Ethnography of the Teaching of Logic

Claude Rosental[1]

***ABSTRACT**: This article provides a frame of analysis for the social study of logic, starting from an ethnographical account of the teaching of logic. This account is based on an inquiry conducted in the department of philosophy of a major University located in the United States. I analyze how the discourse of logic in the classrooms can be constituted as holding by itself, or self-sustaining. I argue, first, that an autonomous language is constituted around a few isolated words, through several strategies of differentiation. I analyze in particular how the dichotomy between formal and informal knowledge can be formed, and how the categories of logical language, ordinary language, and intuition are co-constructed. I then argue that the process of differentiation is supported by an elaborate technology of showing, and that this technology is also used to build demonstrations. I show in particular that, between two steps of reasoning, is a moment of exhibition or de-monstration. Finally, I examine how doubt is managed, targeted and controlled in front of de-monstrations, and how radical doubt can be limited by temporal constraints.*

This paper provides a frame of analysis for the social study of logic, starting from the results of an ethnographical study I have conducted in 1992 on the teaching of logic. The inquiry was carried out in the department of philosophy of a major University located in the United States. Before introducing the outcome of this inquiry, the peculiar problematic and heuristic contexts in which the study was conducted need to be mentioned.

Studying mathematics and logic as an activity is a way of research which has been widely opened by Wittgenstein[2]. In this spirit, Lakatos[3] expressed the idea that the formalist presentation of mathematics, where the theorems are step by step inferred from definitions, may provide an impression of certainty to the extent that the proof is not subject to thorough criticism. For some authors following Lakatos' views like David Bloor[4], that which is considered obvious in

---

[1] Paper presented at the Workshop on Historical Epistemology, Institute for the History of Science, University of Toronto, 1993. A later version of this paper has been published in C. Rosental, *La trame de l'évidence. Sociologie de la démonstration en logique,* Paris, Presses Universitaires de France, 2003, pp. 86-104.

[2] See in particular L. Wittgenstein, *Philosophical investigations* (Oxford: Blackwell, 1953). Note that closer from a phenomenological tradition, Harold Garfinkel showed how rationality could be treated in terms of attitude. Starting from this approach, Bruno Latour and Steve Woolgar have argued that the 'criteria' of compliance with formal logic to characterize scientific activity can't be used as an explanatory resource, but, on the contrary, needs itself to be studied empirically. See B. Latour & S. Woolgar, *La vie de laboratoire* (Paris: La Decouverte, 1988), 149; H. Garfinkel, *Studies in ethnomethodology* (Cambridge: Polity Press, 1984); M. Lynch, *Scientific Practice and Ordinary Action. Ethnomethodology and Social Studies of Science* (Cambridge: Cambridge University Press, 1993).

[3] I. Lakatos, *Proofs and refutations* (Cambridge: Cambridge University Press, 1976).

[4] See in particular D. Bloor, ' Polyhedra and the abominations of Leviticus ', in M. Douglas (ed.), *Essays in the sociology of perception* (London: Routledge, 1982); D. Bloor, *Knowledge and social Imagery* (Chicago: University of Chicago Press, 1991); D. Bloor, ' Wittgenstein and Mannheim on the Sociology of Mathematics ', *Studies in the History and Philosophy of Science*, Vol. 4, No. 2 (1973), 173-191.

mathematics is that which is taken for granted, and the character of obviousness is a social phenomenon[5].

Can we use this representation as a working assumption to give an ethnographical account of logical and mathematical activities? In particular, is it possible to observe *in situ* the constitution of a discourse which is supposed to hold by itself (or, in other words, to have a self-sustaining force)[6], and what is in between steps of reasoning[7]?

Such questions seemed fruitful to me when I started developing a sociology of logic at the beginning of the 1990's[8]. Before carrying out investigations on the work of professional logicians, I found useful to test the relevance of these questions in the context of the teaching of logic. It seemed essential to observe the first steps of apprehension of basic logical objects in order to examine the fundamental process of sedimentation of these objects, to grasp a possible widely shared background of logicians, and to see how it could participate in the constitution of a discipline. It seemed also essential to give a reader who is not familiar with logic the first tools necessary to follow the debates among professional logicians, as it is certainly a major problem encountered by such a sociological description[9].

Besides, the teaching of logic offered a relevant topic in the sense that it was a particular situation where professional logicians could be confronted with naïve individuals, and where the basic principles of logic could be discussed in a way nearly analogous to the way they are discussed in Socrates' dialogues. It was therefore *a priori* possible to go from principles to discussions, and to analyze how discussions could be used to arrive at logical principles.

Where, how and why is logic taught?

The situation in which I carried out my investigation was a course held in the department of philosophy of a major University located in the United States. When I did this study, the only departments of the University which offered complete seminars on logic were the department of

---

[5] Note that some professional mathematicians have addressed this issue from a sociological prospect. See in particular R. De Millo, R. Lipton, A. Perlis, ' Social processes and proofs of theorems and programs ', *Communication of the Association of Computing Machinery*, Vol. 22 (1979), 271-280; P.J. Davis and R. Hersh, ' Rhetoric and Mathematics ' in J. S. Nelson, A. Megill, D. N. Mc Closkey (eds.), *The rhetoric of the human sciences* (Madison: The University of Wisconsin Press, 1987), 53-68.

[6] See in particular R. Carnap, *The logical syntax of language* (London: K. Paul, Trench, Trubner and Co, 1937), and Husserl's position when he speaks of apodictical obviousness of logical principles, in his *Logical Investigations* (New York: Humanities Press, 1970).

[7] This question is also addressed by E. Livingston, *The ethnomethodological foundations of mathematics* (London: Routledge, 1985).

[8] For some recent contributions to the sociology of mathematics (if not logic), see in particular A. Warwick, 'Cambridge Mathematics and Cavendish physics: Cunningham, Campbell and Einstein's relativity theory 1905-1911. Part I: The Uses of Theory', *Studies in History and Philosophy of Science*, Vol. 23, No. 4 (1992), 625-656. D. Mackenzie, 'Negotiating Arithmetic, Constructing Proof: The Sociology of Mathematics and Information Technology', *Social Studies of Science*, Vol. 23 (1993), 37-65.

[9] This may perhaps partly explain why developing a sociology of logic (as well as mathematics) has long been considered as impossible. See in particular K. Mannheim, *Ideology and Utopia* (London: Routledge, 1936).

mathematics and the department of philosophy. The mathematics department offered a research seminar for graduate students on the logic of circuits, which was too black-boxed[10] for the purpose of the current description.

As for the philosophy department, two seminars were offered. The first one was an informal presentation of syllogisms. Its description did not therefore constitute a priority. However, I focussed on the second course as its purpose was to teach the foundations of a logical theory called first order logic, certainly the most well-known stabilized logical theory inside as well as outside the discipline; besides, it did not need any preliminary training in logic, and all the students were beginners.

Between 92 students on the first day, and 78 students on the last day attended the seminar for ten weeks. Among them, 10 were enrolled in the PhD program in the department of philosophy, the other students were undergraduates from all departments at the University.

The access to logic was extremely mediated, as the seminar consisted in three types of presentations and exchanges:

- The plenary session (three times 50 minutes a week) where the Professor provided a monologue interrupted only by several questions.
- The ‘sections’ or ‘discussion groups’ (one hour a week) where exercises and exams were discussed in groups of around twenty students with a teaching assistant (or T. A.). As the general audience was only divided into two groups, this means that only a minority of students was willing to attend these sections.
- The ‘office hours’ where each individual could have a discussion with the Professor or the T. A. (two hours each). In fact, students came essentially before exams and did not generally appear otherwise. However, exams were relatively frequent. They took place after 4 to 6 classes (there were two quiz's, 2 midterms, and one final exam during the term).

I interviewed members of the department, including the Professor, and asked them why this course was taught, and why it was taught in that way. If nobody spoke about the processes of decision in the department, they invoked other reasons. The first reason was that logic was historically important for the foundations of mathematics. Secondly, the teaching of logic was presented as essential for the training of philosophers, as analytical philosophy held an important place in the United States. In particular, the fact that this course was compulsory for students majoring in philosophy (both undergraduates and graduates) was justified by the fact that logical formalism was often used in many philosophical articles[11].

[10] The discussion of such a methodological choice is discussed in B. Latour, *Science in action. How to Follow Scientists and Engineers through Society* (Cambridge (MA): Harvard University Press, 1987).
[11] Note that this argument does not have a consensual scope in the U. S. See for example J. C. Rota, ‘ The pernicious influence of mathematics upon philosophy ’, *Synthese*, Vol. 88 (1991), 168: ‘The fake philosophical terminology of mathematical logic has misled philosophers into believing that mathematical logic deals with the truth in the philosophical sense. But this is a mistake. Mathematical logic does not deal with the truth, but only with the game of truth. The snobbish symbol-dropping one finds nowadays in philosophical papers raises eyebrows among

Besides, the Professor justified the fact that the theory was presented in a stabilized way, i.e. without underlining different points of view of logicians, by saying that it allowed to explain ‘many things without spending a huge amount of time explaining details to students’ who did not want to specialize in logic.

The introduction of the text book that the students had to buy to follow this course also underlined a practical need for logic in everyday life[12]:

‘But in fact logic can be of considerable practical value. All of us, logicians and nonlogicians, students and teachers, are, and must be, concerned to acquire true beliefs about ourselves and the world around us. The emphasis here is on `true' because, as agents constantly interacting with the world, we must care about whether our beliefs accurately picture the world. To be indifferent to the truth of one's beliefs or to think that the truth of a belief is a matter of how one feels about things, rather than how things are, is to court disaster. One who takes such a cavalier attitude toward truth will often end up thinking that such and such is the case when the world is really quite different. And the fact that the world is different will cause him or her at least consternation and very possibly serious harm. For example, one might think, perhaps from some ill-founded principle of Christian charity, that all people are good and kind and then discover that a neighbour has spread malicious gossip about one's personal life or fed rat poison to one's dog.’

The question I would now like to ask is to know to what kind of activity the interactions between the instructors and the students were devoted. First of all, I will show how an autonomous language was constituted, by analyzing the strategies of differentiation developed by the Professor and the T. A.

The construction of a language around a few isolated words

In order to build a logical ‘language’, the instructors presented a few words of ordinary language, ‘and’, ‘or’, ‘there is’, ‘for all’, ‘not’, ‘implies’, ‘equivalent to’, as being ‘logical’ words. These words were not simply ‘used’ as logical words, they were, from the very start, logical. To distinguish them more, the instructors spoke about ‘logical symbols’, and offered a visual support for this distinction, by using specific signs: ‘^’ for ‘and’, ‘v’ for ‘or’, ‘V’ for ‘for all’, ‘]’ for ‘there is’, ‘~’ for ‘not’, ‘->‘ for ‘implies’, ‘<->‘ for ‘equivalent to’.

Besides, to represent all the other words or expressions of ordinary language, they just used letters. For example, the capital letter ‘P’ was used to represent any kind of assertion.

Here is for example a passage where the Professor holds a discussion with the students in order to constitute the dichotomy between logical and non-logical words:

---

mathematicians. It is as if you were at the grocery store and you watched someone trying to pay his bill with Monopoly money.’

[12] M. Bergmann, J. Moor, J. Nelson, *The logic book* (New York: Random House, 1980), 1-2.

'-The Professor: Compare these two sentences. The first one is:

John passed the test
John flew to Europe

John passed the test and flew to Europe

Is it intuitively valid? Yes, obviously. How do we symbolize it in symbolic logic?

P
Q
-
P and Q

Now look at this:

Someone passed the test
Someone flew to Europe

Someone passed the test and flew to Europe

Is it valid?
-A few students: yes.
-Another student: no.
-The Professor: Right, no, because it is not the same person. Why is it that using 'John' makes it valid, and using 'someone' does not?
-A student: Because someone is vague.
-The Professor: No, someone is of a different nature than John, it is a quantifier. You have to distinguish, to see if they behave the same way from a logical point of view. You have 'John' and you have 'someone'.'

Starting from a few specific words of ordinary language, and from letters to represent everything else, the instructors announced to the students that they had a language, which they called formal language. They explained that the main activity of the major part of the course would consist in 'translating' in 'formal language' sentences of 'ordinary language'.

The use of the word 'translating' clearly participated to the presentation of logic as an autonomous language. In practice, the instructors considered as being a logical translation of a sentence any circumlocution being considered as having the same meaning, and using at least one logical symbol.

It is important to note that this 'translation' was not perceived by the instructors as a direct translation, but as an 'analysis' of ordinary language followed by a translation. Analyzing a sentence meant finding a way to express the sentence using logical symbols. This analysis was considered all the more ‘detailed’ or deeper when the periphrasis used more logical symbols.

To illustrate this point, here is a very short extract of the course where the Professor explained to the students the type of analysis he expected from them:

‘We have to do work of analysis, this is not a direct translation. This is what we have to do when we symbolize English sentences. ‘Nobody knows the troubles I have seen’ will be symbolized by: Not (Somebody knows the troubles I have seen); Not (there is an x, x knows the troubles I have seen); Not (there is an x, Px), where Px means: x knows the troubles I have seen; ~(]x Px). Here, I extract the formal structure’.

Hence, the fact of exhibiting a periphrasis using two logical symbols (‘~’ and ‘]’) was described by the Professor as unveiling the form of the sentence. The Professor expressed by this move that a hidden structure was behind the sentence, that this structure needed to be revealed, and that periphrasis using logical symbols served this purpose. Then, logic *appeared* as a language in which the formal structure of sentences could be revealed. In particular, logical symbols became decisive tools that were used to distinguish what is logic from what is not logic, formal and not formal, analyzed and not analyzed.

Let us examine in further detail the constitution of the dichotomy between formal and informal.

The constitution of the dichotomy between formal and informal knowledge

Here is an exchange between the T. A. and two students which is fairly representative of the type of discussions held in this class. Following an exercise found in the text book, the T. A. asked how to symbolize the expression ‘the current President’. The solution proposed by the T. A. consisted in saying ‘a current president’ and adding ‘there is no more than one president’. ‘There is one and no more than one President’ was indeed further analyzed as:

‘there is one, and if there is another, it implies it is the same’, which was symbolized by:
‘there is an x, x is a President, and if there is a y, such that y is a President, then x equals y’; or:
‘there is an x, Px, and if there is a y, Py, then x=y’, where Px means: x is a President; and finally:
‘(]x Px)^((]y Py)-> x=y)’

A student did not understand why it was necessary to add a unicity clause:

‘-Student A: Why do you need that? There can't be more than one President of the U. S.
-T. A.: Well, because it is not really clear from... this is something that you assume, but it is not perfectly determined by this model, uh, that there is only one.
-Student A: Ok, ok. Outside knowledge does not apply?
-T. A.: Right. (silence) Ok, uh...
-Student B: So in that case, you don't have to define what a President is, do you?

-T. A.: Pardon?
-Student B: If you don't apply external knowledge, don't you have to define then what a President is?
-T. A.: Ok, I mean, of course you have to know what you mean by the words. But you don't have to know that there is only one President, or something like that.
-Student A: But I mean, when you are getting an arbitrary model, you know, if you use as universe of discourse 'People of the United States of America', there can only be one President, and that is not something that is external to the concept as... If you don't accept that as good enough, then you can't accept the knowledge of what a President is, because it is going to be ambiguous as, like the President being unique in the U. S.
-T. A.: Well, let us see... Actually yeah, I mean, you have to construct symbolisation keys so that you don't even have to use any knowledge of... You don't have to know... Of course you have to know something, you have to know what is 'a', you have to know what this... But this is a distinction between, uh, the language of logic and the language of meta-logic. We use, to explain something, we use, uh, I don't use the language of logic. Because there is nothing like... So I use the 'is a', right, that means that you understand. But for saying 'current President', I don't have to know what is the current President. (silence)'

Although the explanation of the T. A. might not seem perfectly explicit, the constitution of a dichotomy between formal and informal knowledge (or contextual knowledge) appears clearly in this passage. Expressing 'the current President' by '(a President) and (there is no more than one President)' constituted a logical 'analysis' for the T. A. to the extent that logical symbols were introduced, in a way that corresponded to a certain know-how. The possibility to express a clause of unicity by using logical symbols meant for the T. A. that this data was of formal nature, and that the introduction of this clause allowed to reveal the formal structure of this sentence.

The students didn't however seem to realize that 'formal' was a synonym of this know-how for the instructor. For them, formal seemed to mean something like intrinsic knowledge as opposed to contextual knowledge. As they didn't yet share the same dichotomy between formal and informal, the unicity clause did not seem to them either more formal or more contextual than any other data. Also, in response to a denunciation[13] by the students of the arbitrary character of the exhibition of the unicity clause, a purpose of this exercise was precisely to recenter the dichotomy around this know-how.

However, if such a move was essential to the constitution of the notion of formal knowledge, we have now to analyze in more detail how this constitution went through the construction of 'ordinary language' and 'intuition'.

[13] For a sociology of denunciation, see L. Boltanski, 'La dénonciation', *Actes de la recherche en sciences sociales*, Vol. 51 (1984), 1-39.

The co-construction of logical language, ordinary language, and intuition

The fact that the vocabulary used to speak about logical discourse was called ‘meta-language’ contributed to the constitution of logic as a language. Qualifying ordinary discourse as ‘meta-language’ not only consecrated the categories of both logical and ordinary language, but it also made ordinary language peripheral in its definition of discourse on logic.

Once again, specific expressions and inscriptions were used to build simultaneously both languages, as one can see for example in this passage extracted from the text book[14]:

‘Sometimes, instead of talking of the truth or falsity of a sentence, we shall talk of its *truth value*. If a sentence is true, we shall say that it has the *truth value* T, and, if it is false, we shall say that it has the *truth value* F’.

The dichotomy between logical and ordinary language lied also in the opposition that was made between precision and ambiguity. Ordinary language was presented as a source of confusion, whereas qualities of precision and univocity were attributed to logic[15] (and mathematics), as one can see in this passage where the Professor explained to the students:

‘Natural language is not formal. The interpretation of ‘if’ depends on the context. If I say ‘John’, many people will stand up. Sometimes, ‘if’ means ‘if and only if’, it depends on the context. Modern logic was founded for mathematics and in mathematics, you don't have any ambiguity.’

In a similar way, certain oral justifications of logical reasoning or conventions were differentiated from logical discourse by being called intuitions. In such situations, ‘intuition’ appeared as a first step which did not have the same degree of certainty as formalism, but allowed at least to have access to it. According to the instructors, intuitive argument helped understanding. Yet once this understanding was reached, the intuitive argument and its role in understanding should be forgotten. Once understanding was achieved, credit was given to the formal argument. This process can be seen for example in this passage, where the Professor explained why an implication in which the premises are false should be considered as a true assertion:

‘Let me give you an example that will bring you at least intuition of it: If you love me, I will marry you. Now, if you don't love me, but you marry me, it is clear I did not lie. What I say is consistent with what I said, so we take it to be true. It is our convention and we will take our intuition to take it as a convention.’

---

[14] M. Bergmann, J. Moor & J. Nelson, op. cit. , 5.

[15] Note that the dichotomy is namely challenged by proponents of fuzzy logic. See for example B. Kosko, ‘ Fuzziness Vs. Probability ’, *International Journal of General Systems*, Vol. 17 (1990), 211-240; J. Baldwin & N. Guild, ‘ The resolution of two paradoxes by approximate reasoning using a fuzzy logic ’, *Synthese*, Vol. 44 (1980), 397-420.

It is now clear that the construction of formalism was connected with the construction of intuition. It is important to underline that a constant back and forth move from intuition to formalism allowed to a certain extent to build a desired association between formal reasoning and intuitive reasoning. Indeed, during the course, a constant move between an 'intuitive' and a 'formal' point of view allowed to adapt intuitive and formal reasoning so that they matched to a certain extent. The case of the former convention (in the quote above) is a good example of this process: the characterization of intuitive and formal reasoning were in fact performative in relation with this matching[16].

But now that I have analyzed a few strategies developed to constitute a discourse holding by itself, through the construction of an autonomous language, I would like to underline how the differentiation between logical language and ordinary language was accompanied by a specific way of showing.

A process of differentiation accompanied by a technology of showing

In the second cited passage, the Professor wrote a sentence on the blackboard. He then wrote a periphrasis in which a negation appeared, made a few other circumlocutions, and introduced logical symbols, noting that this progressive visual apparition was the extraction of a formal structure. The Professor showed the different inscriptions, and said what they were. The formal structure appeared by isolating the word 'not', introducing parenthesis, an abbreviation, logical symbols, and juxtaposing the sentences. By such a description, I want to argue that the process of differentiation between formal and informal language was supported by a specific way of showing, using the manipulation and the exhibition of inscriptions[17].

In the passage we are concerned with, the exhibition or de-monstration of formal qualities of the last sentence lied in an ontological statement. The Professor showed the inscriptions, said what they were and in particular attributed formal properties to the last sentence. Hence, this ontological statement substituted itself for an epistemological statement of what formal structure was.

It is important to note that this was the usual approach in the course. The technology of showing used to support the differentiation between formal and informal appeared clearly in many activities.

---

[16] A case of constitution of a similar kind of matching in mathematics is analyzed in A. Pickering & A. Stephanides, 'Constructing Quaternions: On the analysis of conceptual practice', in A. Pickering (ed.), *Science as practice and culture* (Chicago: University of Chicago Press, 1992), 139-167. On this historical case, see also D. Bloor, 'Hamilton and Peacock on the essence of algebra', in H. Mehrtens, H. Bos & I. Schneider (eds.), *Social History of 19th Century Mathematics* (Boston: Birkhauser, 1981), 202-232.

[17] H. R. Alker notes that 'there is a lot to be said for treating and teaching the Frege-Russell program of axiomatic, anti-metaphysical formal, extensional logic as an extraordinarily dialectical, anti-Hegelian exercise, in some ways parallel to Marx and Lenin's efforts to recast Hegelian thought in a materialist vein'. See H. R. Alker, 'Standard logic versus dialectical logic: which is better for scientific political discourse?', paper presented at the 12th World Congress of the International Political Science Association, Rio de Janeiro, (August 8-15 1982), 29.

First, numerous recapitulations called ‘definitions’ were used by the Professor to support this differentiation. Several graphic representations that I have already mentioned, were shown in these definitions, whose repeated visualization tended to accentuate the split between different ontological categories.

Besides, repeated work on symbolization, using sophisticated visualization techniques, led the students both to differentiate logical from non-logical words, and to obtain the ‘apparition’ of formal structures, as one can see on these 3 small exercises:

‘-T. A.: Let us start with ‘Charles is rich and beautiful, but not intelligent’. How do you symbolize it? What is the main operator?
- A student: It is the ‘but’.
- T. A.: Right, very often, it comes after the coma. You have: (...) ^ (...).
[...]
- T. A.: Let us do exercise J. Again, think first what is the main operator. You have ‘but’. But is always translated by ^. You have something like this: (blablabla) ^ (Charles is).
[...]
-T. A.: How do you symbolize ‘Either all are black, or all are red’? The things that are important are: either, all, or, all. [The T. A. underlines the words at the blackboard: either all are black, or all are red]’

Hence, the access of the students to logic was not scheduled by the instructors as a set of interrogations in principle about what formality meant. Instead, a know-how was taught, which consisted in showing a technology of exhibiting logical symbols, and having the students acquiring the corresponding skills. Moreover, in complete correspondence with this approach, the exams accentuated this tendency, as they consisted in asking the same kind of exercises as the ones that were done in class.

From the students' side, this explains why their main activity at home as well as in class consisted in ‘doing exercises’, in practicing to obtain the necessary skills. In particular, the only remedy that the instructors found when students failed exams was encouraging them to ‘work more’, to come at the office hours (where exercises were discussed and corrected most of the time) and to attend the discussion groups so that they got ‘trained’ through exercises.

But now that we have seen that access to logic as a practice was achieved by a process of exhibition, or de-monstration, we have now to see how logical demonstration itself could be supported by a technology of showing.

De-monstration

Even if one does not limit the analysis to formal demonstrations (that is, justifications using completely, or more often partly, inscriptions recognized as logical symbols), one may see how an important technology of showing was developed. For example, in order to show that two sentences have ‘in fact’ the same form, the Professor used the following presentation:

```
    ------------------       ----------------------
If | you love me |     | I will marry you |
    ------------------       ---------------------
            A                         B

  ----------------------                  ------------------
| I will marry you |    only if    | you love me |
  ----------------------                  ------------------
           B                                  A
```

The abundant use of diagrams also was a source of de-monstrations which, in most cases, was more convincing to students than the justification provided in logical symbols. For example, the Professor used a specific sketch in order to show that the sentence ‘Vx (Ax v Bx)’ implied that the universe was reduced to the two sets A and B, and that ‘there was not anything else’. The students seemed to think that ‘the universe’ meant everything that exists, and did not apparently understand how one could speak of a universe whose extension could be arbitrarily decided. In contrast, the Professor explained how this was a matter of free choice by using the following drawing:

```
  ----------------
  |  A  |  B  |     ‘nothing else’
  ----------------
```

By using such a drawing, the Professor showed that it was possible to choose the limits of one's universe with as much ease as it was possible to draw two sets and to write that there was not anything else outside.

The ‘understanding’ of a step of reasoning in a formal demonstration (an equivalence for example) was also supported by showing, or *monstration*[18]. For example, the equivalence between ‘~ (Vx Px)’ and ‘]x ~Px’ was presented by the T. A. the following way:

‘Of course, you just have to take a look at these forms for 2 or 3 minutes, and you will see why they are equivalent. The only thing that you have to do here is simply to change the quantifier and to put negation. It is no problem’.

For the T. A. , a simple look at these two sentences had to lead to the visual apparition of the link of equivalence that united them. The translatory movement of the negation symbol to the right of the quantifier, accompanied with a change for the dual quantifier, constituted a characteristic sign of the equivalence. This process, as the T. A. emphasized, had to take place in

[18] There are many contexts in which the fact of showing supports an ontological claim and in which a sociology of apparitions seems relevant. See in particular E. Claverie, « Voir apparaître. Les "événements" de Medjugorje », *Raisons Pratiques*, vol. 2, 1991, pp. 157-176. L. Daston, ‘Marvelous Facts and Miraculous Evidence in Early Modern Europe’, *Critical Inquiry*, Vol. 18 (1991), 93-124.

a very limited period of time, the time necessary to make sure that the manipulation was an allowed move.

Hence, one sees that this step of reasoning represented a moment where one did not want to give any more justification. The use of exhibition precluded the need for further discourse: This was the intended effect of exhibition.

A demonstration, which was composed of several steps of reasoning, was therefore composed of several exhibitions, several moments of showing, of monstration. The more formal the demonstration was, the more visual was the move between two inscriptions. Exhibition occurred between two steps of reasoning when de-monstrations became more formal and allowed moves were 'postulated', i. e. written for example on the top of the page. This moment of exhibition was localised and restricted in time, and supported by techniques of showing.

Of course, this description holds only to the extent that no one raised an objection, or asked a question, that is to say that no one was unsatisfied by the exhibition and asked for further justification. The situations were indeed numerous where students did not see what the instructors wanted to show them.

Hence, de-monstration appears as a type of discourse which allowed to claim that things were the way they were shown (for example equivalent), without having to explain, if possible, why they were so. In other words, de-monstration proceeded from an ontological claim, not an epistemological one.

But now that we have seen how techniques of showing supported demonstrations, let us examine how a process of exhibition, namely an exemplification, helped support the constitution of a field of application of logical symbolisations.

Exemplification

In order to show that formalism associated with logical symbols captured an essential aspect of reasoning, the instructors had to find and exhibit cases where a form could be revealed 'behind' a reasoning. The power of logical symbolisation was progressively shown by exhibiting such cases and giving them an exemplary dimension.

Hence, one could find for example in the text book an exercise which consisted in trying to formalize 13 small paragraphs written by authors such as David Hume, Democritus, Melissus, Thomas Aquinas, Protagoras, John Stuart Mill, Anaximenes, Charles Darwin, Sigmund Freud, Socrates, and René Descartes[19].

Showing that such authors used reasoning which could be symbolized logically constituted an exemplary point to argue for the success of logic to capture the essence of

[19] M. Bergmann, J. Moor, J. Nelson, op. cit. , 8-10.

reasoning. The fact that the choice of these paragraphs and the question of their representativity were not discussed could even suggest that the list could be extended *ad libitum*, and that these cases constituted examples.

Mutiplicating 'examples' where a form was shown 'behind' a reasoning allowed progressively the instructors to attribute an intrinsic property of power to logic, as one can see for example when the Professor declared:

'Let us examine a few more examples that I will write quickly at the blackboard, so that we can see, again, the expressive power of existential and universal quantifiers.'

An authentic know-how of constituting examples was elaborated by the instructors. Not only did they benefit from the exercises proposed in the text book, but they also had acquired, from their cumulative past experiences, skills to propose 'good examples'. This explains for example why the T. A. could declare: 'By the way, families and numbers are always good examples.'

This know-how appeared very clearly in many exercises on symbolization where the sentences were obviously conceived in close connection with the symbolization that had to be proposed, as in the case of this sentence: 'Every Professor who is an administrator earns more than some Professors who are not administrators.' But if exemplification played a very important role to show the expressive power of logic and constituted an essential aspect of the technology of showing, one needs now to see how this technology had also to be accompanied by a very sophisticated management of doubt.

Targeted doubt: Who has to show what to whom?

In a former passage, we have seen how the Professor tried to show to the students that a proper name was different in nature from a quantifier. In order to do that, the Professor showed two reasonings, asked the students to compare them, and to explain why they were different. In this case, the institution by the instructor of a differentiation between logical and non-logical words was obtained by introducing and guiding the students' doubt. The Professor obtained first that the students saw a difference, and then that they attributed this difference to the opposition between a logical operator and a proper name. In other words, the Professor's questions were targeted and organized in such a way that the students could see what the Professor showed.

This 'lively' type of presentation, where the questions were targeted and where the answers were oriented, was very frequent during the course[20]. For example, it was used to support the claim that logic had a very expressive power to deal with mathematical assertions. At one point in the lecture, the Professor wanted to show that numbers could be expressed in logic, and that the notion of number did not need to be presupposed. In order to do that, he asked the students how they would symbolize the sentence: 'There are at least three students in the class'.

[20] This method can be compared with the narration proposed by Blaise Pascal to lead the reader, through a series of relevant questions, to believe in the existence of god. See B. Pascal, *Oeuvres complètes* (Paris: Gallimard, 1962).

After a long process where the answers of the students were selected and guided, the proposed solution took the following 'form': ']x]y]z (Ax ^ Ay ^ Az ^ ~(x=y) ^ ~(y=z) ^ ~(x=z))'. However, as it sometimes happened, the targeted doubt elaborated by the Professor got suddenly overflowed by a question of a student:

'-The student: How would you say there are 30 students?
-The Professor: Oh! (students laughing for a few seconds). I could say there are 30 students, but it would probably take the whole hour. Ok? (students laugh again for a few seconds). I would have to say: There is an x1, there is an x2, an x3, until x30, and the list of inequalities that are all implied.'

Here, a simple and uncontrolled question of the student suddenly disturbed the progressive elaboration of targeted doubt developed by the Professor to support the main claim. This sole interrogation was sufficient to reverse the argument: In the light of this question, logical symbolisation appeared extremely cumbersome to express a statement quickly made in 'ordinary language'. Students now laughed at the claim concerning the expressive 'power' of logic[21]. Hence, one sees how the demonstration of the Professor could hold only when doubt was highly targeted and controlled.

However, targeted doubt was not only used by the instructors to show properties and the power of first order logic. It was equally used by them to show certain lacks of properties, and limits of this theory. For example, to introduce the necessity of having a logic whose order was higher than first order logic, the Professor asked why the following inference 'did not work':

The Indians are vanishing
Tom is an Indian

Tom is vanishing

The answer proposed by a student saying 'you are using the term 'vanishing' in two different contexts' was just ignored[22] by the Professor who spoke instead about the 'necessity' of an other theory: 'We have to go to a higher order logic to be able to speak about properties of properties'. Hence, if the Professor was willing to show the limits of first order logic, this was in order to show the power of symbolisation of higher order logic. Targeted doubt allowed him to

[21] In a conference organized in 1992 by the Science Studies Program of the University of California San Diego, David Bloor argued that a proof in logic of 2+2=4 uses the same process of material counting than the one used in elementary school. In our case, one sees also that the expression of numbers goes through an indexation of elements which is whether explicitly using numbers (1 to 30), or using an enumeration of elements (a differentiation of elements coupled with an order) by an other visualisation process (for example, lexicographical order 'x, y, z').

[22] On the theme of the use of logic to silence others, see A. Nye, *Words of power: A Feminist Reading of the History of Logic* (London: Routledge, 1990); J. Battali, 'The power and the story', *Postmodern Culture*, Vol. 2, No. 2 (January 1992). For other critical readings, see also S. Restivo, *The social relations of physics, mysticism, and mathematics* (Dordrecht: D. Reidel, 1983); S. Restivo, J.P. von Bendegem, R. Fischer (eds.), *Math worlds : Philosophical and Social Studies of Mathematics and Mathematics Education* (Albany: SUNY, 1993).

support the demonstration, to the extent that it eliminated any potential contextualist interpretation or criticism.

But it is important to insist on the fact that each targeted doubt was first of all a matter of audience. This will appear clearly on the following example, where the Professor explained to the students that the sentence ‘Vx ~Rxx ^ Vx Vy Vz ((Rxy ^ Ryz)-> Rxz) ^ Vx]y Rxy’ needed an infinite universe of discourse to be true. After having proposed a short and hardly formal demonstration, the Professor was interrupted by the T. A. sitting in the class. The T. A. raised an objection, saying that the relation R ‘being a brother’, and the situation of three brothers as universe of discourse, constituted a counter example. After a few minutes of doubt, and the intervention of a student, the Professor finally showed that this situation did not allow the sentence to be true, and therefore did not constitute a counter-example.

At the end of the course, I asked the T. A. how he would formalize the reasoning by recurrence first proposed by the Professor, to show that the universe of discourse was infinite. After a brief attempt, the T. A. told me that the formalization was not obvious, and added with humour: ‘May be there is no demonstration’.

A few minutes later, some students asked him during his office hour why the universe of discourse was infinite. He showed them at the blackboard why there were one, two, three elements in the universe of discourse, and concluded by saying ‘etc’, therefore stepping over the difficulty formerly met to express the existence of an additional element, starting from an arbitrary number of elements in the universe of discourse.

Hence, the expression of doubt appeared highly targeted, depending on whom expressed doubt to whom, and in what circumstances. The same claim could be the object of different configurations of demonstrations and doubt. In particular, the instructors had generally to minimize their own doubt in front of students, and focus the expression of doubt to support the claims they wanted to make.

This case shows us still more clearly that a particular demonstration held only in relation with the targeted doubt it was submitted to. But an important reason why radical doubt could not last in any case was due to the fact that exchanges were restrained by an ineluctably limited allocation of time. This temporal process in which the exchanges took place constitutes the last point which I will analyze.

Temporal process

Many times during the session, the instructors explained to the students that they could discuss in principle any question raised by the students, in particular during the discussion groups and the office hours. In practice, however, as time was limited, the instructors often had to close discussions about the principles of logic. Soon after the beginning of the class, the students generally took this constraint into account and limited their questions on their own. Less than ten students regularly asked questions during the course, whereas the great majority of students never or almost never participated. Some students sometimes tried to find extra time for discussion to express their doubt. For example, one student declared in the middle of a class: ‘Well, I have a general question. Maybe I should ask it after class?’

From the point of view of the instructors, the expression of doubt by the students had to be restrained in order to teach a predetermined number of chapters. The Professor complained: ‘The students have too many questions in this class. Two quarters instead of one would be necessary to do the same program’.

Doubt was essentially oriented around the learning of the technology of symbolisation. The interest of the students around this know-how got constantly stimulated by the exams that were conceived to test their know-how. The constant prospect of an exam instituted a priority for the students to know how to do the exercises. As the exams occurred themselves in a limited time, the students had moreover the priority of acquiring skills which allowed them to answer sufficiently quickly. As a result, the questions were essentially oriented toward this preparation, and many questions were left aside in the context of this priority. A certain complicity was even instituted between the students and the instructors about what questions were legitimate[23]. Moreover, even within the questions about the exercises that could be asked at the exams, certain were given a lower priority rank, as one can see for example when the T. A. stepped from an exercise to another and declared during a discussion group:

‘You can really spend a lot of time on a question like that. If there is something like that on the exam, there won't be any more than one question like that, may be none, so don't spend too much time answering the question.’

Hence, this description ends up with a false paradox, as one sees how temporal limitations were essential to support ‘timeless’ assertions.

Conclusion

The question concerning the way logic can be constituted as a discourse holding by itself in the process of teaching and the issue of knowing what is in between two steps of reasoning, have been treated the following way. In a first moment, we have seen how an autonomous language was constituted around a few isolated words, through several strategies of differentiation. We have seen in particular how the dichotomy between formal and informal knowledge could be formed, and how the categories of logical language, ordinary language, and intuition were co-constructed.

In a second moment, we have examined how the process of differentiation was supported by a technology of showing, and how this same technology was used to build demonstrations. We have seen in particular that what was in between two steps of reasoning was a moment of

[23] This process can be compared to the one described in A. Kleinman, *Patients and Healers in the context of culture* (University of California Press, 1980), concerning the patient-healer relation of occidental medicine in Taiwan, where the healer can refuse to answer to certain questions in particular due to the lack of time.

exhibition or monstration. Finally, we have seen how doubt was managed, targeted and controlled in front of these de-monstrations, and how radical doubt was namely stopped by temporal limitations.

This study offers methodological perspective for the description of the activity of logicians, and in particular, of the controversies around proofs and paradoxes. If one wanted to express it in a few words, one could say that it is helpful, in order to understand how consensus or disensus can be partly obtained, to be particularly sensitive to the theme of the technology of showing developed by logicians when debating around proofs and to focus the analysis on the management of doubt. This means for example being sensitive to the kind of resources a logician mobilizes when he gives a conference, and to the kind of questions individuals ask in the audience, depending in particular on who asks the questions to whom.

The process of elaborating and refereeing a proof can be also examined in the same general perspective. If for Lakatos[24], a demonstration is rigorous only to the extent that it has not been submitted to thorough criticism, the question then is to know what determines the restrictions of applying radical doubt. As these restrictions touch *a priori* the author as well as the readers of the proof, qualifying a proof of being rigorous appears as the product of two distinct practices: the restriction of criticism applied by the writer of the proof on what he has to show (monstration), and the restriction of scepticism applied by the readers of the proof (targeted doubt). One can hope that further empirically grounded work will allow to test the scope of this frame of analysis.

[24] See again I. Lakatos, op. cit.