Quantum corrections to the conductivity and Hall coefficient of a 2D electron gas in a dirty AlGaAs/GaAs/AlGaAs quantum well: transition from diffusive to ballistic regime.

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(Dated: 15 December 2004)

We report an experimental study of the quantum corrections to the longitudinal conductivity and the Hall coefficient of a low mobility, high density two-dimensional two-dimensional electron gas in a AlGaAs/GaAs/AlGaAs quantum well in a wide temperature range (1.5 K - 110 K). This temperature range covers both the diffusive and the ballistic interaction regimes for our samples. It was therefore possible to study the crossover region for the longitudinal conductivity and the Hall effect.

PACS numbers: 73.20.Fz, 73.21.-b, 73.21.Fg

I. INTRODUCTION

At low temperatures the conductivity of a degenerated two-dimensional electron gas (2DEG) is governed by quantum corrections to the Drude conductivity σ_D. In general, these corrections have two principal origins: the weak localization (WL) and the electron-electron (e-e) interaction[1]. Until recently our understanding of the interaction corrections to the conductivity of a 2DEG was based on the seemingly unrelated theories developed for two opposite regimes: the diffusive regime[2] $k_BT\tau/\hbar \ll 1$, and the ballistic regime[3] $k_BT\tau/\hbar \gg 1$. In the diffusive regime the quasi-particle interaction time $\hbar/k_BT$ is larger than the momentum relaxation time τ and two interacting electrons experience multiple impurity scattering. In the ballistic regime the e-e interaction is mediated by a single impurity. Recently, Zala, Narozhny, and Aleiner (ZNA) have developed a new theory of the interaction related corrections to the conductivity[4,5] that bridges the gap between the two theories known previously[2,3]. One of the important conclusions of the new theory is that the interaction corrections to the conductivity in both regimes have a common origin: the coherent scattering of electrons by Freidel oscillations. Conformably to the previous results[2,3], the new theory predicts a logarithmic temperature dependence of the longitudinal conductivity and the Hall coefficient in the diffusive regime, whereas in the ballistic regime the temperature dependence of these parameters becomes linear and T^{-1} respectively.

Despite a surge of experimental activity[6–11] following the publication of the theory[4,5] so far no experiment has been reported where the transition between the two regimes would have been clearly observed. One of the reasons is that the temperature at which the transition is expected to occur is given by $k_BT\tau/\hbar \approx 0.1$, so that in the relatively high-mobility 2D systems that are commonly studied the transition temperature is by far too low ($T < 100$ mK for $\tau > 10^{-11}$ sec). Thus, the ZNA theory has so far been verified only in the intermediate and ballistic regimes[12] ($k_BT\tau/\hbar = 0.1 - 10$).

To shift the transition to higher temperatures one should use low mobility samples (small τ). At the same time high carrier densities N_s are necessary in order to maintain high conductivity and avoid strong localization. Moreover in high density 2D systems the characteristic parameter $r_s = E_C/E_F \propto 1/N_s^{1/2}$, the ratio between Coulomb energy and Fermi energy is small ($r_s < 1$) and hence the effect of e-e interaction is relatively weak. In this case the Fermi liquid interaction constant F_0^σ, the only parameter in the expressions for the quantum corrections to the conductivity in the theory[4], can be calculated explicitly.

In this respect low-mobility high-density systems appear to offer certain advantages for testing the theory[4,5], as compared to high-mobility low-density systems. Indeed not only they provide an opportunity for studying an experimentally accessible temperature transition between the diffusive and the ballistic interaction regimes but also the comparison between the theory and experiment requires no fitting parameters.

The aim of the present work is to experimentally study the interaction related corrections to the conductivity and the Hall coefficient in a broad temperature range covering both the diffusive and ballistic interaction regimes and the transition between them. The experimental results obtained in the weak interaction limit are expected to allow for a parameter free comparison with the ZNA theory.

II. EXPERIMENTAL SET UP

The experimental samples had a 2DEG formed in a narrow (5 nm) AlGaAs/GaAs/AlGaAs quantum well δ-doped in the middle. Such doping results in a low mobility and a high carrier density. A detailed description of the structure can be found in Ref. 13. Two samples from the same wafer have been studied for which similar

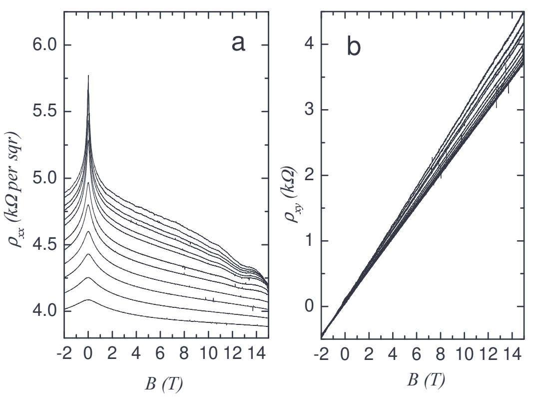

FIG. 1: a) Longitudinal resistivity of the sample at $N_s = 2.56 \times 10^{12}$ cm^{-2} for temperature=1.4 K, 1.9 K, 3.1 K, 4 K, 7.2 K, 10.25 K, 15.45 K, 21.5 K, 31 K, 46.2 K, 62.8 K, 84.5 K and 110 K from top to bottom. b) Hall resistance at the same temperatures (from top to bottom).

results were obtained. Here we present the data obtained for one of the samples with the following parameters at $T = 1.4$ K depending on prior illumination: the electron density $N_s = (2.54 - 3.41) \times 10^{12}$ cm^{-2} and the mobility $\mu = (380 - 560)$ cm^2/Vs. The Hall bar shaped samples were studied between 1.4 K and 110 K in magnetic fields up to 15 T using a superconducting magnet and a VTI cryostat and also a flow cryostat ($T > 5$ K) placed in a 20 T resistive magnet. The data was acquired via a standard four-terminal lock-in technique with the current 10 nA.

Fig. 1 shows the longitudinal and Hall resistances of the sample as a function of magnetic field at temperatures up to 110 K. As can be seen both are strongly temperature dependent. Before analyzing the role of the quantum corrections in the behavior of the transport coefficients shown in Fig. 1, let us estimate the possible contribution from other unrelated temperature dependent factors.

First, since the measurements were performed up to relatively high temperatures, the question of the role of phonon scattering becomes important. In this connection we believe that the following argument can be used. It is well known that in ultra-clean GaAs samples sufficiently high values of mobility are reported even at liquid nitrogen temperatures (see, for example Ref. 14,15, where $\mu = 4 \times 10^5 cm^2/Vs$ at $T = 77$ K). At these temperature the phonon scattering is the dominant scattering mechanism in these samples and yet the mobilities are still a thousand times larger than in our sample. In our experiment, the phonon contribution to the conductivity (and thus its variation with temperature) is thus expected to be around 0.1% at the highest temperature and can be neglected in the entire temperature range.

Now, as can be seen in Fig. 1, the slope of the Hall resistance versus B dependence varies with T at low temperatures but remains practically constant for $T > 20$ K. One might argue that the behavior at low temperatures could be due to a variation of the electron density with temperature. However, we believe that this is not the case. Indeed, from the measurements carried out up to 20 T where the Shubnikov de Haas oscillations are better resolved, we find that the density remains constant at $T < 30$ K. Also we find that the density given by the SdH oscillations is the same as we get from the slope of the Hall resistance at $T > 20$ K where it is T-independent. We conclude therefore that the electron density remains constant in the entire experimental temperature range and all the data presented in Fig. 1 corresponds to $N_s = 2.56 \times 10^{12}$ cm^{-2}.

Having excluded the phonon scattering and the density variation as possible causes of the behavior shown in Fig. 1 we associate the observed temperature dependences with the quantum corrections to the transport coefficients. Our data will be analyzed in the framework of the recent theories[4,5] valid for a degenerated 2DEG ($k_B T \ll E_F$). According to Ref. 13 only one subband is occupied in our quantum wells at $N_s = 2.56 \times 10^{12}$ cm^{-2}. Also $E_F \approx 1000$ K and so the theory[4,5] should apply under our experimental conditions.

III. LONGITUDINAL CONDUCTIVITY AT B=0 T

Let us first describe how the experimental quantum corrections were extracted from the row data and then turn to the analysis of the obtained corrections.

With the magnetic field increasing the MR in Fig. 1a goes through two distinct types of behavior. An abrupt drop of resistance at low fields and then a much weaker magnetic field dependence at higher B. It is easy to show that the possible classical MR described in Ref. 16 can be neglected in our sample. Indeed, the fraction of circling electrons, which in this theory are supposed to cause a deviation from the Drude theory is very small in our samples due to the low electron mobility. This means that the behavior in Fig. 1a must be attributed to quantum interference effects, such as WL and the electron-electron interaction related corrections to the conductivity. As is well known the weak localization is suppressed at magnetic fields larger than $B_{tr} = \hbar/(2el^2)$, where l is the mean free path. In our samples $B_{tr} \approx 1$ T that roughly coincides with the field at which the crossover from the one type of MR to the other takes place. We conclude therefore that the strong MR observed at low fields can be associated with the WL suppression in our samples and that the MR observed at higher fields must be attributed entirely to the e-e interaction effects[2].

The longitudinal conductivity value is a sum of three components: the classical Drude conductivity, the WL contribution and the e-e interaction correction which is supposed to be independent of B as long as $k_B T\tau/\hbar \ll 1$. For the correct evaluation of the interaction related correction at $B = 0$ T, the knowledge of the first two contributions to the conductivity is essential. Unfortunately, in our case there is no direct means of knowing the value of the Drude conductivity σ_D because of a considerable (up to 20%) variation of the zero field conductivity with temperature. Nevertheless there exists an empirical method that can be used for the evaluation of the all three contributions to the conductivity at zero magnetic field. This method has the advantage that one can forgo the usual procedure of fitting the low field data with the theoretical expressions for the WL magneto-resistance[17], thus eliminating a possible source of error at this stage.

As a first step of this method the experimental longitudinal conductivity is obtained by inverting the resistivity tensor using the data shown in Fig. 1. The conductivity can be written as:

$$\sigma_{xx}(T,B) = \frac{\sigma_D}{1+(\omega_c\tau)^2} + \Delta\sigma_{xx}^{WL}(T,B) + \Delta\sigma_{xx}^{ee}(T), \quad (1)$$

where ω_c is the cyclotron frequency, $\delta\sigma_{xx}^{WL}$ and $\delta\sigma_{xx}^{ee}$ are the WL and e-e interaction corrections respectively. The first term corresponds to the classical T-independent Magneto-Conductivity (MC). The e-e interaction correction, B-independent in the diffusive regime[2] is expected to become magnetic field dependent in the opposite ballistic limit. The weak localization corrections dominates at low fields but is suppressed at $(B > B_{tr})$. Therefore, in the diffusive limit and for $B >> B_{tr}$ the shape of the σ vs B dependence will be determined by the first term in Eq. 1 while the e-e interaction correction to the conductivity should only result in a vertical shift of this classical contribution. Indeed, experimentally we find that with the WL completely suppressed at higher magnetic fields the MC corresponding to different temperatures forms parallel vertically shifted traces whose shape is given by the classical term in Eq. 1. However, one can notice that at temperatures $T > 30$ K the shape of the curves begins to deviate slightly from that of the low temperature traces. This change of shape may be the consequence of the interaction correction becoming magnetic field dependent at the crossover from the diffusive to the ballistic regime.

It is possible to determine the momentum relaxation time by fitting the curves for $B > 6$ T using the expression for the classical MC with τ as a fitting parameter. This was done for all the temperatures yielding the average value $\tau = 2.17 \times 10^{-14}$ s with a maximum deviation of $\approx 10\%$. This value of τ corresponds to $\sigma_D \approx 6 \times e^2/h$. Next, in order to eliminate the WL contribution at $B = 0$ the term $\frac{\sigma_D}{1+(\omega_c\tau)^2} + \Delta\sigma_{xx}^{ee}$ was extrapolated for each of the curves down to $B = 0$ T. Finally, to obtain the value of the e-e interaction correction corresponding to a given temperature, the Drude conductivity was subtracted from the corresponding zero field conductivity value obtained at the preceding step. Of course it is well to keep in mind that this method is correct only as long as the e-e correction is B-independent. That means that it is fully reliable only at low temperatures where the MC traces are parallel. At $T > 30$ K it will give rise to an error increasing in proportion to the variation of the shape of the curves. To remedy this we have also used an alternative way to estimate the e-e interaction correction at high T which is as follows. The low temperature zero field WL contribution determined at $T < 30$ K with the method described above was extrapolated to higher temperatures using the common logarithmic law expression[1]. The e-e correction was then obtained by subtracting from the experimental zero field conductivity the WL contribution obtained in this way together with the Drude conductivity.

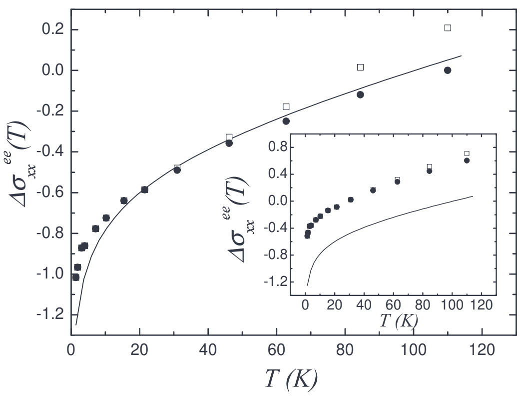

FIG. 2: Experimental temperature dependence of the e-e correction to conductivity obtained by the first method (dots) and by the second method (opened squares), see the text. They are compared to the model of Ref. 4 (solid line). The inset shows the correction obtained using the first approximation of σ_D.

The results obtained by these two different method are presented in the insert to Fig. 2. We realize that none of these two methods is fully accurate yet we suppose that the correct result for the e-e interaction contribution must lie somewhere between these two estimates.

According to Ref. 4 the e-e interaction correction to conductivity is given by the following expressions:

$$\Delta\sigma_{xx}^{ee} = \delta\sigma_C + 3\delta\sigma_T \quad (2)$$

$$\delta\sigma_C = \frac{e^2}{\pi\hbar}\frac{k_BT\tau}{\hbar}\left[1-\frac{3}{8}f(k_BT\tau/\hbar)\right] - \frac{e^2}{2\pi^2\hbar}ln\left[\frac{\hbar}{k_BT\tau}\right];$$

is the charge channel correction and

$$\delta\sigma_T = \frac{F_0^\sigma}{[1+F_0^\sigma]}\frac{e^2}{\pi\hbar}\frac{k_BT\tau}{\hbar}\left[1-\frac{3}{8}t(k_BT\tau/\hbar;F_0^\sigma)\right]$$

$$-\left[1-\frac{1}{F_0^\sigma}ln(1+F_0^\sigma)\right]\frac{e^2}{2\pi^2\hbar}ln\left[\frac{\hbar}{k_BT\tau}\right];$$

is the correction in the triplet channel. The detailed expression of $f(x)$ and $t(x)$ can be found in Ref. 4. It is worth mentioning that for small r_s the interaction constant F_0^σ is an analytical function of parameter r_s (see Ref. 4). In our calculations we used the value $r_s = 0.35$ corresponding to the electron density in our sample.

Inset to Fig. 2 shows the theoretical curve calculated for our system parameters together with the experimental data points. As can be seen there is a systematic shift of the experimental points with respect to the theoretical curve. This shift can be attributed to a not quite accurate evaluation of the Drude conductivity. Indeed, a 10% variation of σ_D brings the experimental data points closer to the corresponding theoretical curve. A variation of this order of magnitude lies within the experimental accuracy with which we determine σ_D and only weakly affect the shape of the theoretical curve. Fig. 2 shows the results obtained using $\tau = 2.33\times10^{-14}$ s ($\sigma_D = 6.5\times e^2/h$). Thus a reasonably good agreement for the entire temperature range which also covers the intermediate regime is found.

Note that contrary to the previous works[6–11] we have used no fitting parameter. Moreover we find that using the interaction constant F_0^σ as a fitting parameter does not result in a better agreement between theory and experiment.

IV. HALL EFFECT

We now turn to the analysis of the Hall data presented in Figure. 1b. According to Ref. 5 the Hall resistivity may be written as:

$$\rho_{xy} = \rho_H^D + \delta\rho_{xy}^C + \delta\rho_{xy}^T \tag{3}$$

where ρ_H^D is the classical Hall resistivity and $\delta\rho_{xy}^C, \delta\rho_{xy}^T$ are the corrections in the charge and triplet channel. These corrections are given as follows:

$$\frac{\delta\rho_{xy}^C}{\rho_H^D} = \frac{2}{\pi}\frac{G_0}{\sigma_D} ln\left(1+\lambda\frac{\hbar}{k_B T\tau}\right) \tag{4}$$

$$\frac{\delta\rho_{xy}^T}{\rho_H^D} = \frac{6}{\pi}\frac{G_0}{\sigma_D} h(F_0^\sigma) ln\left(1+\lambda\frac{\hbar}{k_B T\tau}\right)$$

The detailed expression for $h(x)$ can be found in Ref. 5, $\lambda = \frac{11\pi}{192}$ and the value of ρ_H^D is obtained from the high temperature curves for which $\delta\rho_{xy} \to 0$.

Therefore according to the theory of the e-e interaction[5] one should observe a logarithmic temperature dependence of $\rho_{xy}/\rho_H^D - 1$ in the diffusive regime replaced by a hyperbolic decrease $1/T$ at higher temperatures. Figure 3 shows how this prediction works in our case.

A simple calculation (carried out without any attempt at fitting the experiment) results in the dashed curve.

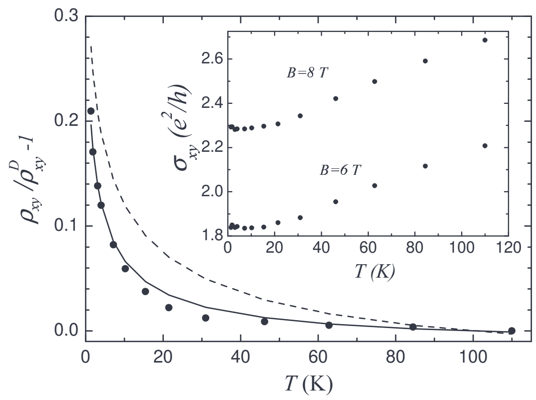

FIG. 3: Temperature dependence of the Hall coefficient (dots) compared to Eq. 3 (dash line) and to Eq. 3 with $\lambda = \frac{3\pi}{192}$ (solid line). The inset shows the transverse conductivity as function of T for two different values of magnetic fields.

In this calculation we used the value $F_0^\sigma \to \frac{1}{2}\frac{r_s}{r_s+\sqrt{2}} = -0.1$, calculated with the expression recommended in the theory[4] for the weak interaction limit. On the whole there is a qualitative agreement between theory and experiment (black dots) but the quantitative agreement is lacking. Using F_0^σ as a fitting parameter does not improve the agreement. Nevertheless we have found, that if the coefficient $\lambda = \frac{11\pi}{192}$ is replaced by $\lambda = \frac{3\pi}{192}$, then the theoretical curve (the solid line) fits the experimental dependence quite well. This result might be related to an anisotropy of electron scattering in the sample which reduces the electron return probability and so weakens the correction at low fields ($\omega_c\tau << 1$). The reduction of the pre-factor λ could just be the way in which this anisotropy reveals itself since the correction is proportional to λ in the ballistic limit.

Finally, in the inset to Figure. 3 we show the experimental data points for the transverse conductivity as a function of temperature for two different values of magnetic field. In our opinion these dependencies can serve as a good illustration for the transition between the diffusive and ballistic interaction regimes. Indeed, there should be no contribution of the WL[2] to the transverse conductivity tensor component. Also, in the diffusive regime[2] $\Delta\sigma_{xy}^{ee} = 0$. Thus, one would expect that $\Delta\sigma_{xy}$ would be T-independent at low T-range which is exactly what we see in the Inset. As for the intermediate and ballistic regime, up to date there have been no predictions concerning $\Delta\sigma_{xy}$. According to Ref. 4 the transition between the diffusive and the ballistic regime occurs at $k_B T\tau/\hbar \approx 0.1$ corresponding to $T \approx 30$ K in our sample. One can see that at about this temperature $\Delta\sigma_{xy}$ starts rapidly increasing. One can conclude that the theoretical results for $\Delta\sigma_{xy}$ in the diffusive limit are no longer valid in the ballistic transport regime. To our knowledge this is the first measurements of the σ_{xy} temperature dependence in the ballistic regime.

V. CONCLUSION

In conclusion, we have observed the transition from the diffusive to the ballistic regime in the weak interaction limit for the longitudinal conductivity, the Hall coefficient, and the Hall conductivity in a high density low mobility 2DEG. We find our experimental results to be in a good qualitative agreement with ZNA theory.

Acknowledgments

We are very grateful to A. Dmitriev, I. Gornyi, V. Tkachenko and S. Studenikin for enlighting discussions. This work was supported by PICS-RFBR (Grant No 1577.), RFBR (Grant No 02-02-16516), NATO, INTAS (Grant No 01-0014), programs "Physics and Technology of Nanostructures" of the Russian ministry of Industry and Science and "Low dimensional quantum structures" of RAS.

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