

## Learning by plotting

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### Abstract

An important characteristic of ict-tools is that they do things more neatly and quickly than we could by hand, and that they do not mind correcting errors. These machines entail less work and more time for things like thinking and learning. If that were all, the advantage would be undeniable. For education however, an ict-tool should do more; it should benefit and promote thinking and learning. A graphics program for educational purposes should offer more than a simple plotting of graphics. The dynamic use of a parameter that is possible in modern software for instance, is a great help for students investigating functions.

### Introduction

When I was a student, a long time ago, there were posters with huge graphs in the classroom, like the one of figure 1, illustrating the importance of mathematics. Doing mathematics at that time often involved, as far as functions were concerned, investigating a function in order to make the graph of that function. The graph was a result of mathematical work. And I liked it, because a graph was something new, different from the algebra which was a major part at that time.

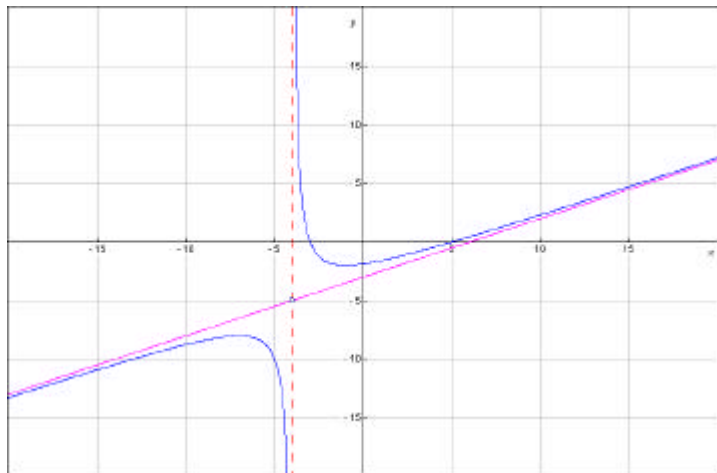


Figure 1

Graph of

$$f(x) = \frac{x^2 - 2x - 15}{2x + 8}$$

Nowadays a plotter, pc or to a certain extent a graphic calculator, gives the result right away. The mathematical work is just typing in a formula. So it is no trouble at all to change the value 8 into 9. This will show a slightly modified graph, but changing it into 5 or 6 the graph will change dramatically. Plotting software via a computer connected to a dataprojector offers a lot of didactical entries and discussions in class.

### Plotting, then and now

The classical way of finding the graph of a function could be given as shown in the next scheme: *function* → *mathematical work* → *graph*

The mathematical work then consists of finding the zeros, extrema, asymptotes, special points etcetera. It costs the student some or much effort and time but the result was a nice plot showing the particularities and even peculiarities of the function.

This mathematical work and the plotting has been taken over by machines and the scheme could be altered into: *graph* → *mathematical activity* → *function*

These mathematical activities come for instance from questions as: why does the graph show what it is showing, how can this be derived from the function? These questions are in fact the same questions as before, only in retrospect. But modern software has not only made plotting an easy job, it also brought the possibility of a dynamic way of plotting. This dynamic plotting enables you to explain in a clear way concepts such as asymptote and parameter. Learning mathematics isn't about learning how to plot a graph but by plotting a graph. I will give now some examples on dynamic plotting.

The best way of course is the use of a computer. On the site [www.vusoft.nl/item2003](http://www.vusoft.nl/item2003) you will find a demoverion of Graphic Calculus (English) and Graphe (French) and also the files of the examples.

### *Example 1 (the fairway)*

In the Netherlands we are dealing with Realistic Math Education, which means that often, but not always, the math is embedded in a realistic situation, that's to say realistic to the students. The first example is like this. It's about a harbour, a harbour at sea so the waterlevel isn't constant but periodically rising and falling. Students can be asked when it is possible for ships to come in or leave. The graphs are plotted slowly from the left to the right. Dynamics in plotting can clarify the situation about inequalities.

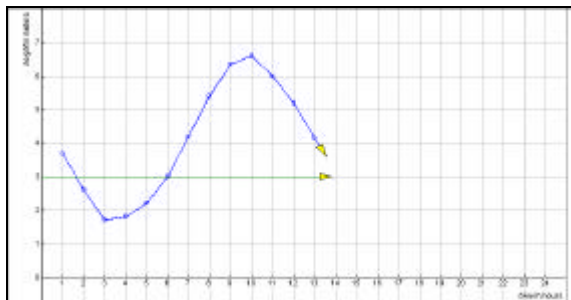


figure 2 the fairway graph

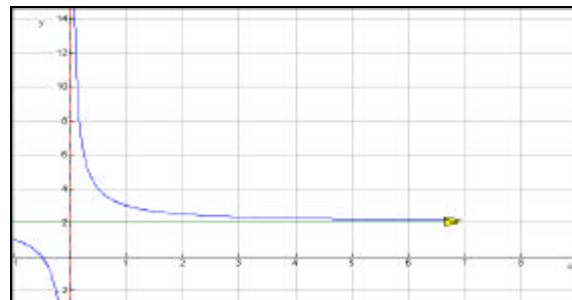


figure 3 the asymptote

### *Example 2 (asymptote)*

The dynamics are also useful in pointing out what is an asymptote

An asymptote isn't so easy, because of the solid state of a plot which usually has the central position where the asymptote is of no importance at all. But the behaviour of a dynamic plot elucidates the question. I ask my students whether the graph and the line meet. Then shifting and zooming, again and again, I will show that, although the graph and the line come nearer, they will never touch. A good image of an asymptote helps to understand.

### **Parameters**

Parameters offer a lot of possibilities for mathematical activities. A distinction can be made between the use as a static and as a dynamic parameter. The static used parameter has values one by one, mostly integers, and the graphs shape a sheaf or a family of graphs. That is why the static parameter is called family parameter. The dynamical used parameter gradually changes and so does the graph. This change can be done by hand with a slider or automatically. That is why this parameter is called sliding parameter (van de Giessen, 2002)

The sliding parameter is a wonderful tool to study functions. In the movement patterns peculiarities will immediately attract attention. Students can be asked to

prove, using the function, what they observed on the graphs. They also can be asked to modify the function in a way that the graphs will show some special properties. I will now give some examples on the use of parameters.

*Example 3 (the sliding graph)*

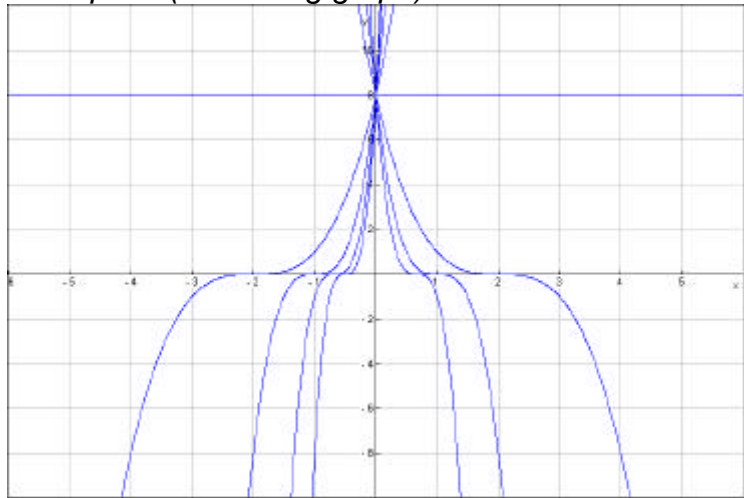


figure 4 the sliding graph

$$f(x) = (2 - ax)^3$$

The function is a simple one, the behaviour all depends on the value of the parameter. Using the sliding parameter it shows that the graph of  $f$  is turning around a point, there is a point of inflexion which moves along the x-axis and this movement slows down as it comes near the origin. I ask my students to prove this using the function. Unusual is the question why the movement is slowing down and so is the request to modify the function in a way that everything is the same but the movement isn't slowing down near the origin. Don't forget that the drawing is showing a sheaf, but in fact there is just one graph moving on the screen.

*Example 4 (two graphs, one parameter)*

The functions  $f(x) = a\sqrt{x}$  and  $g(x) = \sqrt{ax}$  are much alike. For  $a=1$  they are exactly the same. Changing the value of  $a$  the movement of the graph of  $f$  is up and down whereas that of the graph of  $g$  is from the right to the left.

Students are asked to explain the difference in movement from the functions.

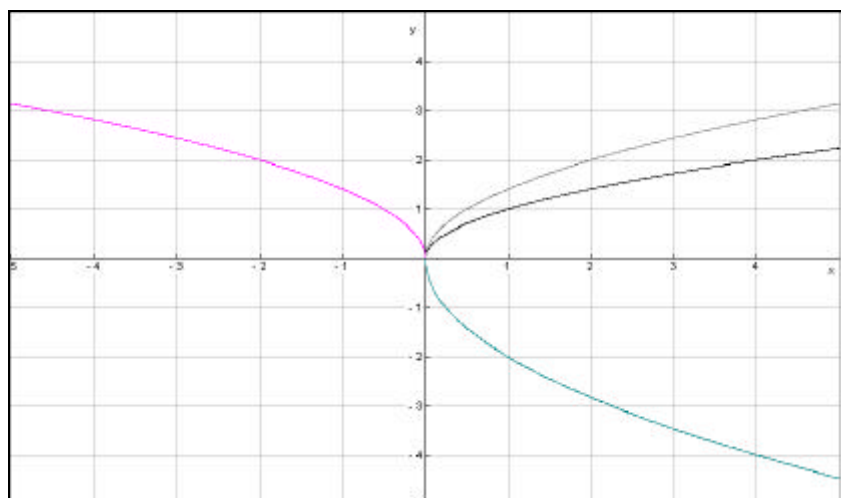


figure 5 two functions, one parameter

*Example 5 (reciprokes)*

In investigating the graphs of  $g(x) = \frac{1}{x^2 + a}$  three types of graphs can be distinguished

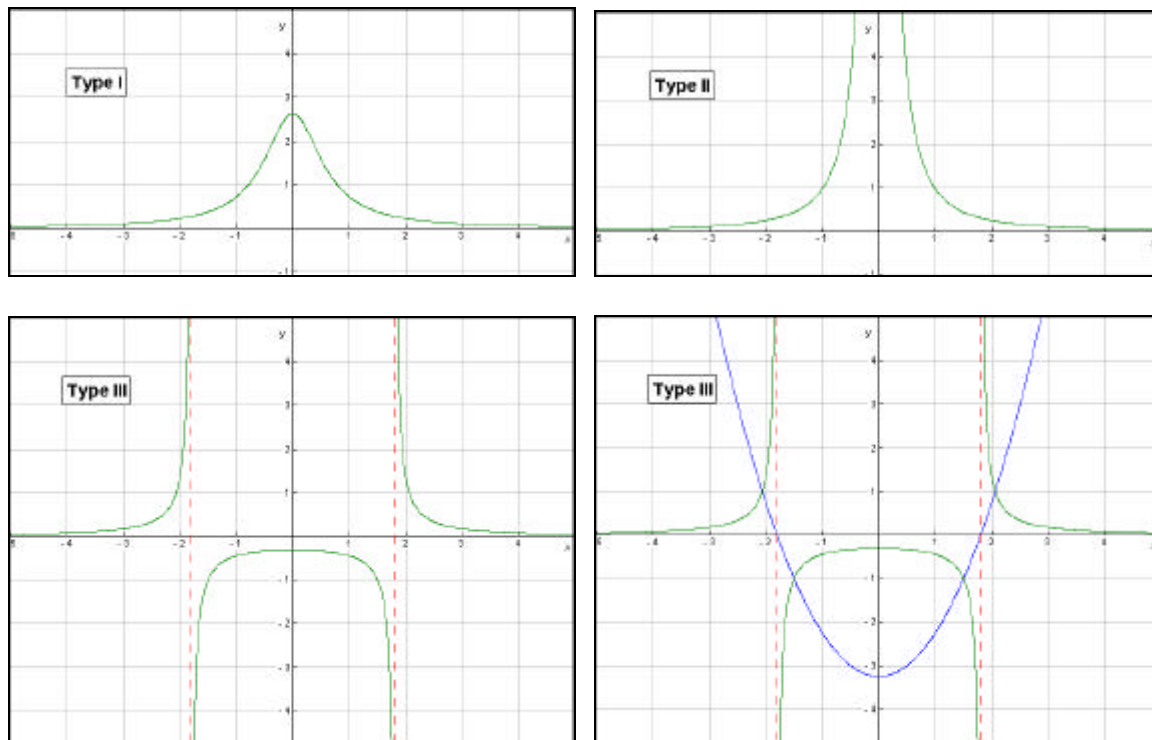


figure 6 the reciprokes

All the properties of the graphs can easily be derived from  $f(x) = x^2 + a$  especially when the graphs are dynamically plotted at the same time.

*Example 6 (the disappearing graph)*

$$f(x) = \sqrt{x^2 - a} \text{ and } g(x) = \sqrt{a - x^2}$$



figure 7 the disappearing graph

A nice example is the one of the disappearing graph. Move the sliding parameter and the graph will disappear and come up again.

To discuss why this happens the student will have to know the difference between a variable and a parameter quite well.

### Example 7 ( the point of no return)

At first sight there is a turning point. But moving the dot along the graph other ideas may rise. Investigation, formulas or graph, will show something different.

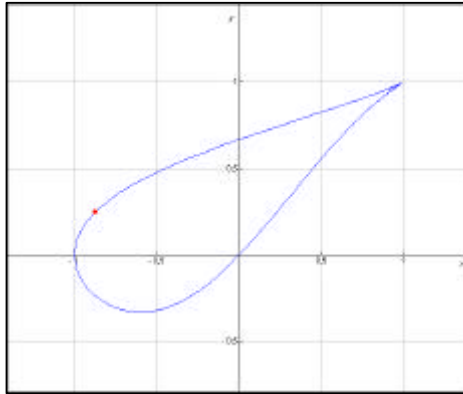


figure 8 the point of no return

$$x = \frac{t^2 - 1}{t^2 + 1}; y = \frac{t^3 - 1}{t^3 + 1}$$

### Concluding remarks

Dynamic plotting gives ways to discover things, could make a lot of things clear, and it is an interesting way of dealing with graphs and functions.

The easy plotting can improve thinking about functions. Students will get an idea about structures and that is much more than a mere plot. The plot isn't the goal, the plot is an intermedium.

In order to do this kind of activities a suitable educational piece of software is required and, for use in the classroom, a dataprojector.

The program should have an interface that allows an intuitive way of working, in my view this means: clear and easy input and edit of formulas, notation of formulas and functions as in the textbooks and is mathematically usual (compare for instance

$f(x) = \frac{x^3 + 7x}{5 - x}$  with  $f(x) = (x^3 + 7x)/(5 - x)$ ), intuitive way to manipulate the (graphs on

the) screen, dynamical options like gradual plotting and dynamic handling of parameters.

But there are some risks. Students nowadays deal with images a lot and may be convinced by images. In mathematics this is almost a sin. Moreover a computer has a limited capacity in computing. So one has to keep using one's brains and common sense and, in a mathematical way, use algebra to accept or reject the suppositions which have come up with the images.

### References

Giessen, C. van de (2002). The visualisation of parameters. In M. Borovcnik & H. Kautschitsch (Eds.), *Technology in mathematics teaching. Proceedings of ICTMT5* (pp. 97-100). Vienna: Oebv&hpt Verlagsgesellschaft.

Blokland, P. van (2002), Graphic Calculus/Graphe Software [www.vusoft.nl/item2003](http://www.vusoft.nl/item2003)