Mode coupling control in a resonant device : application to solid-state ring lasers

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A theoretical and experimental investigation of the effects of mode coupling in a resonant macroscopic quantum device is achieved in the case of a ring laser. In particular, we show both analytically and experimentally that such a device can be used as a rotation sensor provided the effects of mode coupling are controlled, for example through the use of an additional coupling. A possible generalization of this example to the case of another resonant macroscopic quantum device is discussed.

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Devices using macroscopic quantum effects [1] and their associated phenomenon of interference to detect rotations can be arbitrarily divided into two classes [2], namely non-resonant and resonant devices. In devices of the first class, the rotation-induced phase shift is detected by looking at the displacement of an interference pattern. Among such devices are the fiber optic gyroscope [3], the gyromagnetic gyroscope [4], the superfluid gyrometer [5] and the atomic interferometer [6]. In devices of the second class, the rotation is detected through a beat signal. Two examples of resonant and potentially rotation-sensitive devices are the ring laser [7] and the superfluid (e.g. liquid helium or Bose-Einstein condensed gas) in a ring container [8]. In these two examples, it has been shown [9, 10] that a non-linear mode coupling effect plays a crucial role in the system dynamics, usually preventing it from operating in a rotation sensitive regime, unless an additional adequate coupling is set.

For example, in the case of the ring laser, the bidirectional emission regime, required for rotation sensing, can be inhibited because the counter-propagating modes share the same gain medium and can be subject to mode competition. This problem is usually circumvented by choosing for the gain medium a two isotopes mixture of helium and neon and by tuning the laser emission frequency out of resonance with the atoms at rest. Provided the detuning value is bigger than the atomic natural linewidth, the gain medium can then be considered as being inhomogeneously broadened and coexistence of the counter-propagating modes occurs [11]. In the case of solid-state ring lasers, the gain medium is homogeneously broadened, leading to a strongly coupled situation resulting in laser emission in only one direction [10].

We report in this Letter theoretical and experimental investigation of mode coupling control in a solid-state (Nd:YAG) ring laser. The main natural sources of coupling between the counter-propagating modes are identified, and their role in the laser dynamics is discussed. An additional coupling source is introduced in order to ensure the coexistence of the counter-propagating modes. A condition for rotation sensing is then analytically derived, and an experimental confirmation of this theoretical investigation is reported. It is eventually pointed out that the two-level toy model developed in [9] to describe a superfluid placed in a rotating ring container leads to a similar rotation sensitive operation condition, opening the way to a possible generalization of this investigation to the case of other resonant macroscopic quantum devices.

The laser equations are derived in the framework of Maxwell-Bloch theory using abiabatic elimination of the polarization [12]. We assume a single identical mode in each direction of propagation and neglect transverse effects. The total field inside the cavity is taken as the sum of two counter-propagating waves :

$$E(x,t) = \mathrm{Re}\left\{\sum_{p=1}^{2} \tilde{E}_p(t) e^{i(\omega t + \varepsilon_p kx)}\right\} ,$$

where $\varepsilon_p = (-1)^p$ and $k = 2\pi/\lambda$ is the spatial frequency of the emitted modes associated with the longitudinal coordinate x. Using the slowly-varying enveloppe approximation, the following equations are obtained [13] :

$$\frac{\mathrm{d}\tilde{E}_{1,2}}{\mathrm{d}t} = -\frac{\gamma_{1,2}}{2}\tilde{E}_{1,2} + i\frac{\tilde{m}_{1,2}}{2}\tilde{E}_{2,1} + i\varepsilon_{1,2}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\sigma(1-i\delta)}{2T}\left(\tilde{E}_{1,2}\int_0^l N\mathrm{d}x + \tilde{E}_{2,1}\int_0^l Ne^{-2i\varepsilon_{1,2}kx}\mathrm{d}x\right) , \quad (1)$$

$$\frac{\partial N}{\partial t} = W_{\mathrm{th}}(1+\eta) - \frac{N}{T_1} - \frac{aN|E(x,t)|^2}{T_1} , \quad (2)$$

where $\gamma_{1,2}$ are the cavity mode losses per time unit (we will first assume $\gamma_1 = \gamma_2 = \gamma$), σ the stimulated emission cross section, T the cavity round trip time, l the length of the gain medium, $N(x,t)$ the population inversion density function, η the excess of pump power above the threshold value W_{th}, T_1 the population inversion relaxation time and a the saturation parameter. The rotation-induced angular eigenfrequency difference Ω between the counter-propagating modes is linked to the angular velocity $\dot{\theta}$ by the Sagnac formula [14] :

$$\Omega = \frac{8\pi A}{\lambda c T}\dot{\theta} , \quad (3)$$

where A is the area enclosed by the ring cavity and c the speed of light in vacuum. The parameter $S = 4A/(\lambda c T)$ is known as the scale factor of the cavity. The detuning of the cavity from the center of the gain line is defined as $\delta = (\omega_c - \omega_{ab})/\gamma_{ab}$, where $\omega_c = kc$ is the resonance frequency of the modes and ω_{ab} and γ_{ab} are respectively the position of the center and the width of the gain line. For solid-state gain media, δ is usually smaller than 10^{-2}. Its effects (among which is the dispersion of the refractive index) will therefore be neglected in our analysis.

Because of the scattering of light induced by the mirrors and by the amplifying crystal, a fraction of the power of each mode is injected back into the other, resulting in mutual coupling. This effect has been taken into account phenomenologically in equation (1) using the backscattering coefficients $\tilde{m}_{1,2} = me^{i\varepsilon_{1,2}\theta_{1,2}}$. It can cause phase synchronization and intensity stabilization of the counter-propagating modes. The strength of this coupling decreases when the speed of rotation increases, as the difference between the eigenfrequencies of the counter-propagating modes becomes more and more important. Note that even if the backscattering coefficient is usually small (i.e. $m \ll \gamma\eta$), this coupling still has to be taken into account for a correct description of the modes dynamics.

Another source of coupling is caused by the establishment of a population inversion grating in the gain medium, created by the light pattern resulting from the interference between the counter-propagating waves. This coupling corresponds in equation (1) to the term proportional to the spatial Fourier transform of N at the order $2k$, and can be interpreted as resulting from backward diffraction on the grating. When the speed of rotation increases, the contrast of the grating (hence the coupling strength) decreases because of the movement of the light interference pattern and of the inertia of the gain medium. More precisely, under the conditions :

$$|\Omega| \gg \sqrt{\frac{\gamma\eta}{T_1}} , m \qquad \text{and} \qquad \eta \ll 1 , \quad (4)$$

a perturbation method applied to equations (1) and (2) shows that the amplitudes of the counter-propagating modes are coupled through the gain-induced effective coupling coefficient $\tilde{N}$ given by :

$$2\tilde{N} = \frac{\gamma\eta}{1 + \Omega^2 T_1^2} . \quad (5)$$

The presence of the grating hinders the coexistence of the two counter-propagating modes because it results in the cross-saturation parameter being bigger than the self-saturation parameter which leads, in the absence of other coupling source, to single mode (unidirectional) operation [10].

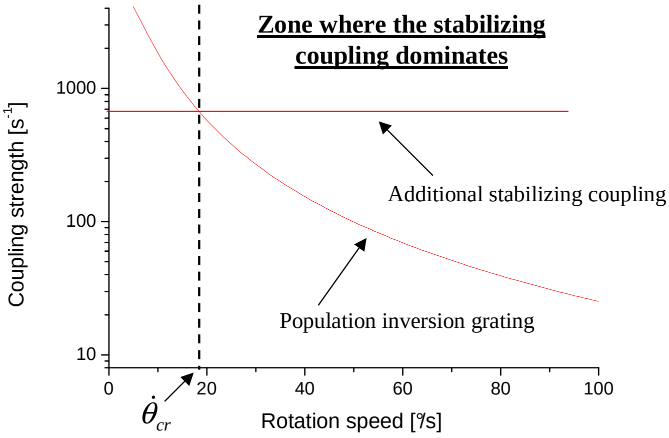

Figure 1: Effective coupling coefficients $\tilde{N}$ and ηK as a function of the rotation speed $\dot{\theta}$ (logarithmic vertical scale). The two ranges of rotation speed are delimited by the value $\dot{\theta}_{cr}$, given by equation (7). Below this value the coupling due to the population inversion grating dominates, while above this value it is the additional stabilizing coupling that dominates.

In order to counteract this effect and to achieve rotation sensitive operation, a stabilizing additional source of coupling is provided to the system. This is done by producing losses which depend on the intensity difference between the counter-propagating modes according to the following law :

$$\gamma_{1,2} = \gamma - \varepsilon_{1,2} K \left(a|\tilde{E}_1|^2 - a|\tilde{E}_2|^2 \right) , \quad (6)$$

where K is chosen to be positive so that the mode with the higher intensity gets the higher losses. The associated effective coupling coefficient is on the order of $K\eta$. A concrete manner of generating such losses will be detailed further in this Letter.

Using typical values of the parameters, we plotted $\tilde{N}$ and $K\eta$ as a function of $\dot{\theta}$ on figure 1. Two ranges of rotation speed rise, delimited by the following critical value :

$$\dot{\theta}_{cr} = \frac{1}{2\pi S T_1}\sqrt{\frac{\gamma}{K} - 1} . \quad (7)$$

In the zone where $\dot{\theta} < \dot{\theta}_{cr}$ the coupling generated by the population inversion grating dominates, while in the zone where $\dot{\theta} > \dot{\theta}_{cr}$ it is the additional stabilizing coupling that dominates. This latter zone turns out to be the zone of rotation sensitive operation, as we will see further.

To proceed, equations (1) and (2) have been solved under the conditions (4). We looked for a solution corresponding to the beat regime, i.e. obeying the following

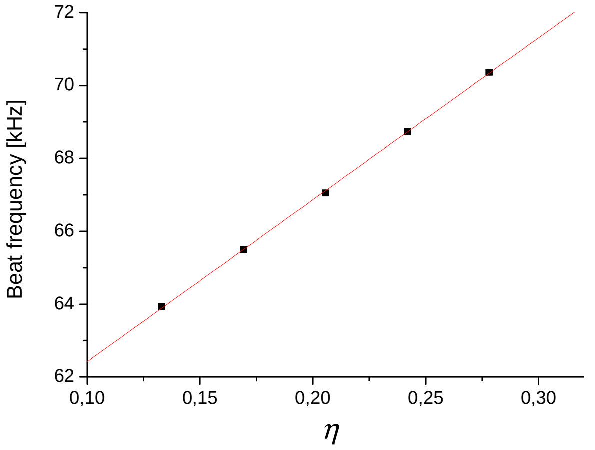

Figure 4: Beat frequency as a function of the pumping rate for $\dot{\theta} = 70°/s$ (error bars are smaller than 100 Hz). The observed lienar shift, in agreement with equation (11), is a direct manifestation of the mode coupling due to the population inversion grating in the solid-state ring laser.

the Sagnac line. This deviation is partly due to the linear coupling induced by backscattering on the mirrors and on the Nd-YAG crystal (Adler pulling) and partly due to the coupling induced by the population inversion grating (this last phenomenon can be interpreted as resulting from Doppler shift on the moving population inversion grating). The beat frequency $< \dot{\Phi} >$ (where $<>$ stands for time averaging) can be expressed analytically under the condition (8), leading to the following formula :

$$< \dot{\Phi} >= \Omega + \frac{m^2 \cos(\theta_1 - \theta_2)}{2\Omega} + \frac{\gamma\eta}{2\Omega T_1} . \qquad (11)$$

This expression is in good agreement with our experimental data for $\dot{\theta} \gtrsim 50°/s$ (see figure 3). The linear dependance of $< \dot{\Phi} >$ on the pump power (represented by the parameter η) for a fixed rotation speed has been checked experimentally. The result is given for $\dot{\theta} = 70°/s$ on figure 4. This is a direct manifestation of the coupling between the counter-propagating modes induced by the population inversion grating in a solid-state ring laser.

Before concluding, it is worth noting that the physical system composed of a superfluid in a ring container has some similarities with the system described in this Letter, at least in the limit of the two-level toy model of [9]. In this model, the condition for the system to be rotation sensitive reads :

$$V_0 > g , \qquad (12)$$

V_0 being the asymmetry energy and g the mean (repulsive) interaction energy per particle in the s-wave state. Condition (12) reflects the fact that a change in the quantum of circulation around the ring, which is a signature of rotation, is more difficult when repulsive interactions are stronger and is easier when the trap is asymmetric. Condition (12) has to be compared with condition (10). In both cases, these conditions express the fact that the system is rotation-sensitive provided that a 'good' additional coupling is stronger than a 'bad' non-linear coupling. We expect that achieving condition (12) in a physical system will lead to new atomic sensors concepts in the same way as achieving condition (10) in an optical system allows the realization of a solid-state ring laser gyroscope.

In conclusion, we have shown that the solid state ring laser provides a good illustration of mode coupling control in a resonant macroscopic quantum device. In particular, rotation sensing has been achieved by the use of a stabilizing mode coupling effect that counteracts the destabilizing effect of the population inversion grating. The simple theoretical model we used is in good agreement with the reported experimental results. This work opens new perspective for the realization of solid state laser gyrometers. In addition, it shows the importance of interplay between mode couplings in a system with periodic boundary conditions. The same concept can be applied in more complex systems like toroidal superfluids, a field in which an important experimental effort is being made at the moment [17].

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