Discrete symmetries in general relativity the dark side of gravity

Frederic Henry-Couannier

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Frédéric Henry-Couannier
Centre de Physique des Particules de Marseille
henry@cppm.in2p3.fr

Abstract

Following the general method described in [1], the space/time exchange, parity and time reversal invariant actions, equations and their conjugated metric solutions are obtained in the context of a general relativistic model modified in order to suitably take into account discrete symmetries. The equations are not covariant however the predictions of the model, in particular its Schwarszchild metric solution in vacuum, only starts to differ from those of General Relativity at the Post-Post-Newtonian order. No coordinate singularity (black hole) arises in the privileged coordinate system where the energy of gravity is also found to vanish. Vacuum energies have no gravitational effects. A flat universe accelerated expansion phase in perfect agreement with observations is also obtained without resorting to inflation nor a cosmological constant and the Pioneer anomalous blue-shift is explained. The Left-Right asymmetry is a natural outcome and the anti-gravitational behavior of anti-particles is an interesting possibility.

I) Negative energies in special relativity

Let us first gather the information we learned from our preliminary investigation of negative energies in special relativity (see [1]).

- All relativistic field equations admit negative energy field solutions.
- Time reversal is the most natural symmetry to provide the link between positive and negative energy fields. Actually, in a non-quantum physics framework we would not have the choice since energy is the time component of a four-vector.
- The unitary operator choice is a usual one which moreover allows to avoid the well known paradoxes associated with time reversal.
- The non coupled positive and negative energy worlds are both perfectly stable and viable. It appears just as a matter of convention to define each one as a positive or negative energy world. It is only when we allow them to interact that new physics is expected to enter into the game.
- Positive and negative energy fields vacuum divergences we encounter after second quantization are unsurprisingly found to be exactly opposite.
- Our world could well be a pure left handed chiral one, making the maximal Parity and charge conjugation violations manifest themselves in any interaction involving Majorana particles.

However,

- If positive and negative energy fields are time reversal conjugated, their Hamiltonian densities or the actions from which they are derived, must also be so. For a scalar field with an Hamiltonian density which is just a squared terms sum and with a positive-definite integration it seems that there is no way for any symmetry to generate a negative Hamiltonian density from a positive definite Hamiltonian density. A possible solution to this issue could only be obtained in the context of general relativity where the metric transformation under discrete symmetries could generate the sign flipping of the general relativistic Hamiltonian densities.
- A trivial cancellation between vacuum divergences is certainly not acceptable since there exists well known accurate experiments showing evidence for such vacuum fluctuations, probably the most famous one being the Casimir effect. But as is well known, vacuum divergences are only a concern when gravity comes into the game. This is because any energy density source must affect the metric through the Einstein equations. Thus, the positive and negative energy worlds must be maximally gravitationally coupled in such a way as to produce exact cancellations of all such vacuum divergences gravitational effects. This was obtained naturally through the mechanism which we shall recall here.
- We for sure know that negative energy states never manifested themselves up to now, strongly suggesting that some kind of barrier is at work between the two worlds. Such barrier should not be artificially introduced but should come naturally as a result of fundamental .i.e. symmetry principles also needed to reach a coherent picture. We noticed that this is not possible as long as we restrict ourselves to non gravitational physics.
- It was shown that allowing both positive and negative energy virtual photons to propagate the electromagnetic interaction simply makes it disappear. We could build a model where the positive and negative energy worlds only interact gravitationally. In such model the gravitational interaction did not vanish.
- If both positive and negative energy worlds are separately stable, it is well known that a generic catastrophic instability arises whenever the positive and negative energy fields are allowed to interact [8][9]. This is because energy conservation does not forbid a positive energy object to absorb an unlimited amount of positive energy from a negative energy object which simultaneously falls into unbounded from below more and more negative energy states. In the quantum version, one says that because the phase space of the final states involving positive and negative energy particles is infinite, the vacuum decay rate into such states is also infinite, leading to a catastrophic instability. By allowing only gravity to propagate the interaction, we restrict the stability issue to gravity. But we will have to show that it is possible to avoid the above disastrous scenario thanks to an enriched model of gravitation properly taking into account discrete symmetries.
- The conjugated metric mechanism and extremum action procedure described in [1] and recalled hereafter gives rise to negative energy density sources in a modified Einstein equation in such a way that vacuum divergences cancellation can really take place. Then, the right handed fields as well as the negative energy ones are expected to live in conjugated metrics. Consequently, such right handed fields acquire reversed energy densities and charges as seen from the point of view of an observer living in the Left handed-world.

We shall try here to further explore in the same vein all possible discrete symmetries.

II) **Conjugated worlds gravitational coupling**

As is well known, general coordinate transformations do not leave the integration four volume invariant so a compensating Jacobi determinant modulus must be introduced to build a scalar integration volume. Then, if Parity and Time reversal transform the general coordinates, this will not affect our scalar actions however if the inertial coordinates ξ^α are also transformed in a non trivial way:

$$\xi^\alpha \xrightarrow{T} \tilde{\xi}_T{}^\alpha$$
$$\xi^\alpha \xrightarrow{P} \tilde{\xi}_P{}^\alpha$$

non trivial in the sense that in general $\tilde{\xi}^\alpha \neq \xi_\alpha$, our metric terms will be affected and our action is not expected to be invariant under P or T . It was a strong assumption (we believe unjustified) in general relativity to consider that discrete transformations only affect general coordinates or inertial coordinates in a trivial way. At the contrary, we find it natural to postulate the existence of a privileged coordinate system where for instance a discrete time reversal transformation applies as $x^0 \to -x^0$ and the inertial coordinates are transformed into another set. Let us anticipate that in this privileged coordinate system the metric will always adopt a diagonal form. Then it appears that for a given general coordinate system there exists not one but two time or parity reversal, conjugated inertial coordinate systems allowing to build, following the usual procedure, two time or parity reversal conjugated metric tensors:

$$g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\beta}{\partial x^\nu}$$
$$\tilde{g}_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \tilde{\xi}^\alpha}{\partial x^\mu} \frac{\partial \tilde{\xi}^\beta}{\partial x^\nu}$$

Then, a new set of fields will couple to our new $\tilde{g}_{\mu\nu}$ metric field and we shall see how such fields will eventually acquire a negative energy density from the point of view of our world**.** We also notice that in such picture, since our P and T symmetries jump from one set of inertial coordinates to another set of inertial coordinates, they keep discrete even in a general relativistic theoretical framework.

Now we have all we need in hands to deduce the modified Einstein equation. Let us first consider I_M , the usual action for matter and radiation in the external gravitational field $g_{\mu\nu}$ and $\tilde{I}_M$ the action for matter and radiation in the external gravitational field $\tilde{g}_{\mu\nu}$. Infinitesimal arbitrary variation of the metric fields produces a variation of our actions which takes the form:

$$\delta I_M + \delta \tilde{I}_M = \frac{1}{2}\int d^4x \sqrt{g(x)} T^{\mu\nu}(x) \delta g_{\mu\nu}(x) + \frac{1}{2}\int d^4x \sqrt{\tilde{g}(x)} \tilde{T}^{\mu\nu}(x) \delta \tilde{g}_{\mu\nu}(x)$$

where we recognize the familiar energy-momentum tensor $T^{\mu\nu}(x)$ for fields living in $g_{\mu\nu}$, for example $A_\mu(x)$, along with $\tilde{T}^{\mu\nu}(x)$ for fields living in $\tilde{g}_{\mu\nu}$, for example $\tilde{A}_\mu(x)$. The latter is obtained by varying $\tilde{g}_{\mu\nu}(x)$ with field $\tilde{A}_\mu(x)$ held fixed. Notice that because $A_\mu(x)$ and

$\tilde{A}_\mu(x)$ don't live in the same metric there is no way for them to interact. The two actions are built in such a way that they are separately general coordinate scalars. But the metric terms are non trivially affected under improper transformations so adding the two pieces is just necessary to obtain a discrete symmetry reversal invariant total action. Both P symmetric and T symmetric of our world positive energy fields will at the end of the story acquire a negative energy density field status from our world point of view.

Though there is no coupling in the usual sense between fields involved in the conjugated actions, $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are necessarily linked since these are symmetry reversal conjugated objects explicitly built out of space-time coordinates. Now, let us postulate that there exists a general coordinate system such that $\tilde{g}_{\mu\nu}$ identifies with $g^{\mu\nu}$. Because the equality is not one between tensors of the same kind it obviously does not apply in arbitrary general coordinate systems. A link is then established between our two metrics which had remained up to now completely independent. But though symmetry reversal conjugated, $A_\mu(x)$ and $\tilde{A}^\mu(x)$ remain completely dynamically independent fields contrary to $g_{\mu\nu}(x)$ and $\tilde{g}_{\mu\nu}(x)$ because they are not explicitly built out of space-time coordinates. In the privileged coordinate system our action variation becomes:

$$\delta I_M + \delta \tilde{I}_M = \frac{1}{2}\int d^4x\sqrt{g(x)}T^{\mu\nu}(x)\delta g_{\mu\nu}(x) + \frac{1}{2}\int d^4x\sqrt{g^{-1}(x)}\tilde{T}^{\mu\nu}(x)\delta g^{\mu\nu}(x)$$

Then, using the relation $\delta g^{\rho\kappa}(x) = -g^{\rho\mu}(x)g^{\nu\kappa}(x)\delta g_{\mu\nu}(x)$ one gets:

$$\delta I_M + \delta \tilde{I}_M = \frac{1}{2}\int d^4x\sqrt{g(x)}T^{\mu\nu}(x)\delta g_{\mu\nu}(x) - \frac{1}{2}\int d^4x\sqrt{g^{-1}(x)}\tilde{T}^{\rho\kappa}(x)g^{\rho\mu}(x)g^{\nu\kappa}(x)\delta g_{\mu\nu}(x)$$

When we eventually require that the total action variation should vanish under arbitrary field variations, the previous matter and radiation action will produce in Einstein gravitational equation positive and negative energy conjugated source terms of the form:

$$\frac{1}{2}\sqrt{g(x)}T_{\mu\nu}(x) - \frac{1}{2}\sqrt{g^{-1}(x)}\tilde{T}^{\mu\nu}(x)$$

For a cosmological constant source term:

$$T_{\mu\nu}(x) = \Lambda g_{\mu\nu}(x), \tilde{T}^{\mu\nu}(x) = \tilde{\Lambda} g_{\mu\nu}(x)$$

we get :

$$\frac{1}{2}g_{\mu\nu}(x)\left[\Lambda\sqrt{g(x)} - \tilde{\Lambda}\sqrt{g^{-1}(x)}\right]$$

The desired cancellation between vacuum energy terms will take place as in flat space time if everywhere:

$$\Lambda = \frac{\sqrt{g^{-1}(x)}}{\sqrt{g(x)}}\tilde{\Lambda} = \frac{1}{g(x)}\tilde{\Lambda}$$

But we can write :

$$\sqrt{g(x)} = \frac{d^4\xi(x)}{d^4x}; \sqrt{g^{-1}(x)} = \frac{d^4\tilde{\xi}(x)}{d^4x}$$
$$\Rightarrow g(x) = \frac{d^4\xi}{d^4\tilde{\xi}}(x)$$

so that the previous condition reads :

$$d^4\xi(x)\Lambda = d^4\tilde{\xi}(x)\tilde{\Lambda}$$

This relation would simply follow from the time (or parity) reversal invariance of a pure constant if the time (parity) reversal conjugated inertial four volumes are supposed to be equal. Not only such cosmological constant terms cancel but also, as can be checked easily for a perfect fluid, all field second quantization vacuum energies vanish as expected from special relativity if and only if:

$$\rho_{vac}(x) = \frac{1}{g(x)}\tilde{\rho}_{vac}(x),\ p_{vac}(x) = \frac{1}{g(x)}\tilde{p}_{vac}(x)$$

or

$$d^4\xi(x)\rho_{vac}(x) = d^4\tilde{\xi}(x)\tilde{\rho}_{vac}(x),\ d^4\xi(x)p_{vac}(x) = d^4\tilde{\xi}(x)\tilde{p}_{vac}(x)$$

If the conjugated inertial four volumes are again supposed to be equal, these relations would also follow from the equality of the conjugated scalar energy densities and pressures as expected from our previous analysis in a Special Relativistic framework.
Indeed, with

$$T_{\mu\nu} = p_{vac}g_{\mu\nu} + (p_{vac} + \rho_{vac})U_\mu U_\nu,\ \tilde{T}^{\mu\nu} = \tilde{p}_{vac}\tilde{g}^{\mu\nu} + (\tilde{p}_{vac} + \tilde{\rho}_{vac})\tilde{U}^\mu\tilde{U}^\nu$$

and $U^\mu, \tilde{U}^\mu$ the velocity four-vectors defined so that:

$$g^{\mu\nu}U_\mu U_\nu = -1$$
$$\tilde{g}^{\mu\nu}\tilde{U}_\mu\tilde{U}_\nu = -1$$

we then have :

$$\tilde{T}^{\mu\nu} = \tilde{p}_{vac}g_{\mu\nu} + (\tilde{p}_{vac} + \tilde{\rho}_{vac})U_\mu U_\nu$$

because $U_\mu = \tilde{U}^\mu$ follows from the fact that the above relations uniquely define the velocity four-vectors. Then the condition for the divergences to vanish is again satisfied :

$$T_{\mu\nu} = \frac{\tilde{T}^{\mu\nu}}{g}$$

or

$$d^4\xi T_{\mu\nu} = d^4\tilde{\xi}\tilde{T}^{\mu\nu}$$

Now the action $I_G + \tilde{I}_G$ for the gravitational field alone reads:

$$I_G + \tilde{I}_G \equiv -\frac{1}{16\pi G}\int d^4x\sqrt{g(x)}R(x) - \frac{1}{16\pi G}\int d^4x\sqrt{\tilde{g}(x)}\tilde{R}(x)$$

where $\tilde{R}(x)$ can be obtained from $R(x)$ through the replacements $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g^{\mu\nu}$. This is manifestly a scalar under general coordinate transformations as well as discrete P or T transformations. Under arbitrary variations of our metrics we get:

$$\delta I_G + \delta\tilde{I}_G \equiv \frac{1}{16\pi G}\int d^4x\sqrt{g(x)}\left[R^{\mu\nu}(x) - \frac{1}{2}g^{\mu\nu}(x)R(x)\right]\delta g_{\mu\nu}(x) + \frac{1}{16\pi G}\int d^4x\sqrt{\tilde{g}(x)}\left[\tilde{R}^{\mu\nu}(x) - \frac{1}{2}\tilde{g}^{\mu\nu}(x)\tilde{R}(x)\right]\delta\tilde{g}_{\mu\nu}(x)$$

But, working again in our privileged coordinate system, we get:

$$\delta I_G + \delta\tilde{I}_G \equiv \frac{1}{16\pi G}\int d^4x\sqrt{g(x)}\left[R^{\mu\nu}(x) - \frac{1}{2}g^{\mu\nu}(x)R(x)\right]\delta g_{\mu\nu}(x) + \frac{1}{16\pi G}\int d^4x\sqrt{g^{-1}(x)}\left[\tilde{R}^{\mu\nu}(x) - \frac{1}{2}\tilde{g}^{\mu\nu}(x)\tilde{R}(x)\right]\delta g^{\mu\nu}(x)$$

It's now time to require the total action to be stationary with respect to arbitrary variations in the metric field. We again make use of the simplifying relation $\delta g^{\rho\kappa}(x) = -g^{\rho\mu}(x)g^{\nu\kappa}(x)\delta g_{\mu\nu}(x)$ to obtain our new gravitational equation:

$$-8\pi G\left(\sqrt{g(x)}T_{\rho\sigma} - \sqrt{g^{-1}(x)}\tilde{T}^{\rho\sigma}\right) = \sqrt{g(x)}\left(R_{\rho\sigma} - \frac{1}{2}g_{\rho\sigma}R\right) - \sqrt{g^{-1}(x)}\left(R^{\rho\sigma} - \frac{1}{2}g^{\rho\sigma}R\right)_{g^{\rho\sigma}\to g_{\rho\sigma},\, g_{\rho\sigma}\to g^{\rho\sigma}}$$

This is of course not a general covariant equation. For example, through the replacement $g^{\mu\nu}(x) \to g_{\mu\nu}(x)$, $g_{\mu\nu}(x) \to g^{\mu\nu}(x)$ the affine connection becomes a quite strange object

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\rho}\left(\frac{\partial g_{\rho\mu}}{\partial x^{\nu}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\rho}}\right) \to \frac{1}{2}g_{\lambda\rho}\left(\frac{\partial g^{\rho\mu}}{\partial x^{\nu}} + \frac{\partial g^{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g^{\mu\nu}}{\partial x^{\rho}}\right)$$

which obviously violates the familiar 'rules' for summing or multiplying tensor-like objects. But it should be kept in mind that the above equation is only valid in our privileged working coordinate system so it is not intended to be generally covariant. However, once our metric solutions $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ have been determined in our privileged coordinate system, these can still be exported to any arbitrary general coordinate system in a straightforward way since these are tensors. Our point of view is that general covariance only needs to be satisfied as a symmetry property of the action we start with. The same for Parity and Time reversal transformations which can still be considered in general relativity as "discrete" symmetries since these are really jumping from one system of inertial coordinates to its conjugated. This is why we required our action to be a scalar under general coordinate transformations plus Parity or time reversal transformations. But our final field equations then follow from both a least action principle and a nontrivial relation between one metric and its discrete symmetry reversal conjugated. This relation involves a privileged coordinate system so that general covariance is not anymore a property of our modified gravitational field equations.

Explaining how a negative energy density can be generated through time reversal, we have now completed the program initiated in section VI [1] and by the way reinforced various results obtained there in the naïve signed volumes approach. We have also achieved the important task of introducing the negative energy density sources in the gravitational equation. The corresponding fields live in the conjugated metric which prevent them from interacting with our world fields except through gravitation. In the presence of a gravitational field at a given space-time point, there exists a locally inertial coordinate system ξ^{α} where our usual metric identifies with the Minkowski one and its first derivatives vanish. There is also another $\tilde{\xi}^{\alpha}$ locally inertial coordinate system where the conjugated metric identifies with the Minkowski one and its first derivatives vanish. At last, considering that we have two metric fields, it is not very surprising that there exists at least locally a third coordinate system where $g^{\mu\nu} = \tilde{g}_{\mu\nu}$. Because the two metrics are not independent the model is not actually bi-metric however. We may alternatively say that the gravitational field is a two-sided object, one side $g_{\mu\nu}$ where we live, and the other side, its inverse $g^{\mu\nu}$ where, from our metric point of view, the negative energy density fields live. Moreover, it is not obvious that such model admits a gravitational wave solution (this is only a meaningful statement in the privileged coordinate system).

It may also be that we could exceptionally find locally inertial coordinate systems satisfying $\tilde{\xi}^{\alpha} = \xi_{\alpha}$ where $g_{\mu\nu} = \tilde{g}_{\mu\nu} = \eta_{\alpha\beta} = \eta^{\alpha\beta}$, the first part of this relation being an equality between tensors must remain locally valid in any general coordinate system. In such case, which does not necessarily follow from a flat metric situation, we may say that there is no gravitational field in an absolute way. This opens the interesting perspective that then fields (particles) may be able to jump from one metric to its conjugated identical metric.

The minus signs needed to interpret our new energy-momentum sources as negative density sources naturally emerge from the extremum action and inverse metric mechanism, starting from a fully general coordinate, time reversal or parity scalar action. Therefore fields living in the reversed world are just seen from our world as negative energy density fields, a statement that we shall soon make more precise. Last but not least, divergences cancel provided vacuum action terms are equal in the conjugated inertial four-volumes. The Schwarzschild solution will be now derived and its physical outcomes explored. This will help us to further clarify in the next paragraph the meaning of the model.

III) The Schwarzschild solution and space reversal

We now want to solve our gravitational equation in the very important physical case of a static isotropic gravitational field. We assume that in our privileged coordinate system where our fundamental relation $g^{\mu\nu} = \tilde{g}_{\mu\nu}$ applies, the solution is isotropic and static. We start from the proper time interval written in the so-called isotropic form:

$$d\tau^2 = B(r)dt^2 - A(r)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

whose Jacobian modulus for our metric tensor is:

$$\sqrt{g\left(x\right)} = A(r)\sqrt{A(r)B(r)}\, r^2 \sin\theta$$

Obviously, in the polar coordinate system, $g^{\mu\nu}$ cannot be identified with a metric $\tilde{g}_{\mu\nu}$ because $g_{\mu\nu}$ is not dimensionless. Moreover, it is natural to require that in our privileged coordinate system, both conjugated metrics should in general possess the same isometries so here be static and isotropic. Thus the polar coordinates are not appropriate. An acceptable system is the Cartesian one where we have:

$$d\tau^2 = B(r)dt^2 - A(r)\left(d\mathbf{x}^2\right)$$
$$\left|\frac{\partial \xi^\alpha}{\partial x^\mu}\right| = A(r)\sqrt{A(r)B(r)}$$
$$d\tilde{\tau}^2 = \frac{1}{B(r)}dt^2 - \frac{1}{A(r)}\left(d\mathbf{x}^2\right)$$
$$\left|\frac{\partial \xi^\alpha}{\partial x^\mu}\right|^{-1} = \frac{1}{A(r)\sqrt{A(r)B(r)}}$$

and the gravitation equation reads (keeping only diagonal terms is enough for our needs):

$$\begin{bmatrix} A\sqrt{AB}G_{xx} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{xx}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{yy} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{yy}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{zz} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{zz}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{tt} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{B^2}G_{tt}\right]_{A\to 1/A;B\to 1/B} \end{bmatrix} = A\sqrt{AB}\begin{bmatrix} T_{xx} \\ T_{yy} \\ T_{zz} \\ T_{tt} \end{bmatrix} - \frac{1}{A\sqrt{AB}}\begin{bmatrix} \widehat{T}_{xx} \\ \widehat{T}_{yy} \\ \widehat{T}_{zz} \\ \widehat{T}_{tt} \end{bmatrix}$$

Where we define the covariant tensor $\widehat{T}_{\mu\nu} = \tilde{T}^{\mu\nu}$, which existence follows from $\tilde{g}^{\mu\nu} = g_{\mu\nu}$ in case of a perfect fluid. Then, because G, T, and $\widehat{\mathrm{T}}$ are tensors:

$$\begin{bmatrix} G_{xx} \\ G_{yy} \\ G_{zz} \\ G_{tt} \end{bmatrix} = C\begin{bmatrix} G_{rr} \\ G_{\theta\theta} \\ G_{\phi\phi} \\ G_{tt} \end{bmatrix}, \begin{bmatrix} T_{xx} \\ T_{yy} \\ T_{zz} \\ T_{tt} \end{bmatrix} = C\begin{bmatrix} T_{rr} \\ T_{\theta\theta} \\ T_{\phi\phi} \\ T_{tt} \end{bmatrix}, \begin{bmatrix} \widehat{T}_{xx} \\ \widehat{T}_{yy} \\ \widehat{T}_{zz} \\ \widehat{T}_{tt} \end{bmatrix} = C\begin{bmatrix} \widehat{T}_{rr} \\ \widehat{T}_{\theta\theta} \\ \widehat{T}_{\phi\phi} \\ \widehat{T}_{tt} \end{bmatrix} \text{ with } C = \begin{bmatrix} \left(\frac{\partial r}{\partial x}\right)^2 & \left(\frac{\partial \theta}{\partial x}\right)^2 & \left(\frac{\partial \phi}{\partial x}\right)^2 & \\ \left(\frac{\partial r}{\partial y}\right)^2 & \left(\frac{\partial \theta}{\partial y}\right)^2 & \left(\frac{\partial \phi}{\partial y}\right)^2 & \\ \left(\frac{\partial r}{\partial z}\right)^2 & \left(\frac{\partial \theta}{\partial z}\right)^2 & \left(\frac{\partial \phi}{\partial z}\right)^2 & \\ & & & 1 \end{bmatrix}$$

Substituting this in the above gravitational equation, the C matrix drops out and one obtains in the polar coordinate system:

$$\begin{bmatrix} A\sqrt{AB}G_{rr} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{rr}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{\theta\theta} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{\theta\theta}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{\phi\phi} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{\phi\phi}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{tt} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{B^2}G_{tt}\right]_{A\to 1/A;B\to 1/B} \end{bmatrix} = A\sqrt{AB}\begin{bmatrix} T_{rr} \\ T_{\theta\theta} \\ T_{\phi\phi} \\ T_{tt} \end{bmatrix} - \frac{1}{A\sqrt{AB}}\begin{bmatrix} \widehat{T}_{rr} \\ \widehat{T}_{\theta\theta} \\ \widehat{T}_{\phi\phi} \\ \widehat{T}_{tt} \end{bmatrix}$$

So we can compute $R_{\mu\nu}(x)-\frac{1}{2}g_{\mu\nu}(x)R(x)$ in the more convenient polar coordinate system, starting from the non vanishing metric tensor components:

$$g_{rr}=A(r),\ g_{\theta\theta}=r^2A(r),\ g_{\phi\phi}=r^2\sin^2\theta A(r),\ g_{tt}=-B(r)$$
$$g^{rr}=A^{-1}(r),\ g^{\theta\theta}=r^{-2}A^{-1}(r),\ g^{\phi\phi}=r^{-2}\sin^{-2}\theta A^{-1}(r),\ g^{tt}=-B^{-1}(r)$$

then get $\left(R^{\rho\sigma}-\frac{1}{2}g^{\rho\sigma}R\right)_{g^{\rho\sigma}\to g_{\rho\sigma},g_{\rho\sigma}\to g^{\rho\sigma}}$ readily in the polar coordinate system where the interpretation of the final solutions is easier. We first compute the non-vanishing components of the affine connection and insert them in the four non-vanishing components of the diagonal Ricci tensor. This yields:

$$R_{rr}=\frac{A''}{A}+\frac{B''}{2B}-\frac{B'^2}{4B^2}-\left(\frac{A'}{A}\right)^2+\frac{A'}{Ar}-\frac{1}{4}\frac{A'}{A}\left(\frac{B'}{B}\right)$$
$$R_{\theta\theta}=\frac{3}{2}\frac{A'r}{A}+\frac{A''r^2}{2A}-\frac{A'^2r^2}{4A^2}+\frac{1}{2}\frac{B'}{B}\left(\frac{A'r^2}{2A}+r\right)$$
$$R_{\phi\phi}=\sin^2\theta R_{\theta\theta}$$
$$R_{tt}=-\frac{1}{2}\frac{B''}{A}-\frac{1}{4}\frac{B'A'}{A^2}+\frac{1}{4}\frac{B'^2}{BA}-\frac{1}{A}\frac{B'}{r}$$

from which the curvature scalar readily follows:

$$R=2\frac{A''}{A^2}+\frac{B''}{AB}-\frac{B'^2}{2AB^2}-\frac{3}{2A}\left(\frac{A'}{A}\right)^2+4\frac{A'}{A^2r}+\frac{1}{2}\frac{A'}{A^2}\left(\frac{B'}{B}\right)+2\frac{B'}{AB}\left(\frac{1}{r}\right)$$

as well as the needed gravitational terms:

$$R_{rr}-\frac{1}{2}g_{rr}R=-\frac{1}{4}\left(\frac{A'}{A}\right)^2-\frac{A'}{Ar}-\frac{B'}{Br}-\frac{1}{2}\frac{A'}{A}\left(\frac{B'}{B}\right)$$
$$\frac{R_{\phi\phi}-\frac{1}{2}g_{\phi\phi}R}{\sin^2\theta}=R_{\theta\theta}-\frac{1}{2}g_{\theta\theta}R=-\frac{1}{2}\frac{A'r}{A}-\frac{A''r^2}{2A}+\frac{A'^2r^2}{2A^2}-\frac{1}{2}\frac{B'}{B}r-\frac{B''r^2}{2B}+\frac{B'^2r^2}{4B^2}$$
$$R_{tt}-\frac{1}{2}g_{tt}R=\frac{BA''}{A^2}-\frac{3B}{4A}\left(\frac{A'}{A}\right)^2+2\frac{BA'}{A^2r}$$

The terms $\left(R^{\rho\sigma}-\frac{1}{2}g^{\rho\sigma}R\right)_{g^{\rho\sigma}\to g_{\rho\sigma},g_{\rho\sigma}\to g^{\rho\sigma}}$ are then simply obtained through $A(r)\to A^{-1}(r),\ B(r)\to B^{-1}(r)$ starting from $R^{\mu\nu}(x)-\frac{1}{2}g^{\mu\nu}(x)R(x)$ where tensor indices were raised using the Cartesian metric components.

$$\left(R^{rr}-\frac{1}{2}g^{rr}R\right)_{A\to 1/A,B\to 1/B}=A^2\left\{-\frac{1}{4}\left(\frac{A'}{A}\right)^2+\frac{A'}{Ar}+\frac{B'}{Br}-\frac{1}{2}\frac{A'}{A}\left(\frac{B'}{B}\right)\right\}$$

$$\frac{\left(R^{\phi\phi}-\frac{1}{2}g^{\phi\phi}R\right)_{A\to 1/A,B\to 1/B}}{\sin^2\theta}=\left(R^{\theta\theta}-\frac{1}{2}g^{\theta\theta}R\right)_{A\to 1/A,B\to 1/B}=A^2\left\{\frac{1}{2}\frac{A'r}{A}+\frac{A''r^2}{2A}-\frac{A'^2r^2}{2A^2}+\frac{1}{2}\frac{B'}{B}r+\frac{B''r^2}{2B}-\frac{3B'^2r^2}{4B^2}\right\}$$

$$\left(R^{tt}-\frac{1}{2}g^{tt}R\right)_{A\to 1/A,B\to 1/B}=B^2\left\{\frac{A}{B}\left(-\frac{A''}{A}+2\frac{A'^2}{A^2}\right)-\frac{3A}{4B}\left(\frac{A'}{A}\right)^2-2\frac{A'}{Br}\right\}$$

In vacuum, when $AB=1$ the space-space equations are trivially satisfied:

$$G_{rr}^{total}=A\sqrt{AB}\left(R_{rr}-\frac{1}{2}g_{rr}R\right)-\frac{1}{A\sqrt{AB}}\left(R^{rr}-\frac{1}{2}g^{rr}R\right)_{A\to 1/A,B\to 1/B}=-2\frac{A}{r}\left(\frac{A'}{A}+\frac{B'}{B}\right)=0$$

$$G_{\theta\theta}^{total}=A\sqrt{AB}\left(R_{\theta\theta}-\frac{1}{2}g_{\theta\theta}R\right)-\frac{1}{A\sqrt{AB}}\left(R^{\theta\theta}-\frac{1}{2}g^{\theta\theta}R\right)_{A\to 1/A,B\to 1/B}=A\left\{-\frac{A'r}{A}-\frac{A''r^2}{A}+\frac{A'^2r^2}{A^2}-\frac{B'}{B}r-\frac{B''r^2}{B}+\frac{B'^2r^2}{B^2}\right\}=0$$

and the time-time equation reads :

$$G_{tt}^{total}=A\sqrt{AB}\left(R_{tt}-\frac{1}{2}g_{tt}R\right)-\frac{1}{A\sqrt{AB}}\left(R^{tt}-\frac{1}{2}g^{tt}R\right)_{A\to 1/A,B\to 1/B}=2B\left\{\frac{A''}{A}+2\frac{A'}{Ar}-\frac{A'^2}{A^2}\right\}=0$$

Substituting $A=e^a$, we will find new Schwarzschild solutions:

$$0=\{ra''+2a'\}=(a'r+a)'\Rightarrow a=\frac{2MG}{r}$$
$$\Rightarrow A=e^{\frac{2MG}{r}}\approx 1+2\frac{MG}{r}+2\frac{M^2G^2}{r^2}$$
$$\Rightarrow B=\frac{1}{A}=e^{-\frac{2MG}{r}}\approx 1-2\frac{MG}{r}+2\frac{M^2G^2}{r^2}-\frac{4}{3}\frac{M^3G^3}{r^3}$$

The metric solutions we get are different from the exact usual ones though in good agreement up to Post-Newtonian order:

$$A=\left(1+\frac{MG}{2r}\right)^4\approx 1+2\frac{MG}{r}+\frac{3}{2}\frac{M^2G^2}{r^2}$$
$$B=\frac{\left(1-\frac{MG}{2r}\right)^2}{\left(1+\frac{MG}{2r}\right)^2}\approx 1-2\frac{MG}{r}+2\frac{M^2G^2}{r^2}-\frac{3}{2}\frac{M^3G^3}{r^3}$$

Notice that our new solutions do not share the pathological behavior at finite radius B=0. The singularities at r=0 are normal since the source mass M confined within an asymptotically null volume becomes an infinite density source. No finite radius (black hole type) singularity arises in the Cartesian isotropic metric. Singularities will again show up if we export our metric solutions to the "standard" form of the metric but we have now good reasons to consider that this is due to a choice of coordinate system which does not respect the isotropy of our configuration for both conjugated metrics. Notice that in standard general relativity, if we are willing to allow the world an unusual topology, it is also possible to find a singularity free coordinate system but this is completely artificial.

The Newtonian potential $\phi(r) = -MG/r$ generated by the central mass with rest energy M living in the metric whose components are A and B is attractive for another object living in the same metric. But an object living in the conjugated metric which components are 1/A, 1/B will feel the repulsive Newtonian potential $\tilde{\phi}(r) = MG/r$. In the same way, it is straightforward to derive the conjugated potentials created by a source mass $\tilde{M}$ in $\tilde{g}_{\mu\nu}$ from the point of view of each metric :

$$\tilde{\phi}(r) = -\tilde{M}G/r \qquad \phi(r) = \tilde{M}G/r$$

Objects living in the same metric attract each other. Objects living in different metrics repel each other. This may be clarified by the following picture where the colored circles stand for the mass sources and the colored squared for the test masses.

Because, the conjugated metrics can be transformed into one another through r → -r, it is natural to identify the discrete transformation involved here to be a parity transformation instead of a time reversal one. Though as we shall later clarify, parity was not expected to generate negative energy one-particle states in a QFT framework, it is not surprising that a 3-dimensional volume reversal should be involved in a parity transformation leading (as well as for time reversal) to a negative energy **density** (see [1]).

IV) Einstein Equivalence Principle

The weak equivalence principle is obviously not menaced by the proposed model since once the metric field solution is established the behavior of matter and radiation living in the metric is described by the same action as in GR. But, because of the non covariance of our modified equation of motion, a violation of the strong equivalence principle seems unavoidable, especially a local Lorentz invariance violation. Indeed, our non covariant Einstein equation is only valid as it stands in a privileged local coordinate system, a locally rest frame. Exported by a Lorentz transformation it is expected to develop frame velocity dependent terms. The alpha Post Newtonian parameters constrain with tremendous accuracies the amount of local Lorentz invariance violation induced but such terms. However, as we shall now show, in our model the local Lorentz violation might only arise at the Post-Post Newtonian level of accuracy. All Post Newtonian parameters are found to be identical to the ones expected from GR, and in good agreement with experimental data. We follow the same notations as in [2] (chapter 9). Assuming there is no source term in the conjugated metric our equation reads:

$$-8\pi G T_{\rho\sigma} = \left(R_{\rho\sigma} - \frac{1}{2} g_{\rho\sigma} R \right) - \frac{1}{g} \left(g_{\rho\mu} g_{\nu\sigma} R_{\mu\nu\, g\to g^{-1}} - \frac{1}{2} g_{\rho\sigma} R_{g\to g^{-1}} \right)$$

Let us transform the equation into a similar form as the one in [2] 9.1.41 p217. It is straightforward to obtain :

$$-8\pi G \left(T_{\rho\sigma} - \frac{1}{2} g_{\rho\sigma} T \right) = R_{\rho\sigma} - \frac{1}{g} g_{\rho\mu} g_{\nu\sigma} R_{\mu\nu\, g\to g^{-1}}$$

In the Parametrised Post-Newtonian formalism, the metric solution are expanded in powers of GM/r :

$$g_{00} = -1 + \overset{2}{g}_{00} + \overset{4}{g}_{00} \quad , \quad g^{00} = -1 + \overset{2}{g}{}^{00} + \overset{4}{g}{}^{00}$$

$$g_{ij} = \delta_{ij} + \overset{2}{g}_{ij} \quad , \quad g^{ij} = \delta^{ij} + \overset{2}{g}{}^{ij}$$

$$g_{i0} = \overset{3}{g}_{i0} \quad , \quad g^{i0} = \overset{3}{g}{}^{i0}$$

From the inverse metric relation we can derive:

$$\overset{2}{g}{}^{00} = -\overset{2}{g}_{00}$$

$$\overset{2}{g}{}^{ij} = -\overset{2}{g}_{ij}$$

$$\overset{3}{g}{}^{i0} = \overset{3}{g}_{i0}$$

To this order, we get the same equations (redefining G→G/2) as in general relativity since :

$$\overset{2}{R}_{ij} - \overset{2}{R}_{ij\, g\to g^{-1}} = 2\overset{2}{R}_{ij}$$

$$\overset{3}{R}_{i0} + \overset{3}{R}_{i0\, g\to g^{-1}} = 2\overset{3}{R}_{i0}$$

$$\overset{2}{R}_{00} - \overset{2}{R}_{00\, g\to g^{-1}} = 2\overset{2}{R}_{00}$$

The Post-Newtonian approximation stops here for g_{ij} et g_{i0} so that the Post Newtonian parameters are identical to those in General relativity for such metric element solutions. We also have :

$$\overset{2}{g}_{ij} = \delta_{ij} \overset{2}{g}_{00}$$

this yields

$$\overset{2}{\sqrt{g}} = \frac{1}{2} \overset{2}{g}_{\mu\nu} \eta^{\mu\nu} = -\frac{1}{2} \overset{2}{g}_{00} + \frac{1}{2} \overset{2}{g}_{ii} = -\frac{1}{2} \overset{2}{g}_{00} + \frac{3}{2} \overset{2}{g}_{00} = \overset{2}{g}_{00}$$

thus

$$1/g = -2\overset{2}{g}_{00}$$

To the next order in g_{00} the following equation has to be satisfied:

$$\overset{4}{R}_{00} - \overset{4}{R}_{00\,g\to g^{-1}} + \overset{2}{\left(\frac{1}{g}\right)}\overset{2}{R}_{00} + 2\,\overset{2}{g}_{00}\,\overset{2}{R}_{00\,g\to g^{-1}} = \frac{1}{2}\left(-8\pi G\left[\overset{2}{T}^{00}+\overset{2}{T}^{ii}\right]+16\pi G\,\overset{2}{g}_{00}\,\overset{0}{T}^{00}\right)$$

where substituting $4\overset{2}{R}_{00} = -8\pi G\overset{0}{T}^{00}$ implies :

$$\overset{4}{R}_{00} - \overset{4}{R}_{00\,g\to g^{-1}} = -8\pi G\frac{1}{2}\left[\overset{2}{T}^{00}+\overset{2}{T}^{ii}\right]$$

but ([2] p217),

$$\overset{4}{R}_{00} = \frac{1}{2}\nabla^2\overset{4}{g}_{00} - \frac{1}{2}\frac{\partial^2\overset{2}{g}_{00}}{\partial t^2} - \frac{1}{2}\overset{2}{g}_{ij}\frac{\partial^2\overset{2}{g}_{00}}{\partial x_i\partial x_j} + \frac{1}{2}\left(\nabla^2\overset{2}{g}_{00}\right)^2$$

now using $\overset{4}{g}^{00} = -\overset{4}{g}_{00} - \left(\overset{2}{g}_{00}\right)^2$ allows to obtain:

$$\overset{4}{R}_{00} - \overset{4}{R}_{00\,g\to g^{-1}} = \frac{1}{2}\nabla^2\overset{4}{g}_{00} - \frac{1}{2}\nabla^2\left(-\overset{4}{g}_{00} - \left(\overset{2}{g}_{00}\right)^2\right) - \frac{\partial^2\overset{2}{g}_{00}}{\partial t^2} = \nabla^2\overset{4}{g}_{00} + \frac{1}{2}\nabla^2\left(\left(\overset{2}{g}_{00}\right)^2\right) - \frac{\partial^2\overset{2}{g}_{00}}{\partial t^2}$$

and finally

$$\nabla^2\overset{4}{g}_{00} = \frac{\partial^2\overset{2}{g}_{00}}{\partial t^2} - \frac{1}{2}\nabla^2\left(\left(\overset{2}{g}_{00}\right)^2\right) - 8\pi G\frac{1}{2}\left[\overset{2}{T}^{00}+\overset{2}{T}^{ii}\right]$$

which is exactly the result we had in GR ([2]9.1.63) provided the G constant is redefined as G→G/2 . This is the ultimate proof that all PN parameters are identical to those in GR and that Local Lorentz invariance violation does not arise at the Post-Newtonian order in the proposed model though it might well appear at the Post-Post-Newtonian order!

V) The Schwarzschild privileged coordinate system

The above determination of the Schwarzschild solution assumed a static and isotropic distribution of the gravitational source. Then the privileged coordinate system was determined to be the isotropic one. In any other coordinate system our non covariant equations would completely loose the simple form they had in such system because the conjugated metric would not satisfy the same isometries. Moreover a diagonal form of the metric seemed mandatory for the simple interpretation of our solutions in terms of a discrete parity transformation. Indeed, in this case transforming in a trivial way the privileged coordinates does not affect the metric. Only the transformation of the inertial coordinates does matter. At last, let us notice an interesting property of the solution we finally got i.e $A = e^{\frac{2MG}{r}}$. Add to M another contribution m and the metric solution simply becomes the product of $e^{\frac{2MG}{r}}$ and $e^{\frac{2mG}{r}}$. Hence, a kind of superposition principle seems here to apply as long as the two involved source distributions share the same isotropy property. Now what about a completely arbitrary and even non static distribution ? Obviously, there is no hope in such case to exhibit a single privileged coordinate system where the metric would stand in the above very simple form. Hence, if we believe that our above results were strong and meaningful ones needing to be generalized to any configuration we are led to postulate that there are as many local privileged

coordinate systems as we need in order to divide our source distribution into several isotropic and static ones, each having its privileged coordinate system where the above Schwarzschild treatment applies. It is allowed and even more tempting to divide our source distribution into point masses since the metric solution due to each component can then be evaluated in vacuum. This is because in such picture a given couple of conjugated metric has as a source only a single point mass and the metric solution is evaluated "outside this point source". Subsequently, all these local metric solutions must be exported into any common coordinate system where remains to be properly analyzed what will be the equation of motion of a given test particle under the influence of as many metric solutions as there are distinct mass point sources in the universe.
There remains however an important issue: for a mass-less particle there is of course no hope to determine any local rest frame even at the macroscopic classical scale.

VI) Gravitational energy in the Schwarzschild solution

For the above Schwarzschild solution we may compute the energy of the gravitational field by performing the difference between the total energy of matter plus gravitation in our privileged coordinate system minus the energy of matter only. According [II] p 302 this is:

$$\int_0^R 4\pi r^2 (1-\sqrt{A(r)B(r)})\rho(r)dr$$

where R stands for the radius of the source object. If this source has zero pressure, $B=\frac{1}{A}$ holds everywhere in the volume, and we find that the energy of gravitation vanishes.

But what about a nonzero pressure source? We just obtained that in our privileged coordinate system which were the rest frame of our zero pressure source (where all its subcomponents are also found to be at rest), energy and momentum of the gravitational field cancel. But we expect preferred frame effects such as local Lorentz invariance violation effects in our model though not at the Post–Newtonian level. So, performing a Lorentz transformation on this gravitational energy-momentum pseudo-Lorentz-vector, it will develop new frame velocity dependent terms in such a way that its components will not vanish anymore. This is not however a real concern since the components of the object we get in this new system cannot anymore be interpreted as representing the energy and momentum of gravity, such interpretation being only valid in the privileged rest frame of the source. We then understand why, for a source with non-vanishing pressure, the above computation that seems to imply that the gravitational energy would not vanish should not be taken serious. This is of course because we cannot anymore consider that such source subcomponents are all at rest. Therefore a single privileged coordinate system cannot be valid as we implicitly assumed and we just have no right to derive in the way we did the Schwarzschild solution in this case.

The gravitational energy associated with a point source mass is well defined and vanishes in its privileged rest frame and all translated frames. For an extended source with all its subcomponents at rest there is still no problem to define and find again a total vanishing energy and momentum thanks to this translation invariance of the theory. But when the pressure does not vanish we are in trouble, because the theory lacks Lorentz invariance and no energy and momentum can be defined for a source with subcomponents moving relative to each other. In this case one has to work with several privileged coordinate systems, there being in each one a null pressure thus a vanishing gravitational energy.

Working in a quasi-Minkowskian coordinate system we may want to show that fields following the geodesics of a given metric are seen from other fields living in the same metric as positive energy density fields while fields living in a given metric are seen as negative energy density fields from the point of view of fields living in the conjugated metric. All these energies will of course be only locally valid concept .i.e defined relative to a given privileged coordinate system. Let us postulate, as we did to study the Schwazschild solution, that locally our privileged coordinate system is a quasi-Minkowskian one where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with the perturbation $h_{\mu\nu}$ vanishing at infinity. The linear term in the geometric side of our new Einstein equation becomes:

$$G^{(1)}{}_{\mu\nu} = \left(R^{(1)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}R_{\lambda\rho}{}^{(1)} \right) - \left(R^{(1)\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\eta^{\lambda\rho}R_{\lambda\rho}{}^{(1)} \right)_{g^{\mu\nu}\to g_{\mu\nu};g_{\mu\nu}\to g^{\mu\nu}}$$

$$= 2\left(R^{(1)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}R_{\lambda\rho}{}^{(1)} \right)$$

so that this equation becomes simply :

$$-8\pi G\tau_{\mu\nu} = -8\pi G(t_{\mu\nu} + \sqrt{g}T_{\mu\nu} - \frac{1}{\sqrt{g}}\tilde{T}^{\mu\nu}) = G^{(1)}{}_{\mu\nu} = 2\left(R^{(1)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}R_{\lambda\rho}{}^{(1)} \right)$$

As in general relativity, the term on the right hand side satisfies the linearised Bianchi identity and is a Lorentz tensor. Therefore, the left hand side term could still be interpreted as a total radiation, matter and gravitation energy-momentum pseudo(general and Lorentz)-tensor conserved in the usual sense. In the weak field approximation, this would just be a conservation law of matter and radiation only which simplifies to:

$$\partial_\mu G^{\mu\nu(1)} = \partial_\mu(-\tilde{T}^{\rho\sigma}\eta_{\rho\mu}\eta_{\nu\sigma} + T^{\mu\nu}) = 0$$

The latter equation and all the previous ones follow from the Einstein equation of the model which we obtained by requiring that the variation of our action induced by the metric variation $\delta g_{\mu\nu}$ should vanish. Hence the conserved tensors we finally obtained are defined from the point of view of the $g_{\mu\nu}$ metric. We would obviously have obtained a similar result from the point of view of the conjugated metric, deriving in a similar way our equations expressed versus this metric. Now, what this equation seems to tell us is that we have a pseudo-tensor, conserved in the usual sense which energy and momentum components read:

$$T^{00} - \tilde{T}^{00} \ , \quad T^{0i} + \tilde{T}^{0i}$$
.

Hence the matter and radiation fields have their energy density reversed from the other metric point of view while their momentum keeps the same. This is perfectly coherent with what a Unitary time reversal or parity scenario led us to expect: energy density reverses but momentum is invariant under time reversal or parity transformations. Notice however that neither $\tau^{\mu\nu}$ nor $t^{\mu\nu}$, nor even $-\tilde{T}^{\rho\sigma}\eta_{\rho\mu}\eta_{\nu\sigma} + T^{\mu\nu}$ are Lorentz tensors as was the case in general relativity which reminds us that these are only valid as they stand in the single local rest frame privileged coordinate system where we already know that $t^{\mu\nu}$ vanishes as well as all momenta (not only T^{0i} and $\tilde{T}^{0i}$ since all subcomponents must also be at rest for such privileged coordinate system to remain valid).

Therefore these energy and momentum here defined and only meaningful with respect to a particular local couple of Schwarzschild metric solutions do not happen to be very useful concepts! This makes us suspect that such a local metric solution is not an appropriate background for various matter and radiation fields to interact with each other through common electromagnetic, weak or strong interactions, which is only possible if one can show that these are able to exchange globally well defined, at least under Lorentz transformations, energy and momentum quantities. Fortunately, we shall soon show that another couple of conjugated metrics will be the ideal candidate to play this role, in a global privileged coordinate system where both energy and momentum Lorentz tensors will be well defined. This additional global metric will thus provide a satisfactory common background for various fields to meet (interact with) each other.

VII) **Cosmology and time reversal**

We already derived a couple of parity conjugated purely spatial solutions in a privileged local rest-frame. We will know show that another couple of time reversal conjugated purely time dependent solutions can be derived from a new couple of conjugated actions. Our first task is to find a coordinate system where both metrics would appear in the spatially homogeneous and isotropic form appropriate for cosmology. Again the most general suitable working privileged system is here the Cartesian one:

$$d\tau^2 = c^2(t)dt^2 - a^2(t)f(k,r)d\mathbf{x}^2$$

where c(t) and a(t) are dimensionless scale factors. The FRW metric in polar coordinates and standard form reads:

$$d\tau^2 = c^2(t)dt^2 - a^2(t)\left\{\frac{dr'^2}{1-kr'^2} + r'^2 d\theta^2 + r'^2\sin^2\theta d\phi^2\right\}$$

The corresponding isotropic form reads:

$$d\tau^2 = c^2(t)dt^2 - a^2(t)f(k,r)\left\{dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right\}$$

The change of variable from r to $r' : r'^2 = f(k,r)r^2$ is such that :

$$\frac{1}{1-kr'^2} = \frac{1}{1-kf(k,r)r^2} = \frac{1}{\left(1+\frac{r}{2f(k,r)}\frac{df(k,r)}{dr}\right)^2}$$

Thus $f(k,r)$ is solution of the differential equation :

$$-kf(k,r)r^2 = \frac{r^2}{4}\left(\frac{f'(k,r)}{f(k,r)}\right)^2 + r\frac{f'(k,r)}{f(k,r)}$$

The only solution such that $f^{-1}(k,r) = f(k',r)$ (both metrics are required to be isotropic and homogeneous) with $k = \pm 1, 0; k' = \pm 1, 0$ is obtained for $k = k' = 0$ and $f(k,r) = 1$ so that in our coordinate system we get:

$$d\tau^2 = \left[c^2(t) = B(t)\right]dt^2 - \left[a^2(t) = A(t)\right]d\mathbf{x}^2$$

We see that our model predicts without resorting to inflation that our 3-dimensional space is flat. We then follow the same method as for the Schwarzschild solution to get in the polar coordinate system our modified cosmological Einstein equations:

$$G_{tr}^{\ tot} = A\sqrt{AB}\left(\frac{\dot{A}'}{A} - \frac{A'\dot{A}}{A^2} - \frac{1}{2}\frac{B'}{B}\frac{\dot{A}}{A}\right) - \frac{AB}{A\sqrt{AB}}\left(\left(\frac{-\dot{A}'}{A} + \frac{2A'\dot{A}}{A^2}\right) - \frac{A'\dot{A}}{A^2} - \frac{1}{2}\frac{B'}{B}\frac{\dot{A}}{A}\right) = 0$$

$$G_{rr}^{\ tot} = +A\sqrt{AB}\left\{\frac{\ddot{A}}{B} - \frac{1}{4}\frac{\dot{A}}{B}\left(\frac{\dot{A}}{A}\right) - \frac{1}{2}\frac{\dot{A}}{B}\left(\frac{\dot{B}}{B}\right)\right\} - \frac{A^2}{A\sqrt{AB}}\left\{-B\frac{\ddot{A}}{A^2} + 2B\frac{\dot{A}^2}{A^3} - \frac{1}{4}B\frac{\dot{A}}{A^2}\left(\frac{\dot{A}}{A}\right) - \frac{1}{2}B\frac{\dot{A}}{A^2}\left(\frac{\dot{B}}{B}\right)\right\} = -\frac{8\pi G}{3}\left(A\sqrt{AB}pA - \frac{A^2}{A\sqrt{AB}}\frac{\tilde{p}}{A}\right)$$

$$G^{tot}_{\ \ tt} = A\sqrt{AB}\left\{-\frac{3}{4}\left(\frac{\dot{A}}{A}\right)^2\right\} - \frac{B^2}{A\sqrt{AB}}\left\{-\frac{3}{4}\left(\frac{\dot{A}}{A}\right)^2\right\} = -\frac{8\pi G}{3}\left(A\sqrt{AB}\rho B - \frac{B^2}{A\sqrt{AB}}\frac{\tilde{\rho}}{B}\right)$$

We know need to get our purely time dependent solutions (this by the way insures that the mixed space-time equation is satisfied). Such solutions will obviously be Parity invariant since they do not at all depend on the spatial r coordinate. On the other hand the symmetry requirement $A \to 1/A, B \to 1/B$ when t reverses is a natural one for the metric solutions to be time reversal conjugated.

The privileged coordinate system such that $B^2 = A^3$ satisfies this requirement. Moreover, both time-time and space-space equations are coherent with a cold-cold universe (vanishing pressures) whose metrics do not exchange matter i.e. $\rho \approx \frac{M}{a^3}, \tilde{\rho} \approx a^3 M$. Indeed, our equations then take the simple form:

$$\frac{\ddot{A}}{A} - \left(\frac{\dot{A}}{A}\right)^2 = 0$$

$$-\frac{3}{4}\left(\frac{\dot{A}}{A}\right)^2 = -\frac{8\pi G}{3}M$$

And both the space-space cosmological equation and time-time one (after derivation) lead to the extremely simple result:

$$\frac{\ddot{A}}{\dot{A}} = \frac{\dot{A}}{A}$$

with solutions: $A = e^t, B = e^{3t/2}$ in the privileged coordinate system. We check that A,B→$1/A, 1/B$ under t→-t : these are time reversal conjugated and P invariant solutions.

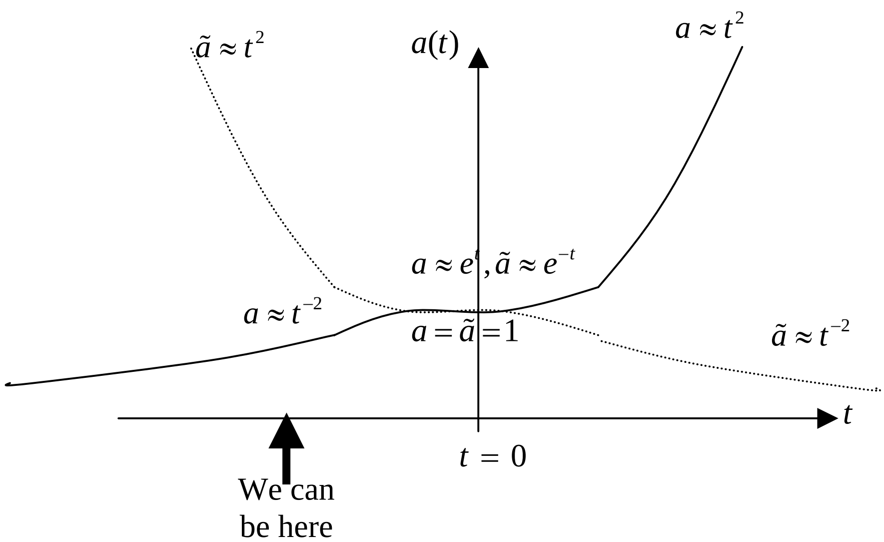
We could also determine in [1] another solution satisfying this strong requirement though the cold cold state is only approximately valid in particular ranges of the scale factor. The privileged coordinate system was the conformal time one where B=A . Quite interestingly B=A is also a requirement for the space-time exchange metric studied below. Moreover, B=A is invariant under Lorentz transformation. The scale factor evolution was driven by the following differential equations in the three particular domains:

$$a << 1 \Rightarrow \ddot{a} \approx \frac{3}{2}\frac{\dot{a}^2}{a} \Rightarrow a \approx \frac{1}{(t)^2} \text{ where } t < 0$$

$$a \approx 1 \Rightarrow \ddot{a} \approx \frac{\dot{a}^2}{a} \Rightarrow a \approx e^t$$

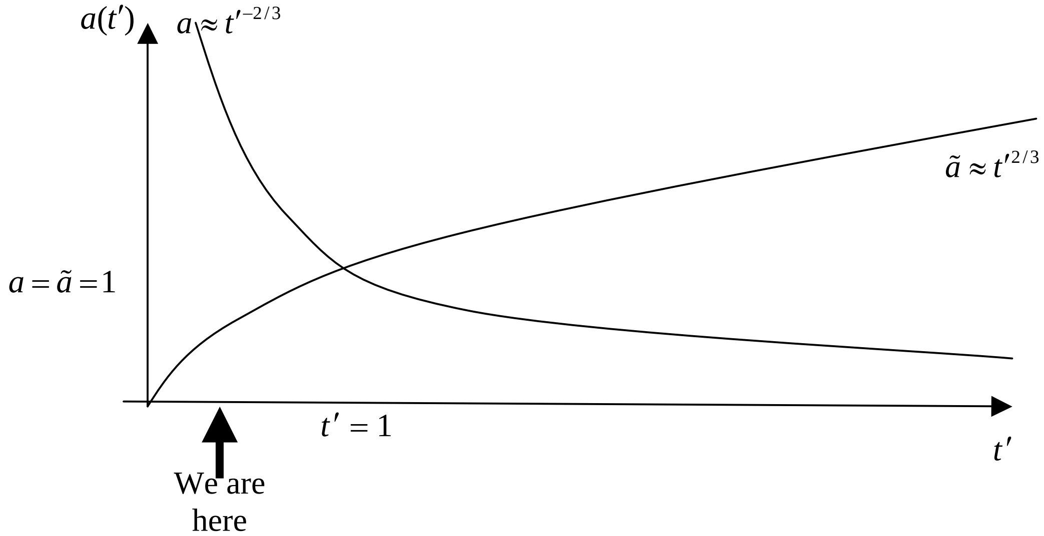
$$a >> 1 \Rightarrow \ddot{a} \approx \frac{1}{2}\frac{\dot{a}^2}{a} \Rightarrow a \approx t^2 \text{ where t } > 0$$

Thus we can check that t→-t implies $\frac{1}{t^2}$ → t^2 but also $e^t \to e^{-t}$ as required in the privileged coordinate system! We could also derive the evolution in the familiar comoving coordinate system, but we shall here follow another way assuming that we have already the correct coordinate system for an observer: a system where "photons are in free-fall" but not rest galaxies. Hence the photons wavelength keep unchanged during their path (require $d\tau = 0$ on the light path) while our atomic standards time intervals may contract or expand (require $d\boldsymbol{x} = 0$) . The evolution of the effective scale factor is then the following:



When our metric expands, time scales of the reference atoms are contracted while photons keep unaffected, resulting in an observable red-shift of their frequency. The two universes will cross each other and time reversal will occur in the future. Then, if instead of what we have in the picture, the two metrics exchange their roles our universe may start contracting.

Now let us have a closer look at the other exponential solution. In this case, the metric of the observer is necessarily the familiar comoving metric for rest galaxies while the photon wavelengths follow the metric evolution because the exponential behavior is observationally ruled out. Thus, the comoving scale factors of the conjugated metrics follow a $t'^{2/3}; t'^{-2/3}$ evolution which we might represent as follows:



At this point, we can just notice that either this solution stands for another independent couple of conjugated universes, or a non-trivial combination of the latter and the former is the correct answer. From the observational point of view, the $a \approx t'^{2/3}$ evolution alone seems already ruled out for our universe while both the $a \approx t^2$ alone and the combined $a \approx t^2 t^{-2/3} = t^{4/3}$ are very interesting. The combination would work only because t=0 implies t'=1.

In any case, it is natural to consider that the global privileged coordinate system involved here is the system where the CMB appears isotropic. Let us stress that our new couple of cosmological metric solutions does not imply any local gravitational interaction between objects but only a global one between the two conjugated universes. Localized objects contributing to a global energy density of the universe may be considered here to follow the geodesics of a slowly evolving locally non dynamical background described by the cosmological metric. Neglecting this slow evolution, just as in a Minkowski space we may define there the energy and momentum Lorentz tensors conserved as usual. Not all particles can be assumed to be at rest with respect to the global privileged coordinate system so that a momentum can be defined here for each particle. Since they do not interact locally, the energy sign from a given metric point of view of objects living on the other side of this couple of conjugated metric does not really matter. But for sure they contribute with an opposite sign to the global energy densities and pressures as can be read in the above equations.

There remains to be determined the exact form of our source terms in order to have the exact (not only approximate) compatibility between the evolutions derived from our space-space and time-time equations. It might even be that we could only clarify the issue by postulating a non conservation of matter in each metric through possible matter-energy exchanges between the conjugated metrics so that we would need new physics to enter into the game. However, with B=A and in the relativistic universe where $\rho = 3p = \frac{C}{A^2}, \tilde{\rho} = 3\tilde{p} = A^2 C$ and C is an arbitrary constant, the source terms vanish and a stationary solution trivially satisfies our equations.

VIII) **Stability issues**

In general relativity, the energy of gravitational radiation is always positive. Therefore negative energy sources are problematic for a negative energy particle would catastrophically fall to an infinite negative energy by radiating a positive infinite gravitational energy [8][9]. But for the time being, in none of our privileged coordinate systems such gravitational energy emission is a concern since there, gravitational waves cannot be accepted as solutions. Indeed, we could only determine stationary and cosmological solutions. Hence our model is absolutely non-local for the time being since there is nothing such as a graviton to carry the gravitational interaction.

The instability is usually also clearly seen in the phenomenology of a positive energy mass interacting with a negative energy mass through an usual interaction propagated by positive energy virtual interaction particles. The negative energy object is being attracted by the positive energy object, the latter being repulsed by the former. They then accelerate together for ever this resulting in an obviously instable picture! But in our model the local gravitational interaction between two masses living on different sides of a couple of conjugated metrics exhibits no such instability since they just repel each other. Yet, from the point of view of one of the two metrics, this is really the interaction between a positive energy mass and a negative energy mass.

Now as we shall see, another kind of symmetry than Parity or Time reversal should be found if we want to exhibit a solution involving both space and time coordinates.

IX) Space time exchange and the tachyonic metric

The latest Lorentz group representation which now also needs to be rehabilitated is the tachyonic one. In an isotropic configuration, the symmetry linking this representation to the bradyonic ones is naturally the transformation which exchanges the roles of space and time coordinates [11][12][13]. Quite impressively, another couple of solutions depending symmetrically on both space and time coordinates can be determined. Under $x \leftrightarrow t$, we expect: $B \to \tilde{B} = -A, A \to \tilde{A} = -B$ and all off diagonal mixed space-time terms left invariant. Hence, the transformation simply exchanges the role of the metric terms. The signature itself flips therefore the object living in the conjugated metric must appear as a tachyonic object from the point of view of the original metric [11][12][13]. If such metric terms are also Parity and Time reversal invariant then the complete action is obtained by adding to the usual one its $x \leftrightarrow t$ conjugated image. Then, following the same method as above, we can deduce in a straightforward way the differential equations satisfied by this new couple of conjugated metrics.

$A = B$ obviously satisfies the mixed space-time equation. Indeed, since $\delta \tilde{g}_{rt} = \delta g_{rt}$, the extremum action principle implies the trivial cancellation:

$$G_{tr} + G_{tr\,A,B=A-->-A,-A} = A^2\left(\frac{\dot{A}'}{A} - \frac{A'\dot{A}}{A^2} - \frac{1}{2}\frac{A'}{A}\frac{\dot{A}}{A}\right) - A^2\left(\frac{\dot{A}'}{A} - \frac{A'\dot{A}}{A^2} - \frac{1}{2}\frac{A'}{A}\frac{\dot{A}}{A}\right) = 0$$

$\delta \tilde{g}_{tt} = \delta g_{rr}$ in the same way allows to derive in vacuum the two remaining equivalent equations:

$$G_{rr} + G_{tt\,A,B-->-B,-A} = 0$$
$$G_{tt} + G_{rr\,A,B-->-B,-A} = 0$$

For example, the left-hand side of the former reads:

$$A\sqrt{AB}\left\{-\frac{1}{4}\left(\frac{A'}{A}\right)^2-\frac{A'}{Ar}-\frac{B'}{Br}-\frac{1}{2}\frac{A'}{A}\left(\frac{B'}{B}\right)\right\}_{B=A}$$

$$+A\sqrt{AB}\left\{\frac{\ddot{A}}{B}-\frac{1}{4}\frac{\dot{A}}{B}\left(\frac{\dot{A}}{A}\right)-\frac{1}{2}\frac{\dot{A}}{B}\left(\frac{\dot{B}}{B}\right)\right\}_{B=A}$$

$$-A\sqrt{AB}\left\{\frac{B''}{B}-\frac{3}{4}\left(\frac{B'}{B}\right)^2+2\frac{B'}{Br}-\frac{3}{4}\frac{B}{A}\left(\frac{\dot{B}}{B}\right)^2\right\}_{B=A-->-A}$$

$$=A^2\left\{-4\frac{A'}{Ar}+\frac{\ddot{A}}{A}-\frac{A''}{A}\right\}$$

This yields:

$$-4\frac{A'}{r}+\ddot{A}-A''=0$$

Thus the metric solution depending on both space and time coordinates must now satisfy a second order linear equation. We can express A(r,t) as a Fourier sum:

$$A(r,t)=\int a(k)U(k,r)\cos(kt)dk$$

where we are allowed to keep only the cosine terms since the result should be even in the time coordinate in order to satisfy the time reversal invariance requirement. Then, the equation satisfied by U(k,r) is of the form:

$$U''+4\frac{U'}{r}+k^2U=0$$

This is the equation of the radial part R divided by r of spherical free waves with l=1 [17] [18]. The most general free waves solution can thus be written as an infinite superposition of Bessel functions[17]:

$$A(r,t)=\int_0^\infty a(k)\frac{J_{3/2}(kr)}{r^{3/2}}\cos(kt)dk$$

And with $a(k)=k^{3/2}$ an invariant solution under both parity, time reversal and space-time exchange transformations is obtained [19]:

$$A(r,t)\propto\frac{1}{\left(r^2-t^2\right)^2}$$

It is very striking that there exists such a set ($a(k)=k^{3/2}$) of Fourier coefficients and this makes us confident that the approach is on the right way! Let us express the most straightforward interpretation of such solution. Objects following light-like ($r^2-t^2=0$) paths feel a singular metric while objects following time-like (resp space-like) paths are considered to feel a positive (resp negative) signature metric in the privileged coordinate system.

X) Discussion of our solutions and possible source terms

The rehabilitation of the Lorentz group discrete symmetries: time reversal, space reversal and space/time exchange and the new understanding of its representations in terms of fields living in various conjugated isotropic metrics now has reached a satisfactory level. We could determine:

1) Many couples of local stationary parity conjugated metric solutions, one for each individual static and isotropic source, may be one for each mass point:

$$A = e^{\frac{1}{r}}, B = e^{-\frac{1}{r}} \xrightarrow{r \to -r} \tilde{A} = e^{-\frac{1}{r}}, \tilde{B} = e^{\frac{1}{r}}$$

2) Two possible couples of global time reversal conjugated homogeneous and flat metric solutions for the universe as a whole:

$$A = e^{t}, B = e^{3t/2} \xrightarrow{t \to -t} \tilde{A} = e^{-t}, \tilde{B} = e^{-3t/2}$$

and

$$\text{t<0, } A = B << 1 \Rightarrow A = B \approx \frac{1}{t^4} \xrightarrow{t \to -t} \tilde{A} = \tilde{B} \approx t^4$$

$$A = B \approx 1 \Rightarrow A = B \approx e^{2t} \xrightarrow{t \to -t} \tilde{A} = \tilde{B} \approx e^{-2t}$$

$$\text{t>0, } A = B >> 1 \Rightarrow A = B \approx t^4 \xrightarrow{t \to -t} \tilde{A} = \tilde{B} \approx \frac{1}{t^4}$$

3) Many local couples of space/time exchange conjugated metrics:

$$A = B = \frac{1}{\left(r^2 - t^2\right)^2} \xrightarrow{r,t \to t,r} \frac{1}{\left(r^2 - t^2\right)^2} = -\tilde{B} = -\tilde{A}$$

The solutions are surprisingly simple as are the equations from which they follow. To be complete however, there remains to be clarified which kind of source may enter our equations and lead to a solution that still satisfies the symmetry manifest in the free solution. In the simpler case of time reversal or parity conjugated metrics, respectively only purely r or t dependent terms would have been allowed to enter on the source side. For the stationary case, as we already suggested, the impulse source term $\delta(r)$ is all we need to compute each independent gravitational field, and get the final metric expression as a result of a matrix multiplication (after a transformation from the various privileged coordinate systems into a common single one) superposition principle. For the cosmological case, we could determine apart from the empty universe (an impulse source term $\delta(t)$ universe?) non trivial possible source terms such that the time reversal symmetry is respected, for example the sources for a cold-cold evolving universe or a hot-hot stationary universe. In the same way, we are now tempted to consider that only a source 'compatible' with the space/time exchange symmetry of the allowed solutions should be seriously considered for "the equation of the third kind".
The source term might read:

$$\left(-B\sqrt{AB} \left(\frac{8\pi G}{3} \tilde{T}_{00} = S(r,t) \right) \right)_{B=A \to -A}$$

where we want to avoid the presence of the couple of space/time exchange conjugated metrics except in $B\sqrt{AB}$ since we believe that the linearity of our equation is a strong feature of this model and must be preserved. Indeed, we then obtain:

$$-4\frac{A'}{r}+\ddot{A}-A''=AS(r,t)$$

And again we are allowed to express A(r,t) as a Fourier sum:

$$A(r,t)=\int a(k)U(k,r)\cos(kt)dk$$

The differential equation satisfied by U(k,r) follows directly:

$$U''+4\frac{U'}{r}+\left(k^2+S(r,t)\right)U=0$$

Or, making the replacement U=R/r as in [17]:

$$R''+2\frac{R'}{r}+\left(\frac{-2}{r^2}+k^2+S(r,t)\right)R=0$$

which we recognize to be the equation of spherical waves with l=1 in a time dependent central isotropic potential $-S(r,t)$. Depending on the exact form of this potential, particularly its repulsive or attractive behavior, the solutions may possess a continuous and/or discrete spectrum of energy eigen-values [17][18]. Indeed our equation is just the familiar Schroedinger one in presence of a potential. In case we obtain a discrete energy spectrum, quite interestingly, a profound relationship might well arise between general relativity and quantum mechanics! Anyway, of course this will be valid only if we are able to find the Fourier weights that make the final sum satisfy the required space/time exchange symmetry.

Let us identify and discuss the three basic possible source terms that come to our mind:

- The impulse distribution $S(r,t)=\delta(r)\delta(t)$ source is of certainly of great importance since this is presumably the basic gravitational waves source. This is a good new since we had several metrics, but no gravitational wave solution for the time being in our privileged coordinate systems. Indeed, because this solution only depends on combinations of $r-t$ and $r+t$ we are tempted to regard it as a kind of green function G(r,t) for the above differential equation with source $\delta(r)\delta(t)$ and obtain its Fourier transforms, the associated propagators. We'd better identify $A(r,t)=\left|\frac{1}{r^2-t^2}\right|^2$ to be the squared modulus of a field a(r,t)=$\frac{1}{r^2-t^2}$ which ingoing and outgoing propagators are the Fourier transforms of this expression. We can alter this solution without loosing the good behavior under the space-time exchange symmetry writing it as:

$$A(r,t)\propto\frac{1}{\left(r^2-t^2-i\lambda^2\right)\left(r^2-t^2+i\lambda^2\right)}$$

which makes the singularity in the metric disappear, a good reason to believe in the imaginary term (probably originated from another source in our equation, see below). Another good reason is that using Cauchy's theorem, the integrals can be solved in close form to give us a massive or massless propagator if we take the limit $\lambda \to 0$:

$$\hat{a}(E,p) = \frac{\delta(E \pm p)}{p}$$

Where p is positive for an outgoing propagator and negative for an ingoing propagator and the sign of E is fixed (a stability requirement!). Thus the propagator of the gravitational field is always on-shell, meaning that it is probably not a quantum field! Indeed, the QFT time ordering has obviously no effect for a commutative field resulting into the disappearing of the Theta functions and eventually the above on-shell propagator in the same way as if (though this is here a physically much stronger statement) we were adding as in [1] the contribution of positive and negative energy propagators. As a consequence the arrow of time would also disappear for this gravitational field. The absence of a virtual graviton to carry the gravitational force implied by the above Schwarzschild solution simply means that this particular local (as opposed to global) gravitational interaction is un-propagated i.e instantaneous in the privileged coordinate system.

- Distribution sources of the kind $S(r,t) = \delta(r \pm t)$ seem ideally suited to obtain the gravitational field generated by mass-less objects, for example electromagnetic waves. This is also a very good new since such kind of sources cannot be handled in the equations satisfied by the Parity conjugated couple of metrics because it is impossible to exhibit a privileged system where a mass-less object would be stationary. Here again the program can be successfully carried on for any electromagnetic source distribution, making use of several privileged coordinate systems and transporting the various solutions into a single coordinate system where we shall postulate that a multiplicative superposition principle again applies.

- The constant distribution $S(r,t) = 1/\lambda^2$, leading to the following form of our equation:

$$R'' + 2\frac{R'}{r} + \left(\frac{-2}{r^2} + k^2 + \frac{1}{\lambda^2}\right)R = 0$$

We can notice that quite interestingly, a cosmological source term such as :

$$S(r,t) = \frac{8\pi G}{3} T_{00}(r,t) = \frac{8\pi G}{3}\left(\frac{M}{A^{3/2}{}_{\cos mo}} B_{\cos mo}\right) = \frac{8\pi G}{3} M$$

amounts to a constant distribution thanks to $B_{\cos mo} = A^{3/2}{}_{\cos mo}$. If we are lucky the mass of our graviton might come with the solution of our equation in presence of this constant source.

X) Antiparticles and parity

We could show in [1] that a purely Left-handed Lagrangian involving only the combination $\psi_L(x) + \psi_{cL}(x)$ could satisfactorily describe all known physics (at least kinetic and interaction terms) and provide an interesting explanation for maximal parity violation. We already explained why we do not believe that fields living in the time reversal conjugated metric could interact locally with our fields except through global gravity. This is because, most probably, fields cannot meet each other even if they live on the same side of a couple of parity conjugated metric. We mean that they would really need to live on the same side of the global time reversal conjugated metrics for the usual electromagnetic, weak and strong interactions to take place. This is because globally well defined energy and momentum are certainly needed if we would like to impose the conservation of such quantities at each vertex as always in quantum field theory.

On the other hand, in principle, the parity conjugated fields living in the parity conjugated metric can still share the global metric with Left handed fields $\psi_L(x)$ and interact with them. It is easy to show that in this case, the parity conjugated field acquires a reversed charge allowing us to identify it with $\psi_{cL}(x)$. This was demonstrated in [1] section VI.1 in the naïve Jacobi determinant approach. It was also foreseen that this field would acquire a negative energy density, it being understood that this energy is only defined with respect to the couple of local parity conjugated metric. The field living in the parity conjugated metric will interact anti-gravitationally with the particle field on the other side of the couple of conjugated metrics. Thus, in our model it is possible (though not mandatory) to obtain as in [10] that antimatter interacts anti-gravitationally with matter though they are found to possess the same sign of the energy at each vertex, energy defined with respect to the global couple of conjugated metrics where they both live on the same side.

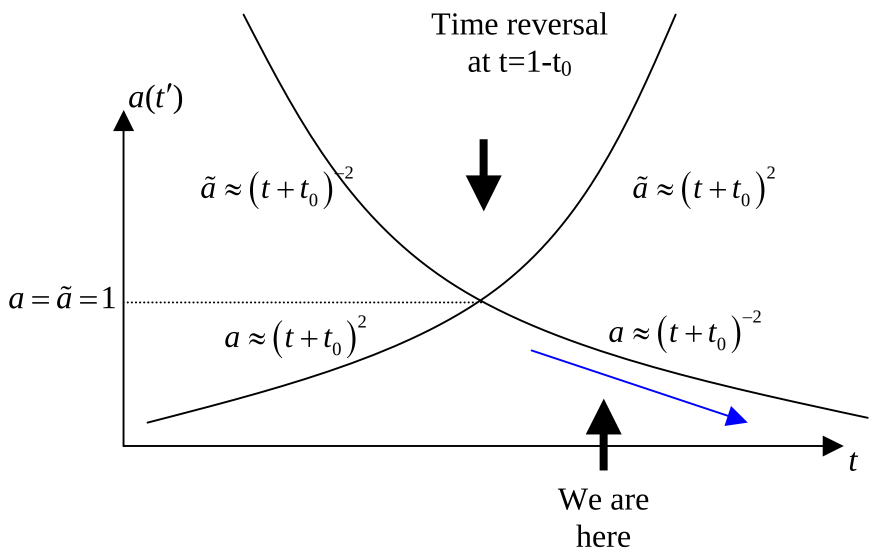
XI) Phenomenological consequences

If we are wishing to investigate the observational outcomes of our model, there remains to be clarified, once the various metric solutions have been obtained by solving our equations in their privileged coordinate systems, how a particle test will behave in such a "many metric background". Eventually, there must remain a single object playing the role of a metric for a given test particle that will follow its geodesics. Indeed, we cannot imagine that a particle follows the geodesics of many metrics at the same time: this is clearly nonsense at least if we keep on working with a single coordinate system (we might be tempted to work with many 'parallel' coordinate systems) since the space or time intervals deduced according to these various metrics disagree. It remains that from the resolution of our fundamental equations we have many metric-like objects in the sense that each one alone could play the role of a metric. But the unique genuine metric for a test particle has to be a combination of these objects. Therefore an exhaustive study of mathematical structures allowing to combine various metric-like objects into a genuine metric should first be carried on. However, we already remarked that there is a kind of superposition principle that seems valid in the form of a product law for metric matrix elements at least for the exponential solutions associated with two local stationary source distributions sharing the same center of isotropy, for example two neighbor mass sources. Of course, this can only work if we can take apart the signature minus signs in order to get a unit element for the matrix product law in place of the Minkowski matrix. This in turn will certainly require in the near future that we work with imaginary inertial coordinates and more generally complex coordinates and fields. But just to be able to derive observational predictions in the simplest case let us first start from a reasonable working hypothesis: one can combine in the same way, i.e a simple product law of their matrix elements (except the signature), any set of metric-like objects sharing the same isotropy center.

In the vicinity of a massive body, the gravitational field is presumably the superposition of :

- The cosmological solution
- The Schwarzschild solution
- The gravitational field generated by gravitational waves

We already noticed that our Schwarzschild solution only differs from that of GR at the PPN order. The interesting new feature is that there is no suppression effect of the cosmological gravitational field by the local gravitational field generated by the massive body. Hence, the signal from an object at several Astronomical Units as are the Pioneer aircrafts should appear very slightly cosmologically red-shifted. This very tiny effect has actually been measured to a very good precision with the expected magnitude (short light travel compared to the cosmological distances involved in the usual red-shift observations) but the wrong sign! Actually a blue-shift is being observe. But this also can be accounted for in this model as soon as we realize that locally the conjugated metrics (cosmological scale factor combined with the local gravitational field) can cross each over earlier in the presence of a gravitational source and this can produce the exchange of the two metrics. Hence the two conjugated metric exchange their roles in the volume around the source where the following plots describe the evolution of the scale factor:



Because the exchange takes place inside a volume we are still in a red-shift regime outside but in a blue shift regime inside the volume: there is as a phase transition at the frontier delimiting the domain where the local gravitational field is dominant and the outside where cosmology dominates. It was believed that expansion only applies on the largest scales leaving the smallest scales rigid. This was a conventional General Relativity picture working thanks to the well known GR suppression effect. Here the picture is only modified at small scales where structures are not rigid but get contracted.

Now if the photon emitted by a distant galaxy has been red-shifted along its million light years travel toward us and blue-shifted only in the vicinity of our galaxy we also expect a negligible impact on the magnitude vs red-shift relation for such distant objects thanks to the smooth behavior of the scale metrics near the crossing region.

The third kind of gravitational effect is generated by gravitational waves. For a punctual stationary body $\delta(r)$ we get the generated complex gravitational field by integrating the Green function :

$$a(r) = \alpha \int \frac{dt}{r^2 - t^2 + i\lambda^2}$$

with α a new multiplicative constant.
The integrand has two simple poles:

$$t_\pm = \pm^{1/4}\sqrt[4]{r^4 + \lambda^4}\, e^{\frac{i}{2}Arctg\left(\lambda^2/r^2\right)}$$

Integrating in the upper half plane we shall need the residue of the pole t_+:

$$\mathrm{Re}\, s(t_+) = \frac{1}{\left[d\left(r^2 - t^2 + i\lambda^2\right)/dt\right]_{t=t_+}} = -\frac{1}{2^{1/4}\sqrt[4]{r^4 + \lambda^4}\, e^{\frac{i}{2}Arctg\left(\lambda^2/r^2\right)}}$$

therefore:

$$a(r) = 2i\pi \mathrm{Res}(t_+)$$

and the real metric reads:

$$1 + \left|a(r)\right|^2 = 1 + \alpha\pi^2 \frac{1}{\sqrt{r^4 + \lambda^4}}$$

which should arise as an anomalous acceleration when r is becoming larger that λ .

XIII) Conclusion

We could settle down here the foundations for a new gravitation theory. This theory is essentially general relativity enriched to take into account in the various discrete symmetries involved in the structure of the Lorentz group i.e. linking its fundamental representations. We started to explore the phenomenological implications and could obtain already interesting results.

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