multiple functions: compensation, protection and interface for generators. In most cases these devices generate current harmonics which can induce resonance and stability problems in the power system and for that reason the installation of filters or additional generators and compensators is required. A better knowledge on the harmonic generation and interaction could improve the control system, optimise the filters and make the harmonic rejection more efficient.

There are several techniques used for converter systems modelling [1]. They can be situated in the frequency domain or in the time domain.

In the time-domain the converter is presented by a set of differential equations. When the equations are solved, the currents and the voltages spectra are obtained by the application of a Fourier transform. The main disadvantages

of the time-domain analysis are the need of a long simulation time and the errors caused by the step time and the persistence of the transient process when the time-frequency transform is applied. Moreover, the time-domain models are not suitable for the study of the harmonic propagation and interactions, because they can not give an analytical expression of the harmonics.

The simplest model in the frequency-domain is the current source model. Because the converter acts as an injection current source, this model considers it as a known source of harmonic currents, presented by their magnitude and phase.

Another technique of modelling is to present the converter by its Norton equivalent. Introducing the Norton admittance matrix, the model takes into account the relationship between converter input currents and its terminal voltages.

Those two techniques are simple, but not sufficiently accurate, as they do not reflect the interaction between the two sides of the converter. However, they are used for modelling and analysis of multi-converter systems.

In order to take the harmonic interactions into account, more detailed models are necessary. Such a model is the transfer function model, which uses two transfer functions to relate the two sides of the converter. Another method proposed in [2] describes the converter by a set of nonlinear equations solved by the Newton's method. These models are characterized by better precision, but are too complicated for being applied for harmonic analysis when the system contains multiple converters.

We can therefore divide the existing techniques of modelling into two categories: simple but not reliable, and accurate but too complicated. The technique of modelling used in this paper attempts to be a compromise between the two categories. It takes the harmonic interactions into account and in the same time can model systems composed by several converters.

The proposed method is based on the periodicity of the converter variables in steady state. Previous researches on

this area have been already made [3], but they use the linearization around the nominal ac voltage and do not take into account the disturbance of the network voltage and the interharmonics.

The considered system in the presented method is described by differential equations, which are converted in the frequency domain. Being periodic signals, the currents and the voltages are described by terms of Fourier series and then presented by vectors of harmonics. The passive elements and the switching functions are presented by matrices. The resolution of the matrix equations gives the spectrum of the converter voltages and currents.

For better understanding and comprehension, the presentation of the proposed method starts with the study of the harmonic transfer throughout the components of a converter: the switching elements and the passive components (resistors, inductors and capacitors). The modelling algorithm is then illustrated by the example of a single phase full wave rectifier and the possibilities to realise converter models with different command modes or distorted voltage network are discussed. Finally, a more complicated system composed of a single phase rectifier associated with a single phase inverter is modelled and analysed.

## 2. HARMONIC PROPAGATION STUDY

### 2.A Harmonic propagation throughout a switching element

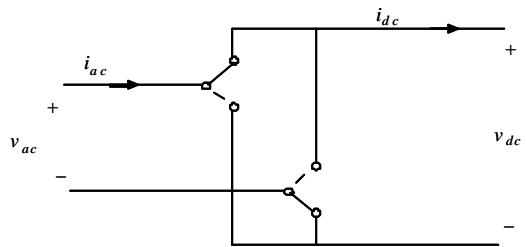


Figure 1 Presentation of a switching process

For the switching process presented in figure 1, the relation between the ac and the dc currents is given by the switching function  $u(t)$ :

$$i_{dc}(t) = u(t)i_{ac}(t)$$

The frequency-domain expression of this equation is obtained by the use of Fourier series:

$$\sum_{k=-\infty}^{\infty} \langle i_{dc} \rangle_k e^{jk\omega t} = \left( \sum_{k=-\infty}^{\infty} \langle i_{ac} \rangle_k e^{jk\omega t} \right) \left( \sum_{i=-\infty}^{\infty} \langle u \rangle_i e^{j\omega t} \right) = \sum_{k=-\infty}^{\infty} \left( \sum_{i=-\infty}^{\infty} \langle u \rangle_{k-i} \langle i_{ac} \rangle_i \right) e^{jk\omega t}$$

$$\langle i_{dc} \rangle_k = \sum_{i=-\infty}^{\infty} \langle u \rangle_{k-i} \langle i_{ac} \rangle_i$$

The last equation can be expressed in a matrix form:

$$\begin{bmatrix} \cdot \\ \langle i_{dc} \rangle_{k-1} \\ \langle i_{dc} \rangle_k \\ \langle i_{dc} \rangle_{k+1} \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \langle u \rangle_0 & \langle u \rangle_{-1} & \langle u \rangle_{-2} & \cdot & \cdot \\ \langle u \rangle_1 & \langle u \rangle_0 & \langle u \rangle_{-1} & \cdot & \cdot \\ \langle u \rangle_2 & \langle u \rangle_1 & \langle u \rangle_0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \langle i_{ac} \rangle_{k-1} \\ \langle i_{ac} \rangle_k \\ \langle i_{ac} \rangle_{k+1} \\ \cdot \end{bmatrix}$$

The transfer matrix via switching element has a Toeplitz structure (the elements situated on the same diagonal are equal) and represents a linear time periodic system [4]. When a harmonic at the frequency  $f$  is applied to the system, it generates a multiple of harmonics at the frequencies  $f \pm n f_0$ , where  $f_0$  is the fundamental frequency of the switching function and  $n$  is an integer  $n = 1, 2, 3, \dots$ .

From the matrix expression of the switching process, it can be concluded that the switching elements not only modify the amplitude and the phase of the harmonics, but also their frequencies. They generate new harmonic components.

### 2.B Harmonic propagation throughout a passive element

In order to study the harmonic transfer via passive elements, the case of an inductor is considered. The relation between current and voltage harmonics is given by the formulae:

$$v = L \frac{di}{dt} \Rightarrow \langle v \rangle_k = L \left\langle \frac{di}{dt} \right\rangle_k = L \frac{d \langle i \rangle_k}{dt} + jk\omega L \langle i \rangle_k$$

where  $k$  is the harmonic rank.

As the converter is considered in its steady state, the harmonic magnitude and phase do not vary with the time:

$$\langle i \rangle_k = const \Rightarrow \langle v \rangle_k = jk\omega L \langle i \rangle_k$$

and the dependence between the voltage and current harmonics can be expressed in the following matrix form:

$$\begin{bmatrix} \cdot \\ \langle v \rangle_{k-1} \\ \langle v \rangle_k \\ \langle v \rangle_{k+1} \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot & 0 & 0 & 0 & 0 \\ 0 & j(k-1)\omega L & 0 & 0 & 0 \\ 0 & 0 & jk\omega L & 0 & 0 \\ 0 & 0 & 0 & j(k+1)\omega L & 0 \\ 0 & 0 & 0 & 0 & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \langle i \rangle_{k-1} \\ \langle i \rangle_k \\ \langle i \rangle_{k+1} \\ \cdot \end{bmatrix}$$

The transfer matrix represents a linear time-invariant system: it contains non-zero elements only in its main diagonal. It means that a current harmonic at frequency  $f$  generates a voltage harmonic at the same frequency or vice versa and there is no frequency coupling between the harmonics.

In the same manner it can be proved that the transfer matrices via a capacitor and an inductor are also linear time-invariant systems.

From the structure of the transfer matrix it can be verified that passive elements modify the harmonic magnitudes and phases, but not their frequencies. They do not generate new harmonic components

### 3. MODELLING AND SIMULATIONS

The models of two converter structures are presented in this section. The first one is a single phase full wave rectifier and the other represents an association between two single phase converters- a rectifier and an inverter. The models rely upon the following assumptions:

- The converter is in steady state and operate in continuous mode
- The switching elements are ideal
- The passive elements are linear
- The network voltage is ideal

Some particularities should be noticed. First of all, the converter modelling is possible only if the switching function and the input voltage are periodic. Second, in order to obtain correct results, the vectors and the matrices are built with the negative and the positive terms of the Fourier series decomposition. Finally, a predetermination of the appeared harmonics is logically effectuated. This allows a reduction in the matrices' size and the computation time, and also avoids division by zero when inverting matrices.

#### 3.A Modelling of single phasefull wave rectifier

In order to illustrate the method, the simple example of single phase self-commuted rectifier has been taken.

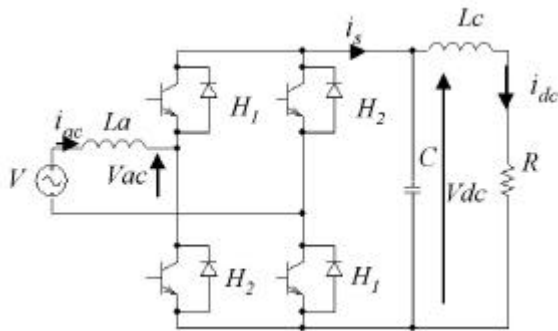


Figure 2 Single phase rectifier

The converter is described by the following equations:

$$\begin{cases} L_a \frac{di_{ac}(t)}{dt} = V(t) - V_{ac}(t) = V(t) - u(t)V_{dc}(t) \\ C \frac{dV_{dc}(t)}{dt} = i_s(t) - i_{dc}(t) = u(t)i_{ac}(t) - i_{dc}(t) \\ L_c \frac{di_{dc}(t)}{dt} = V_{dc}(t) - Ri_{dc}(t) \end{cases}$$

In steady state these equations can be converted in the frequency-domain and presented in the following matrix form:

$$\begin{cases} [L_a][I_{ac}] = [V] - [U][V_{dc}] \\ [C][V_{dc}] = [U]^T [I_{ac}] - [I_{dc}] \\ [L_c][I_{dc}] = [V_{dc}] - [R][I_{dc}] \end{cases}$$

where the matrix U contains the non-zero terms of the Fourier series decomposition of the switching function

u(t). As this function has a rectangular form, only odd-rank harmonics are presented in the matrix structure:

$$[U] = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \langle u \rangle_1 & \langle u \rangle_{-1} & \langle u \rangle_{-3} \\ \cdot & \langle u \rangle_3 & \langle u \rangle_1 & \langle u \rangle_{-1} & \cdot \\ \cdot & \langle u \rangle_5 & \langle u \rangle_3 & \langle u \rangle_1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

As the network voltage is supposed ideal only the terms corresponding to the fundamental are different from zero. Via the switching function, they generate harmonics of even rank on the dc side, which in turn, passing through the switching function, generate odd-rank harmonics on the ac current and voltage.

As a result the converter harmonics ranks at the dc side are even and those of the ac side odd:

$$\begin{aligned} r_{ac} &= [\cdot \cdot \cdot -5 \ -3 \ -1 \ 1 \ 3 \ 5 \ \cdot \cdot] \\ r_{dc} &= [\cdot \cdot \cdot -4 \ -2 \ 0 \ 2 \ 4 \ \cdot \cdot] \end{aligned}$$

Therefore, the vectors of the converter state variables are:

$$\begin{aligned} [I_{ac}] &= [\cdot \cdot \langle I_{ac} \rangle_{-3} \ \langle I_{ac} \rangle_{-1} \ \langle I_{ac} \rangle_1 \ \langle I_{ac} \rangle_3 \ \cdot \cdot]^T \\ [V_{dc}] &= [\cdot \cdot \langle V_{dc} \rangle_{-4} \ \langle V_{dc} \rangle_{-2} \ \langle V_{dc} \rangle_0 \ \langle V_{dc} \rangle_2 \ \langle V_{dc} \rangle_4 \ \cdot \cdot]^T \\ [I_{dc}] &= [\cdot \cdot \langle I_{dc} \rangle_{-4} \ \langle I_{dc} \rangle_{-2} \ \langle I_{dc} \rangle_0 \ \langle I_{dc} \rangle_2 \ \langle I_{dc} \rangle_4 \ \cdot \cdot]^T \end{aligned}$$

and the matrices corresponding to the passive elements are diagonal:

$$[L_a] = \text{diag}(j\omega_0 r_{ac} L_a)$$

$$[L_c] = \text{diag}(j\omega_0 r_{dc} L_c)$$

$$[C] = \text{diag}(j\omega_0 r_{dc} C)$$

$$[R] = \text{diag}(R)$$

The model is implemented in Matlab software, the number of considered harmonics is limited to rank 40. When the matrix equations are solved, the wave forms of the currents and the voltages are reconstructed and compared with Matlab Simulink model (indicated with index s). As it can be seen in fig.3 the two models give the same results, the difference between them is due to the limitation of the harmonics number.

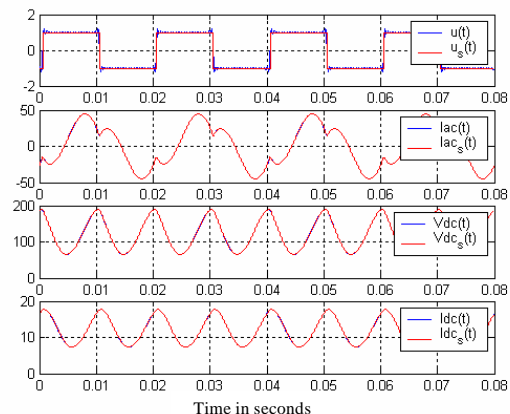


Figure 3 Comparison between the linear time periodic model and Matlab Simulink model of a single phase self-commuted rectifier

The same algorithm is applied into a three phase full wave rectifier and single phase PWM rectifier and the results are confirmed by time-domain simulations [5]. When the network voltage is distorted, the modelling of the converter is still possible. For example if the voltage network is:

$$V(t) = 240\sin(w_0t) + 50\sin(2.6w_0t)$$

the same matrix equations have to be solved, the harmonics ranks of the two sides of the converter being:

$$r_{ac} = [ \dots -4.6 \quad -2.6 \quad -0.6 \quad 1.4 \quad 3.4 \quad \dots ]$$

$$r_{dc} = [ \dots -3.6 \quad -1.6 \quad 0.4 \quad 2.4 \quad 4.4 \quad \dots ]$$

**3.B Harmonic interactions between two converters**

The proposed structure of modelling consists of two converters - a current rectifier and a voltage inverter (fig.4).

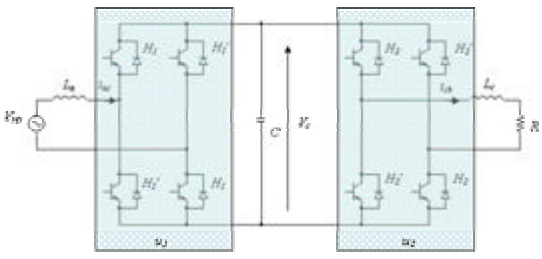


Figure 4 Association rectifier- inverter

In the frequency domain the considered system is described by the following equations:

$$\begin{cases} L_a [I_{ac}] = [V_{res}] - [U_1][V_c] \\ C[V_c] = [U_1][I_{ac}] - [U_2][I_{ch}] \\ L_c [I_{ch}] = [U_2][V_c] - [R][I_{ch}] \end{cases}$$

When the two converters operate at the same frequency, the harmonics on the ac sides are odd-rank, and the voltage harmonics on the continuous bus are even rank. The vectors and the matrices are built as explained in paragraph 3A. The model gives the same results as a Matlab Simulink model of the same structure.

When the converters switch at different frequencies, the first appeared frequency in the state variables vectors is equal to the least mean common multiple of the two switching functions frequencies. For example, if the rectifier commutates at 50 Hz and the inverter at 60Hz, the ranks of the harmonics on the two sides of the converter, considering the frequency of 50Hz as fundamental, are:

$$r_{ac} [ \dots -1.4 \quad -1 \quad -0.6 \quad -0.2 \quad 0.2 \quad 0.6 \quad 1 \quad 1.4 \quad \dots ]$$

$$r_{dc} = [ \dots -1.2 \quad -0.8 \quad -0.4 \quad 0 \quad 0.4 \quad 0.8 \quad 1.2 \quad \dots ]$$

The vectors of the converter variables are:

$$[I_{ac}] = [ \dots <I_{ac}>_{-0.6} \quad <I_{ac}>_{-0.2} \quad <I_{ac}>_{0.2} \quad <I_{ac}>_{0.6} \quad \dots ]^T$$

$$[V_c] = [ \dots <V_{dc}>_{-0.8} \quad <V_{dc}>_{-0.4} \quad <V_{dc}>_0 \quad <V_{dc}>_{0.4} \quad <V_{dc}>_{0.8} \quad \dots ]^T$$

$$[I_{ch}] = [ \dots <I_{ch}>_{-0.6} \quad <I_{ch}>_{-0.2} \quad <I_{ch}>_{0.2} \quad <I_{ch}>_{0.6} \quad \dots ]^T$$

and the transfer matrices are built in the same way as it is already explained in paragraph 3.A.

The results are confirmed by time simulation (fig.5):

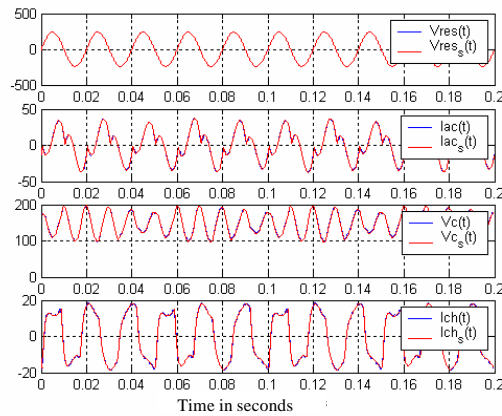


Figure 5 Comparison between the linear time periodic model and Matlab Simulink model of an association rectifier-inverter

**4. CONCLUSION**

Simple and reliable, the presented method is useful to understand power systems behavior with switching components. It is able to assess harmonics and interharmonics and study the interactions between converters and power system. As a consequence it can be applied to improve power system analysis, control techniques and filter design.

**5. REFERENCES**

[1] Task Force on Harmonics Modeling and Simulation, IEEE PES Harmonic Working Group: "Characteristics and modeling of harmonic sources- power electronic devices", IEEE Transactions on Power Delivery, vol.16, no.4, Oct. 2001; p.791-800

[2] Bathurst-G, Smith-B, Watson-N, Arrillaga-J: "Modelling of HVDC transmission systems in the Harmonic Domain", IEEE Transactions on Power Delivery, Vol.14, No.3, July 1999

[3] Mollerstedt-E., Bernhardsson-B.: "A harmonic transfer function model for a diode converter train", 2000-IEEE-Power-Engineering-Society-Winter-Meeting.-Conference-Proceedings-Cat.-No.00CH37077. 2000: 957-62 vol.2

[4] Prater-J, Loeffler-C: "Analysis and design of periodically time-varying IIR filters, with application to transmultiplexing", IEEE transactions on signal processing, vol.40, no11, November 1992

[5] Ignatova-V: "Etudes des interactions harmoniques des compensateurs d'électronique de puissance dans un réseau confiné" Master thesis, Laboratory of Electrical Engineering of Grenoble, Juny 2003