

Anomalous Hall Effect due to the spin chirality in the Kagomé lattice

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We consider a model for a two dimensional electron gas moving on a kagomé lattice and locally coupled to a chiral magnetic texture. We show that the transverse conductivity σ_{xy} does not vanish even if spin-orbit coupling is not present and it may exhibit unusual behavior. Model parameters are the chirality, the number of conduction electrons and the amplitude of the local coupling. Upon varying these parameters, a topological transition characterized by change of the band Chern numbers occur. As a consequence, σ_{xy} can be quantized, proportional to the chirality or have a non monotonic behavior upon varying these parameters.

I. INTRODUCTION

In ferromagnetic systems, there are two contributions to the transverse resistivity ρ_{xy} : one is due to the usual Lorentz force acting on the electrons when a magnetic field is applied, $R_0 B$, the second one, $R_s M$, is proportional to the magnetization of the ferromagnet. This is called the anomalous Hall effect.

The origin of this anomalous Hall effect has long been controversial and both extrinsic (impurities) or intrinsic mechanisms were discussed. Karplus and Luttinger¹ proposed that this effect is a consequence of spin-orbit interaction in metallic ferromagnets. Then it was argued that impurities give the main contribution to the anomalous Hall effect and usually two mechanisms, both due to spin-orbit coupling, contribute: one is known as side-jump mechanism^{2,3} and it predicts that the Hall resistivity R_s is proportional to the longitudinal resistivity ρ . The second one^{4,5} is the skew-scattering mechanism, which gives a contribution proportional to ρ^2 .

In the recent years several groups measured anomalous Hall effect in various systems which cannot be attributed to usual mechanisms (skew-scattering or side-jump). A new intrinsic mechanism, related to non-collinear spin configuration, with a ferromagnetic component, was first proposed for manganites^{6,7,8}: in these systems deviation from collinearity is a consequence of the competition between double exchange, superexchange and spin-orbit interactions. Then it was proposed that a similar mechanism works in spin glass systems where spin configuration is highly non coplanar^{9,10}: in the weak coupling limit it was shown that the Hall resistivity is proportional to the uniform chirality parameter with a sign which depends on the details of the band structure. This chirality parameter was defined by Tatara and Kawamura⁹ for three spins at sites i, j, k as a scalar quantity: $\chi = \langle \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) \rangle$, being non-zero if the spins are non-coplanar. Very recently several groups confirmed the existence of such a contribution in typical AuFe spin glasses^{11,12}.

In non-disordered systems, the same mechanism can work as soon as the ordered magnetic structure is non-coplanar. This is realized in several pyrochlore compounds such as $\text{Nd}_2\text{Mo}_2\text{O}_7$ for which Taguchi et al¹³ pro-

posed that an anomalous contribution to the Hall effect is related to the umbrella structure of both Nd and Mo moments in the long range ordered phase. Detailed studies of this system and other pyrochlores ($\text{Sm}_x\text{Y}_{1-x}\text{Mo}_2\text{O}_7$, $\text{Nd}_x\text{Y}_{1-x}\text{Mo}_2\text{O}_7$) have been performed^{13,14,15} leading to some controversy: using neutron diffraction experiments in magnetic field it is possible to calculate the T and H dependence of both Mo and Nd chiral order parameter and in Ref. 15 it was concluded that the chiral mechanism alone cannot explain the T-dependence of the Hall coefficient in pyrochlores.

From the theoretical point of view, this mechanism is often called "Berry phase contribution" because a non-vanishing spin chirality is associated with a non-vanishing Berry phase for conduction electrons coupled via local Hund's exchange interaction to the spins. Ohgushi et al¹⁶ calculated the Hall effect in a Kagomé lattice with a non-coplanar long range spin structure. They have shown that, in the adiabatic limit, when the electron conduction spins are aligned with localized spins at each site of the lattice (this corresponds to infinite Hund's coupling), the Berry phase contribution to Hall conductivity is quantized for some values of the band filling. In this paper, we study a model which extrapolates between this strong coupling limit and the weak coupling case studied by Tatara and Kawamura⁹. We show also that the Berry phase contribution does not depend only on the chirality, but also on the strength of the local Hund's coupling and on the band filling.

II. MODEL

A. Hamiltonian

The aim of this work is to show how the transport properties of electrons are influenced by two different contributions. First, the electrons are restricted to move on a lattice and are therefore experiencing its geometry. This results in a peculiar band structure. Second, each electron has its spin locally coupled to a given distribution of magnetic moments on each site of the lattice.

Both effects are taken into account in the following Hamiltonian:

$$\mathcal{H} = \sum_{\langle i,j \rangle, \sigma} t_{ij} \left(c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right) - J \sum_i c_{i\alpha}^\dagger (\boldsymbol{\sigma}_{\alpha\beta} \cdot \mathbf{S}_i) c_{i\beta}. \quad (1)$$

The first term describes the electrons moving on the lattice: t_{ij} is the hopping integral between two neighboring sites i and j ; $c_{i\sigma}^\dagger$ and $c_{i\sigma}$ are the creation and annihilation operators of an electron with spin σ on the site i . The second part of the Hamiltonian couples the electron spin to a local moment on each site.

The coupling constant to each local moment \mathbf{S}_i is J , and these moments are treated below as classical variables. $\boldsymbol{\sigma}_{\alpha\beta}$ are the Pauli matrices. The underlying lattice is the Kagomé lattice, depicted in Fig. 1. It is a two dimensional tiling of corner sharing triangles, described as a triangular lattice of triangles throughout this article.

This Hamiltonian has already been discussed in the limit of $J \rightarrow \infty$ by Ohgushi et al.¹⁶ In this limit the two $\sigma = \uparrow, \downarrow$ bands are infinitely splitted and the model describes a fully polarized electron gas subject to a modulation of a fictitious magnetic field, corresponding to the molecular field associated with the magnetic texture. The comparison with the present work will be done later on. In a general case of finite J , the calculations must be done numerically. It is worth noting that the sign of J is unimportant in this classical treatment since changing J to $-J$ is equivalent to an exchange of \uparrow and \downarrow spin states.

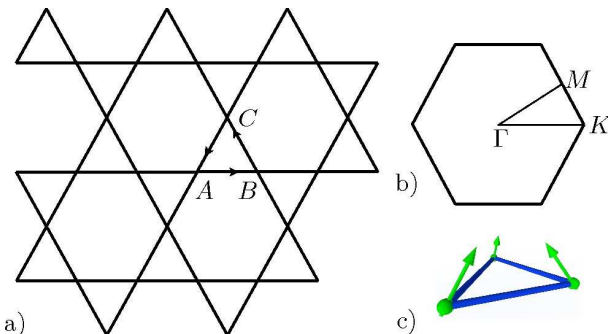


FIG. 1: (a) The Kagomé lattice is described as a triangular lattice of triangles. Its Bravais vectors are $a_1 = (1, 0)$, $a_2 = (-1/2, \sqrt{3}/2)$, and $a_3 = (1/2, \sqrt{3}/2)$, connecting respectively, the points A to B , B to C , and C to A . (b) The first Brillouin zone is a hexagon with the corners located at $k = \pm(2\pi/3) a_1$, $k = \pm(2\pi/3) a_2$ and $k = \pm(2\pi/3) a_3$. (c) The umbrella structure on the triangular cell of the kagomé lattice.

B. Parameters and motivation

We consider the spin texture as a 'parameter' of the model. The local moments \mathbf{S}_i are described classically, and the chosen magnetic phase is periodic, thus allowing to work in the reciprocal space. The choice of the magnetic arrangement is motivated by two reasons. First,

we are interested in describing the anomalous transport properties of the type already addressed in previous works (see Refs. [14,16,17]) in which it has been argued that the transverse conductivity may be related to the chirality of the magnetic phase. Second, an unusual mechanism of the AHE has recently been proposed, based on an unconventional chiral magnetic ordering due to geometrical frustration^{14,18,19,20} but with controversial interpretations. We therefore consider the present model as a simple model to address these problems and clearly identify each relevant contribution.

We consider the magnetic phase which has been obtained by studying a pure spin model with the anisotropic Dzyaloshinskii-Moriya interactions on the Kagomé lattice²¹. It consists of an umbrella of three spins per unit cell of the Kagomé lattice (see Fig. 1). Each umbrella can be described by the spherical coordinates of the three spins $(\pi/6, \theta)$, $(5\pi/6, \theta)$ and $(-\pi/2, \theta)$. The angle θ ranges from 0 (all the spins are perpendicular to plane, and the corresponding ordering is ferromagnetic) to π . It is worth noting that the usual three sublattice planar magnetic phase belonging to the ground state manifold of the kagomé antiferromagnet, and commonly denoted as the "q = 0 phase" corresponds to $\theta = \pi/2$.

In between, all phases are possible and chiral, where chirality is defined as the mixed product of three spins on a plaquette

$$\chi_{ijk} = \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) = \frac{3\sqrt{3}}{2} \cos \theta \sin^2 \theta. \quad (2)$$

All these phases ($\theta \in [0, \pi]$) are therefore translation-invariant but do not have time reversal symmetry.

We consider the local spins as classical spins with length M and we introduce in eq. 1 an effective coupling constant $J_0 = JM$ which allows to rewrite the hamiltonian as

$$\mathcal{H} = \sum_{\langle i,j \rangle, \sigma} t_{ij} \left(c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right) - J_0 \sum_i c_{i\alpha}^\dagger (\boldsymbol{\sigma}_{\alpha\beta} \cdot \mathbf{n}_i) c_{i\beta} \quad (3)$$

where \mathbf{n}_i is a unit vector collinear to the local moment \mathbf{S}_i .

Setting $t = |t_{ij}|$ as the energy unit, our free parameters are the angle θ , parametrizing the chirality of the magnetic texture, and the value of J_0 . The Fermi level, through the filling factor p of the bands, can also be varied. Each of these variables gives a different contribution as will be discussed in the next sections.

C. Physical quantities

Before calculating any physical observables, we have to diagonalize the Hamiltonian of Eq. 1. For this purpose, the Hamiltonian is rewritten in the reciprocal space as

$$\mathcal{H} = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger h_{\mathbf{k}} \Psi_{\mathbf{k}} + h.c., \quad (4)$$

with $\Psi_{\mathbf{k}} = (c_{A\mathbf{k}\uparrow}, c_{B\mathbf{k}\uparrow}, c_{C\mathbf{k}\uparrow}, c_{A\mathbf{k}\downarrow}, c_{B\mathbf{k}\downarrow}, c_{C\mathbf{k}\downarrow})$, A, B and C are the corners of the kagomé lattice unit cell (Fig. 1-(a)) and

$$c_{A\mathbf{k}\sigma} = \sum_j c_{A,j,\sigma} e^{i\mathbf{k}\cdot\mathbf{r}_{ij}}, \quad (5)$$

$$h_{\mathbf{k}} = \begin{pmatrix} -J_0 \cos \theta & p_k^1 & p_k^3 & -J_0 \sin \theta e^{i\frac{\pi}{6}} & 0 & 0 \\ p_k^1 & -J_0 \cos \theta & p_k^2 & 0 & -J_0 \sin \theta e^{i\frac{5\pi}{6}} & 0 \\ p_k^3 & p_k^2 & -J_0 \cos \theta & 0 & 0 & iJ_0 \sin \theta \\ -J_0 \sin \theta e^{-i\frac{\pi}{6}} & 0 & 0 & J_0 \cos \theta & p_k^1 & p_k^3 \\ 0 & -J_0 \sin \theta e^{-i\frac{5\pi}{6}} & 0 & p_k^1 & J_0 \cos \theta & p_k^2 \\ 0 & 0 & -iJ_0 \sin \theta & p_k^3 & p_k^2 & J_0 \cos \theta \end{pmatrix}, \quad (6)$$

with $t_{ij} = t$ and $p_{\mathbf{k}}^i = 2t \cos(\mathbf{k} \cdot \mathbf{a}_i)$.

To calculate the Anomalous Hall Effect, we use the expression of the off-diagonal conductivity using the following Kubo formula in the limit of disorder-free electron gas²²

$$\sigma_{xy}(\omega) = \frac{e^2 \hbar}{S} \sum_{n \neq m} \sum_{\mathbf{k}} (f_{n\mathbf{k}} - f_{m\mathbf{k}}) \times \frac{\langle n, \mathbf{k} | v_x | m, \mathbf{k} \rangle \langle n, \mathbf{k} | v_y | m, \mathbf{k} \rangle}{(\varepsilon_{n\mathbf{k}} - \varepsilon_{m\mathbf{k}})(\varepsilon_{n\mathbf{k}} - \varepsilon_{m\mathbf{k}} - \omega)}, \quad (7)$$

where S is the surface of the unit cell, $\varepsilon_{n\mathbf{k}}$ is the eigenvalue of matrix (6) corresponding to the energy of n th band for the wave vector \mathbf{k} , with the eigenvector $|n, \mathbf{k}\rangle$. Here $v_i = \frac{1}{\hbar} \frac{\partial h_{\mathbf{k}}}{\partial k_i}$ is the velocity of electrons ($i = x, y, z$). Taking the static limit of $\omega \rightarrow 0$, Eq. (7) becomes

$$\sigma_{xy} = \frac{e^2 \hbar}{S} \sum_{n, \mathbf{k}} f_{n, \mathbf{k}} \sum_{m \neq n} \frac{(v_x)^{nm}(v_y)^{nm} - (v_x)^{mn}(v_y)^{mn}}{(\varepsilon_{n, \mathbf{k}} - \varepsilon_{m, \mathbf{k}})^2} = \frac{e^2}{\hbar S} \sum_{n, \mathbf{k}} f_{n, \mathbf{k}} [\nabla_{\mathbf{k}} \times \mathbf{A}_{n, \mathbf{k}}]_z, \quad (8)$$

where $\mathbf{A}_{n\mathbf{k}} = -i \langle n\mathbf{k} | \nabla_{\mathbf{k}} | n\mathbf{k} \rangle$ is the geometric vector potential and $f_{n\mathbf{k}}$ is the Fermi-Dirac distribution function.

The expression (8) is similar to that obtained by Thouless *et al.*²³ in the context of the Quantized Hall effect in two dimensions. In particular, when the band is completely filled, the circulation of $\mathbf{A}_{n\mathbf{k}}$ or the flux of Berry curvature defined as $\Omega_{n\mathbf{k}} = \nabla_{\mathbf{k}} \times \mathbf{A}_{n\mathbf{k}}$ over the first Brillouin zone is equal to $2\pi i$ times an integer^{23,24} called the Chern number ν_n . Expression (8) of the off-diagonal conductivity finally becomes

$$\sigma_{xy} = \frac{e^2}{\hbar S} \sum_{n\mathbf{k}} f_{n\mathbf{k}} \Omega_{n\mathbf{k}} = \frac{e^2}{h} \sum_n \nu_n, \quad (9)$$

where $\nu_n = \frac{1}{2\pi} \int f_{n\mathbf{k}} \Omega_{n\mathbf{k}} d^2\mathbf{k}$. As we shall see, this implies that the Hall conductivity is quantized when the Fermi level lies in a gap.

is the Fourier transform of $c_{A,j,\sigma}$ and $r_{Aj} = r_A + R_j$ with R_j the lattice vector and r_A the position of the moment \mathbf{S}_A in the unit cell. $h_{\mathbf{k}}$ is a 6×6 matrix given by

The Chern numbers have the following properties: (i) the sum over all bands is equal to 0, (ii) if for some peculiar values of parameters of the model, the energy bands m and n cross each others at some point of the Brillouin zone, their respective Chern number obey the following conservation rule

$$(\nu_m + \nu_n)_b = (\nu_m + \nu_n)_a \quad (10)$$

where the indices b, a refer to the values of the Chern number before and after the bands are touching each other.²⁵ We will check each of these conservation rules in the next sections.

It should be noticed that the sum over \mathbf{k} in Eq. (9) runs over all occupied states. This may seem unusual as in metallic systems we expect that only the quasiparticles near the Fermi surface contribute to transport properties. However, it has been recently shown by Haldane²⁶ that Eq. (9) can be reconciled with this point of view and reduced to a sum over the states near the Fermi energy.

III. NUMERICAL RESULTS

This section is divided in two parts. In the first one, the band structure is calculated for different angles θ and coupling constants J_0 . The evidences for two different regimes of couplings are given, illustrated by the variation of Chern numbers. In the second one, we focus on the transverse conductivity at zero temperature. It is shown that the chirality may be a relevant quantity for describing the transverse conductivity ($\sigma \propto \chi$) in some ranges of coupling only for a half-filled band.

A. Band structure and associated Chern numbers

The energy spectrum of the Kagomé lattice in the absence of exchange interaction, is characterized by one flat band at $E = 2t$ and two dispersive bands, which

touch each other at the \mathbf{k} -point K of the Brillouin zone as shown in Fig. 2. This peculiarity has already been addressed in the context of spin models as well as for the metallic systems²⁷.

Once the coupling constant J_0 is nonzero, the energy spectrum splits in two parts due to the spin dependant potential. For very large values of J_0 , the spectrum is divided into two groups of three bands, which is the case studied by Ohgushi *et al.*¹⁶. In between, for nonzero but not too large values of J_0 , the spectrum can only be computed numerically, except for the special cases of $\theta = 0$ or π , or for general θ at high symmetry points. For these intermediate values of J_0 , the splitting of the spectrum depends qualitatively on two mechanisms.

First, the coupling J_0 gradually separates each group of three bands taking them degenerate for $J_0 = 0$ to fully separated for infinite J_0 . Second, within each group of three bands, point like degeneracies are lifted (see Fig. 2, points Γ and K) when switching on J_0 , and finally restored for $J_0 \rightarrow \infty$ (see Ref. 16). The numerical calculation of the spectrum shows that it is either gapless for small values of J_0 , or has gaps for higher values.

When gaps open, it always occurs at the M point of the Brillouin zone. This allows to compute analytically the critical value of the coupling as a function of chirality parameterized by the angle θ ,

$$J_c(\theta) = \pm \frac{2}{\sqrt{1 + 3 \cos^2 \theta}}. \quad (11)$$

Using Eq. (11) we can distinguish between two different regimes (Fig. 2) depending on the value of J_0 as compare to $J_c(\theta)$. These regimes are characterized by a particular organization of the band structure as shown in Figs. 2-(b) for $J_0 = t$ and 2-(c) for $J_0 = 2t$ and different Chern numbers. For $J_0 > J_c(\theta)$, the Chern numbers associated to each band are given by $-1, 0, 1, 1, 0, -1$ from the lower to the upper band. Those numbers were obtained by Ohgushi *et al.*¹⁶ in the limit of $J_0 \rightarrow \infty$. When $J_0 < J_c$, the Chern numbers associated to this configuration are $-1, 3, -2, -2, 3, -1$ given in the same order as in the previous case. The global sum rule $\sum_n \nu_n = 0$ is always verified as well as the local one around the critical coupling where two pairs of bands are crossing each other. It reads as

$$\nu_2 + \nu_3 = 1 \quad \text{and} \quad \nu_4 + \nu_5 = 1 \quad (12)$$

below and above the critical coupling J_c , respectively. The corresponding phase diagram is shown on figure 3.

B. Off-diagonal conductivity at $T = 0$

The off-diagonal conductivity is computed using Eq. (7) at $T = 0$ for different values of the chirality θ and the amplitude of the exchange interaction J_0 . As shown in Fig. 4 (the corresponding energy spectrum is presented in Figure 2,(b) and 2,(c)) at $T = 0$, the Hall

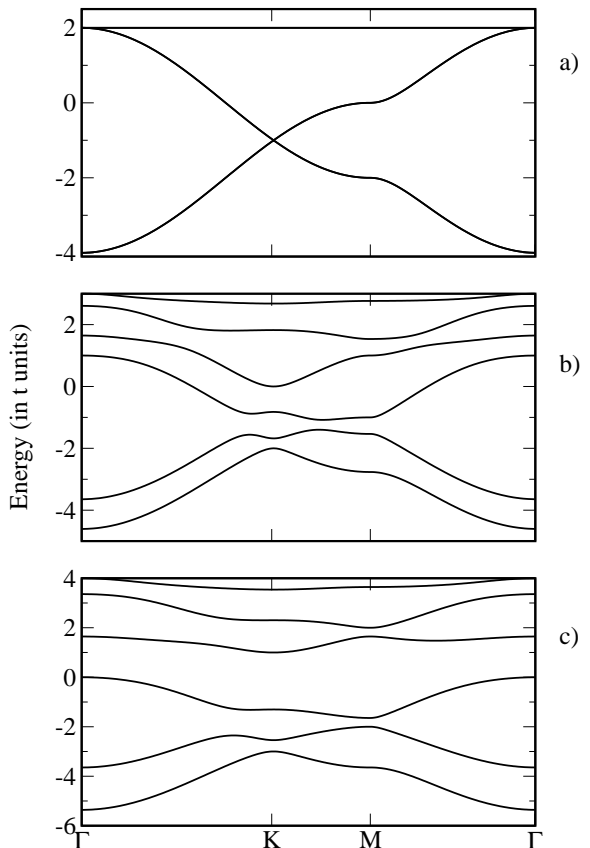


FIG. 2: (a) Energy spectrum of (14) calculated for $J_0 = 0$. Each band is twice degenerate due to the spin degeneracy. (b) Energy spectrum calculated for $J_0 = t$ and $\theta = \pi/3$. (c) Energy spectrum calculated for $J_0 = 2t$ and $\theta = \pi/3$. The critical value J_c is equal to $4t/\sqrt{7} \approx 1.51 t$

conductivity is quantized when the Fermi level is lying in the gap. The value of the plateaus depends explicitly on the Chern number of the filled bands. When the Fermi level is in a band, the sum (9) can be reformulated in terms of a sum over the Fermi level as shown by Haldane²⁶ recently. One has to note that when the Fermi level and θ are fixed, any variation of J_0 around $J_c(\theta)$ may induce a jump in the Hall conductivity. For instance with a filling factor $p = 1/3$ (Fig. 5-a), the Fermi level always lies in a gap but the off-diagonal conductivity jumps from $-e^2/h$ for $J_0 > J_c$ to $2e^2/h$ for $J_0 < J_c$.

A similar jump can be obtained if θ is changed, while J_0 is constant: such a change in θ can for example be induced by application of an external magnetic field perpendicular to the kagomé plane. In this case, large change of Hall conductivity (even sign change) can be induced by magnetic field.

This is not the case for $p = 1/2$. For that filling factor, the Hall conductivity varies smoothly when J_0 passes through the point $J_0 = 3t/2$ which does not depend on θ (Fig. 5-b). When $J < 3t/2$, the Fermi level crosses

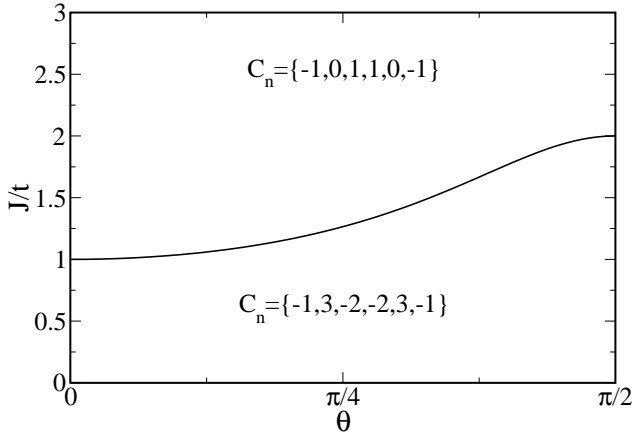


FIG. 3: The phase diagram in $J_0 - \theta$ variables. The black line is given by Eq. (11) and corresponds to the vanishing gap between the bands 2 and 3 (the gap between the bands 4 and 5 is also vanishing).

the bands three and four resulting to a nonzero Hall conductivity. When $J > 3t/2$, the Fermi level is in the gap separating the band three and four and the off-diagonal conductivity is given by the sum off the Chern number of the first three bands which gives zero.

Fig 5 shows the variation of the Hall conductivity as a function of J_0 for filling factors $1/4$ (Fig. 5-c) and $1/5$ (Fig. 5-d): in both cases the Fermi level is not in a gap and Hall conductivity does not exhibit any jump.

From these comments, it is clear that the chirality is never a relevant parameter for the Hall conductivity when the Fermi level lies in a gap, because σ_{xy} is then quantized and obviously, not proportional to the chirality.

Conversely, it is possible to choose a filling factor such that the Fermi level is not within a gap. In Fig. 6, we present σ_{xy} versus θ for the filling factor $p = 1/2$ (a) and (b) and different values of J_0 . For this filling factor, the Fermi level is not in a gap if $J_0 < 3t/2$. Scaling the conductivity by its maximal value in each case shows that it is proportional to chirality in a large range of coupling J_0 , but also for large values of the chirality. This is somewhat unexpected as the previous studies relating σ_{xy} to χ were restricted to small values of χ^9 . However, for different values of the band filling this is no longer true as shown on figure 6 for $p = 1/4$ (6-c) and $p = 1/5$ (6-d): for these filling factors, σ_{xy} is sensitive to the variation of the Chern numbers when J_0 varies. In fact we found that the Hall conductivity is proportional to chirality only for $p = 1/2$ and small enough J_0 .

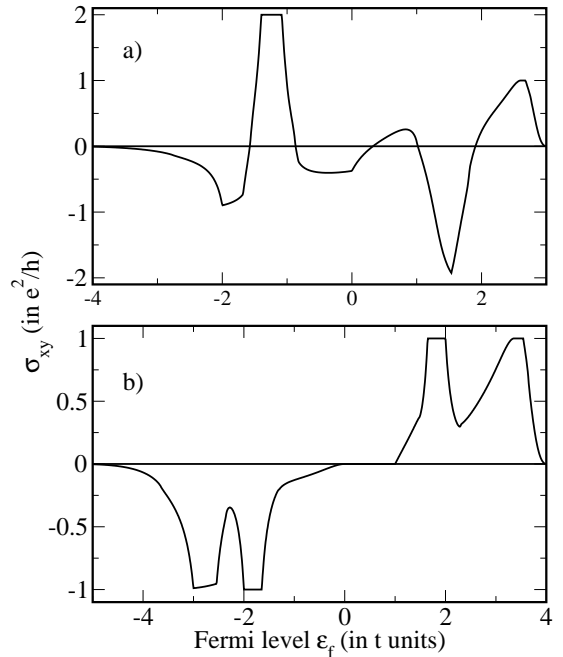


FIG. 4: Off-diagonal conductivity evaluated at $T = 0$ as a function of ϵ_f for $\theta = \pi/3$ and (a) $J_0 = 2t$ (b) $J_0 = t$. In both cases, σ_{xy} is quantized when the Fermi level is lying in a gap. The values of the plateaus depend strongly on the topology of the energy spectrum.

IV. DISCUSSION

The model studied in this paper cannot be directly applied to materials, in which the Anomalous Hall Effect attributed to the spin chirality has been observed.

However, some interesting features have been obtained in this model. This work shows that transverse conductivity may be proportional to chirality in a strong coupling regime, extending previous results obtained in the weak coupling case⁹ but this occurs only for a half filled band. It is also shown that the dependence of σ_{xy} is not only determined by the chirality but also depends strongly on the exchange coupling parameter and on the band filling factor. σ_{xy} can present a large variety of behaviors: non-monotonic variation, sign change or plateaus can be observed. Thus, even if the underlying magnetic phase chirality is the main origin of this intrinsic anomalous Hall effect, it is far from being entirely determined by the chirality.

We suggest that in real systems, variation of coupling J can be induced by temperature: the temperature acts on the system in many different ways. It acts on the magnetic configuration of the localized spins \mathbf{S}_i by changing the amplitude of the magnetization, and on the position of the Fermi level in the band through the Fermi-Dirac distribution.

We assume that the magnitude of each local moment

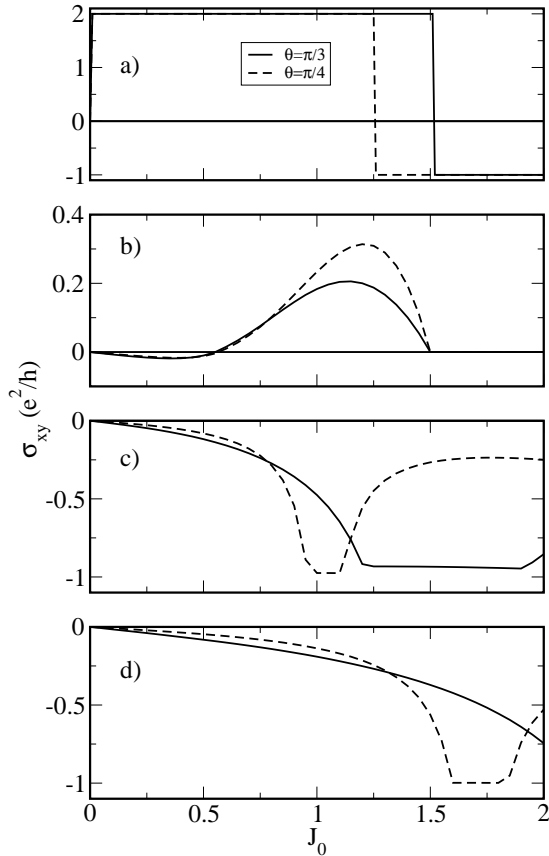


FIG. 5: Off-diagonal conductivity $\sigma_{xy}(J)$ evaluated at $T = 0$ for a filling factor $p = 1/3$ (a), $p = 1/2$ (b), $p = 1/4$ (c) and $p = 1/5$ (d) and $\theta = \pi/3$ (continuous lines) and $\theta = \pi/4$ (dashed lines).

follows the mean field relation

$$M(T) = |\mathbf{S}_i| = M_0 \sqrt{1 - \frac{T}{T_c}}. \quad (13)$$

where T_c is the critical temperature. This can be described by introducing in eq. 1 a temperature dependent coupling constant

$$J(T) = J_0 \sqrt{1 - \frac{T}{T_c}}. \quad (14)$$

where $J_0 = JM_0$ is the zero temperature exchange constant and J_0 and T_c are supposed to be independent.

Consequently, decreasing the temperature from T_c to $T = 0$ is equivalent to increase the exchange coupling from $J = 0$ to $J = J_0$. This increase of J can be considered as done at zero temperature since in a large range of parameters, the additional effect of temperature (i.e. the T -dependence of the Fermi function in Eq. (8)), is significant only when the Fermi level is very close to a band edge.

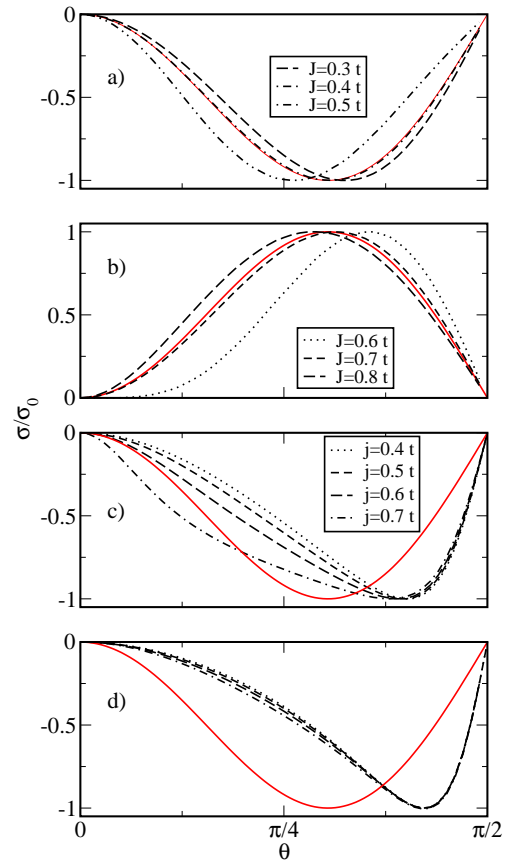


FIG. 6: σ_{xy}/σ_0 with $\sigma_0 = \max(|\sigma_{xy}|)$ versus θ angle, calculated for a filling factor $p = 1/2$ (a) and (b) and $p = 1/4$ (c) and $p = 1/5$ (d) and different values of J_0 . The continuous line represents the chirality given by (9). The Hall conductivity is represented by the dashed lines for different values of the parameter J_0 . We remark that σ_{xy} changes sign when J_0 tends to $0.6t$. That change is related to the position of the Fermi level within the band but not associated to change of the Chern numbers. It is remarkable that the conductivity is more or less proportional to the chirality in a large range of couplings, up to $J_0 \approx t$ as well as for large χ values. For $p = 1/4$ and $p = 1/5$, one can show that the off-diagonal conductivity is not proportional to the chirality even for small values of J_0 .

Qualitatively, this means that when temperature is decreased, one moves vertically from the bottom to the top of the phase diagram in Fig. 3: if J_0 is large enough, the temperature decrease produces at some peculiar temperature T_* such that $J(T_*) = J_c(T = 0)$, a change in conductivity due to the change of the Chern numbers. From this observation it follows that σ_{xy} can show abrupt changes with temperature. It is worth noting that during that process, in the frame of our model, the magnetic texture characterized by its θ angle has *not* moved. Of course, a change in θ can also be responsible for a sign change.

To be more specific, we consider several examples presented in Fig. 7.

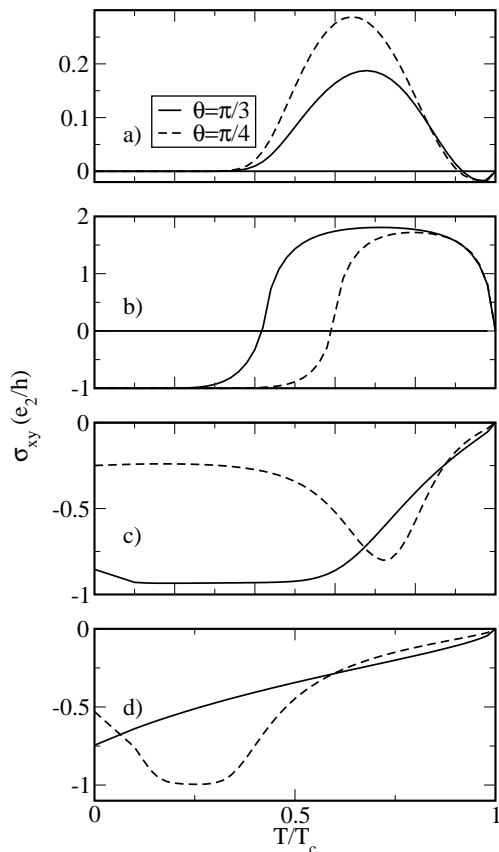


FIG. 7: Off-diagonal conductivity $\sigma_{xy}(T)$ for a filling factor $p = 1/2$ (a), $p = 1/3$ (b), $p = 1/4$ (c) and $p = 1/5$ (d) and $\theta = \pi/3$ (continuous lines) and $\theta = \pi/4$ (dashed lines).

We start with the filling factor $p = 1/2$, the critical temperature $T_c = 0.1t$ and $J(T = 0) = 2t$ (Fig. 7-a). With this choice of parameters, the Fermi level lies in a gap when $J_0 > 3t/2$, which means that at low temperatures (i.e., for large J_0), the system is insulating. When the temperature is increased, σ_{xy} remains equal to zero until T reaches the temperature T_1 for which $J(T_1) = 3t/2$. Above this temperature, the gap closes, making the transverse conductivity increasing continuously. It reaches an extremum which is a function of the texture angle θ . Just below T_c , σ_{xy} changes sign because of a subtle balance between the states giving positive and negative contributions to the conductivity,

namely, the states from bands 3 and 4 (negative contribution) and the states from band 2 (positive contribution). These contributions may be globally explained by the associated Chern numbers of these bands, at small J_0 : $\nu_3, \nu_4 = -2$ and $\nu_2 = 3$.

For the filling factor $p = 1/3$ (Fig. 7-b), the behavior is completely different. The transverse conductivity is quantized and finite at zero temperature as the Fermi level is located in a gap. When the temperature increases and reaches the value T_* defined by $J(T_*) = J_c(\theta)$, for which gaps are finite but sufficiently small to authorize the interband processes, σ_{xy} continuously increases. Upon increasing temperature, the conductivity increases and then changes its sign. This change of sign is not of the same origin as we considered for $p = 1/2$. In the first case, the Chern numbers are fixed but the temperature changes the balance between the weights of each band. In the second example, bands 3 and 4 get crossed thus changing their Chern numbers. The variation of Chern numbers produces the sign change in the transverse conductivity. Due to thermal fluctuations, the extremum of conductivity does not reach its quantized value of 2 (in units of e^2/h) but saturates at 1.8. The conductivity finally decreases down to zero when T reaches T_c , as it should be.

It is also possible to choose the filling factors that place the Fermi level at zero temperature within a band. In these cases, there is no generic behavior, and the angle of the spin texture plays an important role. Some results are shown on figure 7 for the different filling factors $p = 1/4$ (c) and $p = 1/5$ (d).

Thus, these results show that in the mean field approximation a large variety of behaviors are possible in our model. The next step would be to study a model closer to the experimental situation of pyrochlores, where the variation of magnetic structure with temperature and applied field has been studied by neutrons experiments¹⁵, allowing to take into account the real crystallographic and magnetic structures of these systems.

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