EPJ manuscript No.
(will be inserted by the editor)

Seniority scenario for the $^{68-72}$Zn and $^{66-68}$Ni B(E2)⇑ difference

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Received: date / Revised version: date

Abstract. In the seniority scheme the reduced B(E2: $0^+_1 \to 2^+_1$) (B(E2)⇑) transition probability of even-even nuclei can be related to the product of the number of particles by the number of holes of the valence space. This very simple expression is used to analyze at the same time the experimental B(E2)⇑ values of $^{56-68}$Ni and those of $^{62-72}$Zn. The evolution of these B(E2)⇑ values with neutron number fits in with a scenario involving p-n interaction.

PACS. 21.10.-k Properties of nuclei : nuclear energy levels – 23.20.-g Electromagnetic transitions – 21.30.-x Nuclear forces – 21.60.-n Nuclear structure models and methods – 27.40.+z 39≤A≤58 – 27.50.+e 59≤A≤89

1 Introduction

The B(E2: $0^+_1 \to 2^+_1$) reduced transition probability, B(E2)⇑, is correlated to the possibilities to get a 2^+_1 state from excitations in the single-particle spectrum underlying the 0^+_1 one. Its value is then very sensitive to the (sub-)shell structure. Low B(E2)⇑ values are obtained for doubly closed-shell nuclei in which any excitation requires to overcome the gap. High values are reached at mid-shell where the product of the number of valence particles by the number of single-particle levels available for the excitations is maximal. In order to study the shell structure of neutron-rich nuclei with $N \simeq 40$, the B(E2)⇑ have been measured in the ^{68}Ni [1] and ^{72}Zn nuclei [2] by coulomb excitation experiments performed at Ganil. The results put light on an opposite behavior of the B(E2)⇑ evolution above N=38 between the Ni and the Zn isotopes. This difference is the subject of numerous research studies. Recently, experiments concerning the B(E2)⇑ values in the two next neutron rich isotopes, ^{74}Zn and ^{70}Ni have been performed at Ganil [3] and at REX-ISOLDE [4], but their results are not yet available. Two theoretical papers discussing the B(E2)⇑ of the Ni isotopes only in relation with a N=40 gap and its size have been published [5,6]. In the present work, on the basis of simple calculations performed within the seniority framework and assuming different (sub-)shell structure, we propose an overall interpretation of the evolution of the B(E2)⇑ curves from the Ni up to the Zr isotopes. The p-n interaction plays a large part in our scenario (see [7]) which, in this paper, is detailed for the Ni and Zn isotopes only.

2 A simple formula for B(E2)⇑

In fig.1 are shown, for N between 20 and 42, the known experimental B(E2)⇑ values of the Ca, Ti, Cr, Ni and Zn isotopes. For N between 38 and 40, the B(E2)⇑ value decreases in the Ni but increases in the Zn isotopes. This is explained in ref. [2] as due to an increase of the deformation with the neutron number N in the Zn isotopes. Consequently, the number of single-particle levels in the neutron valence space increases which increases the B(E2)⇑ . Moreover, the onset of deformation allows the quadrupole excitations in ^{70,72}Zn to avoid the hindering of their 1p-1h component due to the parity of the $\nu 1g9/2$ orbital [2]. On the contrary, in the spherical ^{68}Ni (N=40, Z=28) the 1p-1h excitations are hindered and a very low B(E2)⇑ value is observed [2],

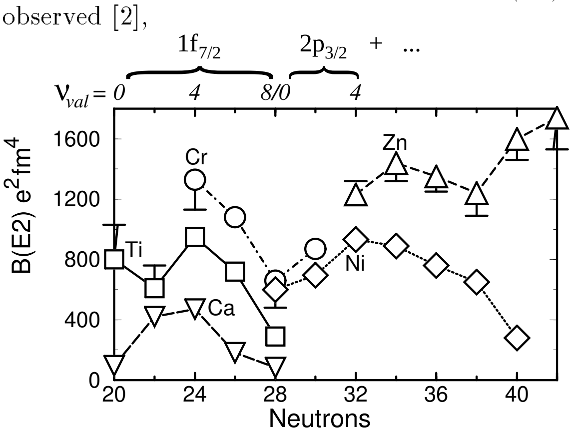

Fig. 1. Experimental B(E2)⇑ values of Ca, Ti, Cr, Ni and Zn isotopes from [8]. Only one half of the error bar, when larger than the symbols, is drawn. The number of valence neutrons (ν_{val})in the $\nu 1f_{7/2}$ and in $\nu 2p_{3/2}$ are indicated at the top.

In fig.2 we show the $R_4 = E(4^+_1)/E(2^+_1)$ ratios for the Ni and the Zn isotopes. The near-magic and the vibrator limits of ref.[9] are also indicated. For N between 30 and 42 the striking parallelism of the Zn and Ni curves indicates a constant difference of deformation between the

Ni isotopes and the Zn ones. For N between 38 and 40, the R_4 values are decreasing, nearly down to the magic limit in the Ni isotopes and down to the near-magic limit (R_4=2.0) in the Zn ones. Fig.2 indicates thus a decreasing deformation in ^{64,68}Ni and ^{66,70}Zn. Even if one may have reservations about this conclusion, because the R4 ratio brings into play two more units of spin than the B(E2)$\Uparrow$, no doubt is possible about the rather small deformation of ^{70}Zn and ^{68}Ni. For such nuclei the single-particle levels evolve slowly as a function of the deformation (cf. a Nilsson diagram) and there are as few single-particle levels going up and crossing the Fermi level than going down. Therefore an increase of deformation in ^{66,70}Zn – unless it is a considerable one, which the Zn R_4 ratios totally exclude – would not increase the single-particle level density, no more than the B(E2)$\Uparrow$ value. Another explanation has thus to be found.

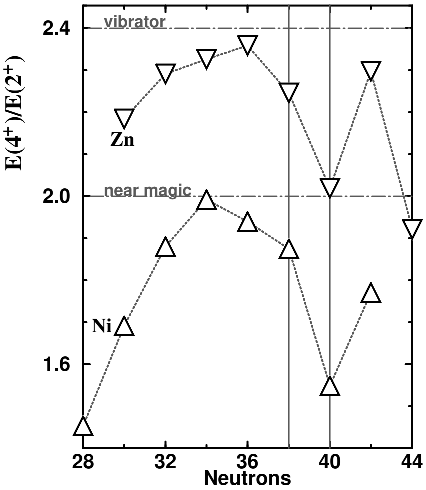

Fig. 2. Experimental $E(4_1^+)/E(2_1^+)$ values for the Ni and Zn isotopes (dotted gray lines and symbols) obtained from the ENSDF file [10]; near-magic and vibrators nuclei limits from [9] (dot-dashed lines.)

One can note in fig.1 the common parabolic profile of the B(E2)$\Uparrow$ curves, as obtained in the seniority scheme, and for N between 32 and 38 the similarity of the Ni and Zn curves. This leads us to assume that protons as well as neutrons are paired in the Ni as they are in the Zn isotopes and to apply, in the search of new clues, the seniority scheme. It is worth noting that to consider paired neutrons excludes that the neutrons 1p-1h excitations could play in the B(E2)$\Uparrow$ value the part assumed in [2].

In the seniority scheme, reducing the nucleus to $N_{val.}$ valence particles (neutrons or protons) interacting in a single j-shell ($1f_{7/2}$ for example) only by pairing, it can be shown [11] that :

$$\mathrm{B(E2)}\Uparrow_{seniority} \propto \mathrm{B(E2)}\Uparrow_{approx.} \quad (1)$$

$$\text{where} \quad \mathrm{B(E2)}\Uparrow_{approx.} = \frac{N_{val.}}{2} \times \left(\frac{\Omega_j}{2} - \frac{N_{val.}}{2}\right) \quad (2)$$

$$= N_{part.} \times N_{hol.}. \quad (3)$$

$N_{part.}$ ($N_{part.} = \frac{N_{val.}}{2}$), $N_{hol.}$ and $\Omega_j = (2j+1)$ are the number of particle pairs, the number of hole pairs and twice the number of levels in the single-j shell. The profile of B(E2)$\Uparrow_{approx.}$ is parabolic, with a maximum at mid-shell $N_{val.}= \Omega_j/2$ and two symmetrical minima, one at the beginning of the shell $N_{val.}$=0, the other at its end $N_{val.}=\Omega_j$. A known example of such a profile is given in fig.1 as a function of the $\nu_{val.}$ valence neutrons by the B(E2)$\Uparrow$ curve of the Ca isotopes series,which is a textbook case for the seniority model [9,12]. One can also see in fig.1 that the Ni B(E2)$\Uparrow$ curve is, for N between 28 and 38, as regular as the Ca one. And yet, up to three different neutron single j-shells are filled consecutively when going from ^{58}Ni to ^{68}Ni. In first approximation we can then consider these single-j shells as different members of one large and composite valence shell, each contributing with the same weight to the B(E2)$\Uparrow$ value. Having only in mind to analyze the B(E2)$\Uparrow$ curves in a very phenomenological way[(1)], we will extend the expression of B(E2)$\Uparrow_{approx.}$ obtained in the seniority scheme to such a composite valence shell by replacing in (2) Ω_j by $\Omega = \sum \Omega_j$. This is equivalent to considering a composite shell as a large single-j shell. In this approach the regular profile up to N=38 of the Ni and Zn B(E2)$\Uparrow$ curves (see fig.1) results from the existence of a lower and an upper spacing between single-particle orbitals (a gap if large or a sub-shell closure if small) confining the several single-j shells in one valence shell. Any sharp discontinuity, such as in the Zn B(E2)$\Uparrow$ curve at N=38, can be interpreted as due either to the end of a neutron valence at its upper spacing, or to the disappearance of one of the spacings. Indeed, if the upper (or lower) spacing disappears, the valence shell will be enlarged to the shell above (or below) it. Immediately $N_{hol.}$ (or $N_{part.}$) will increase, the B(E2)$\Uparrow$ value, related to their product, will suddenly be enhanced. But such a change will also suddenly increase the deformation, and we have excluded this. The Zn B(E2)$\Uparrow$ curve highlights then a spacing at N=38. A careful look to the experimental Ni B(E2)$\Uparrow$ curve reveals one feature which can be also related to a spacing at N=38 : the value at N=38 is nearly equal to the one at N=28. It is worth noting that the B(E2)$\Uparrow$ curves in the Ge (Z=32) and Se (Z=34), as in the Zn isotopes, are increasing for N between 38 and 40 (and not decreasing as in the Ni). For the Se, N=38 is even a minimum. The hypothesis of a N=38 sub-shell closure explains then more B(E2)$\Uparrow$ features of this region than the one of a doubly magic ^{68}Ni. It remains to explain why the ^{68}Ni B(E2)$\Uparrow$ value is the lowest one of the Ni series, N=28 and 38 included. To do so, within our hypothesis, we need more information about the expected behavior of the B(E2)$\Uparrow$ curves of this region. In the next section we thus compare experimental B(E2)$\Uparrow$ curves with the ones calculated with various proton and neutron valence spaces.

3 Comparison with calculations

We take into account both type of particles by summing the proton and the neutron contributions. Replacing in

[(1)] It is important to note that the extension of the single j-shell case to composite valence shell, through the quasi-spin formalism (without weighting) or the generalized seniority one, is the basis of the boson models [13] widely used to describe collective states.

eqs.(3) $N_{part.}$, $N_{hol.}$ by $\nu_{part.}$, $\nu_{hol.}$ for the neutrons particle and hole pairs, and by $\pi_{part.}$, $\pi_{hol.}$ their equivalents in proton, we have computed the following expression :

$$\mathrm{B(E2)}\Uparrow_{gen.} = [(\nu_{part.} \times \nu_{hol.}) + (\pi_{part.} \times \pi_{hol.})] \qquad (4)$$

where $\nu_{part.}$, $\nu_{hol.}$, $\pi_{part.}$ and $\pi_{hol.}$ are calculated accordingly to the proton and neutron valence spaces.

We have performed the calculations for Z=28 and 30 (varying N between 28 and 50) and also for Z=20 (20≤N≤28). These last ones are used to normalize our results to the $_{20}$Ca experimental values. In the Ca isotopes there is only one possibility for the proton and neutron valence space, the $\pi 1f_{7/2}$ and $\nu 1f_{7/2}$ single-j shells. On the contrary, when N, as Z, are above 28 (neutrons in $^{56-68}$Ni and $^{62-72}$Zn and protons only for the $_{30}$Zn isotopes) it exists two possibilities of valence space : either the full 28-50 major shell (MS), or a sub-shell (sS) resulting from a sub-shell closure. As previously mentioned, the sub-shell closure is assumed to take place at N=38 for neutrons. For protons, accordingly to the B(E2)⇑ ratio of two consecutive isotones and to the B(E2)⇑ curves of isotonic series, it is assumed at Z=40 [7]. The neutron and proton valence spaces used to calculate the curves, drawn with different lines in fig.3, are given in the legends. The Z=28 curve and the two Z=30 ones present a minimum at N=38 (as stressed by the legend) accordingly to the N=38 sub-shell closure assumption. A major shell for the neutron valence space would have given a maximum for N=38-40 (see ref.[7]).

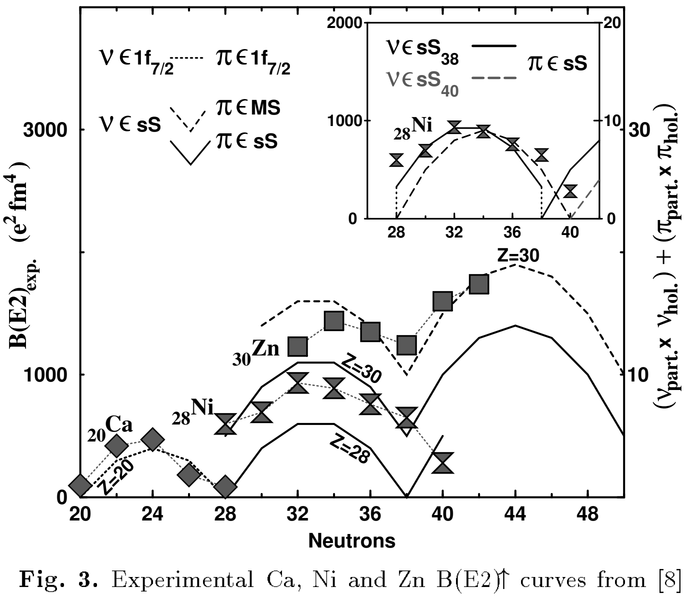

Fig. 3. Experimental Ca, Ni and Zn B(E2)⇑ curves from [8] (symbols, dotted gray lines) and calculated ones with different proton and neutron valence spaces (see text and legend). Inset: experimental Ni B(E2)⇑ values and calculated profiles with a proton sub-shell closure and a neutron one located either at N=38 (shifted curve, black line) or at N=40 (dashed gray line).

These calculations provide a frame of "smooth behaviors" highlighting the significant deviations of the experimental curves. Fig.3 makes evident that the experimental Ni curve lies higher than expected, very near the lowest Z=30 curve. As proton magic nuclei, their B(E2)⇑ should be null for N=28, like for the ^{48}Ca, and the other values should lie on the Z=28 curve which prolongates the Z=20 one. The Ni experimental B(E2)⇑ value at N=40 which was considered "abnormal" since lower those at N=28 and N=38, is in fact the only normal one, near the Z=28 calculated value. All other Ni B(E2)⇑ values are too high, increased from N=28 to N=38 by a phenomenon disappearing at N=40. This is more probably due to the protons than to the neutrons. Indeed, a neutron effect would evolve with N and change the shape of the Ni B(E2)⇑ curve. But, in the insert of fig.3 one can see the perfect agreement along four points for N between 30 and 36 of the experimental profile and the one calculated with a N=38 neutron sub-shell closure (sS_{38}, solid black line). This latter curve has been shifted up to underline the agreement and to illustrate the hypothesis of a constant enhancement. The curve obtained under the assumption of a N=40 neutron sub-shell closure (sS_{40}, dashed gray line) exhibits indeed another profile and does not allow to obtain the previous agreement. The enhancement appears mainly due to the protons, except at N=28 and N=38 were the slight disagreement can be related to a neutron effect, as discussed in [7]. On the other hand, in the insert of fig.3, one can note that the N=40 neutron sub-shell closure assumption gives a null B(E2)⇑ value for $^{68}_{28}$Ni, lower than the experimental one. Therefore, the hindering of the quadrupole excitation assumed in [2] would go in the wrong way for reproducing the ^{68}Ni data in a N=40 sub-shell closure hypothesis. The decrease in the Ni isotopes between N=38 and 40 is really misleading, it dissimulates a N=38 neutron sub-shell closure behavior and a proton effect.

The agreement between experiment and calculations is more visible in the Zn (excellent above N=34) than in the Ni isotopes thanks to the experimental minimum at N=38. Nevertheless, between N=32 and 34 the Zn experimental curve passes from near the curve obtained with proton and neutron sub-shell closures to the curve obtained with a major-shell for the proton valence space, where it stays.

In both Ni and Zn B(E2)⇑ curves, the proton contribution is increased for some neutron numbers. The change of the proton valence space between the lightest Zn isotopes and the heaviest ones can be interpreted as the vanishing of the proton sub-shell closure. All this calls to mind a proton-neutron interaction effect. Indeed, it has been pointed out [14,15] that the p-n interaction can be strong enough to reduce (and even to eradicate) the Z=40 sub-shell gaps in the heavy (N≥60) Zr, Mo and Ru isotopes [14–16], this gap reduction going with promotion of protons over it and of neutrons into higher orbitals [14–16].

4 Anchors of our interpretation

Our interpretation of the Ni and Zn B(E2)⇑ curves is built firstly on a neutron sub-shell closure at N=38. A N=38 spacing has been obtained between the $1f_{5/2}$ and $2p_{1/2}$ orbitals in [17], as shown in the insets of fig.4. One can note that this single-particle level spectrum offers at N=40 the possibility for a second energy spacing.

We involve also in our interpretation the p-n interaction. It is worth noting that the $1f_{7/2}$ (for particle number ranging between 22 and 28), $1f_{5/2}$ and $1g_{9/2}$ orbitals surrounding N=38 and 40 can allow the p-n interaction to start [7], since they make possible a large overlap between the proton and neutron orbitals [18]. The N=40 spacing, if existing for the Ni isotopes only, could complete our interpretation of the difference between the B(E2)⇑ curves in ^{66,68}Ni and ^{68,72}Zn. In addition to the Z=28 gap, two consecutive spacings between $\nu 1f_{5/2}$ to $\nu 1g_{9/2}$ would represent an insuperable obstacle for the p-n interaction unable to act in the Ni isotopes as it does in the Zn isotopes after N=38.

The presence of a N=38 neutron sub-shell closure in both the Ni and Zn isotopes followed by, in the Ni isotopes only, a N=40 energy spacing is confirmed by the energies of the two first excited states in the Ni and Zn odd isotopes with N=39 (drawn in fig.4a) and 41 (fig.4b). In fig.4 we have called E_{38} and E_{40} the difference of excitation energy related respectively to e_{38}, the N=38 spacing between $1f_{5/2}$ and $2p_{1/2}$, and to e_{40}, the N=40 one between $2p_{1/2}$ and $1g_{9/2}$. We can see on fig.4 that e_{38} and e_{40} are both active in the odd-Ni isotopes : E_{38} and E_{40} are so large that the ^{67,69}Ni excitation spectra, with only 2 excited states below 1MeV (900keV for 69Ni) have twice smaller density than in their corresponding odd-Zn isotopes. Indeed, in ^{69}Zn E_{38} is 100keV smaller and E_{40} is twice smaller than in ^{67}Ni$^{(2)}$. In ^{71}Zn the $1g_{9/2}$ orbital is so close to $2p_{1/2}$ that 1f5/2 (too far) is not anymore involved in the first excited states; E_{38} can not anymore be deduced. From the N=40 spacing, the B(E2)⇑ value of the even ^{70}Ni can be expected around 400e^{2}fm^4 (on the sS_{40} curve of the inset of fig. 3), near the ^{68}Ni B(E2)⇑ value. On another hand, in ^{70}Ni two neutrons are occupying $\nu 1g_{9/2}$. The p-n interaction could restart, eradicate the N=40 subshell closure. The N=38 sub-shell closure would then be the lower limit of the neutron valence space and the ^{70}Ni B(E2)⇑ value will lie around 800e^{2}fm^4 (on the sS_{38} curve of the inset) or even above if the proton promotion restarts also. It is worth noting that the ^{68}Ni B(E2)⇑ value lies exactly between the two curves of the inset as a transitional point would. Recent lifetime measurements [19] give lower $^{58-64}$Ni B(E2)⇑ values than in [8]. If confirmed, such values –which are in agreement with a N=38 sub-shell closure without even involving proton contribution – put with the values at N=28 and 38, which are in agreement with N=40 sub-shell closure, would extend this transitional behavior to the whole Ni isotopic series.

The Ni and Zn B(E2)⇑ curves are analyzed using calculations performed within the seniority scheme. They are shown to follow a N=38 sub-shell closure behavior with deviations interpreted as coming from a proton contribution enlarged by the p-n interaction. The low B(E2)⇑ value in ^{68}Ni is explained by the Z=28 gap which, added to the N=38-40 sub-shell spacings, prevents the p-n interaction to promote protons over the Z=28 gap (as it does in ^{56,66}Ni) together with (as in ^{64,72}Zn) neutrons over the N=40 energy spacing. The ^{70}Ni B(E2)⇑ value is expected either near the ^{68}Ni one or twice larger if the p-n interaction can restart despite the Z=28 and N=38-40 gaps. Indeed, in ^{66,72}Zn the p-n interaction is found strong enough to suppress the Z=40 proton sub-shell closure. The B(E2)⇑ value in the heavier even Zn isotopes is expected to increase up to N=44 then to decrease.

$^{(2)}$ The decrease of E_{38} and E_{40} from Ni to Zn odd-isotopes is also probably due to the p-n interaction

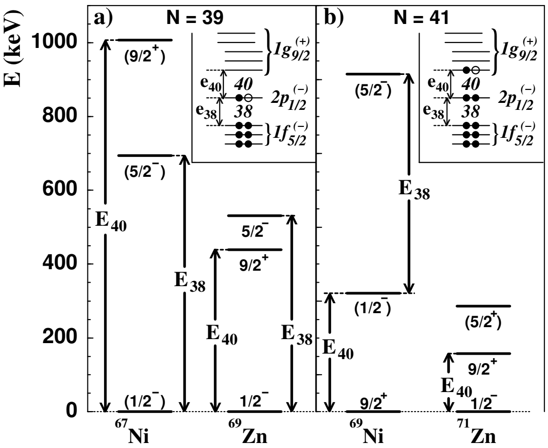

Fig. 4. Experimental excitation energies from [10] of the first three states in a) ^{67}Ni and ^{69}Zn, b) ^{69}Ni and ^{71}Zn. Insets: Neutron single-particle level order from [17] and occupation for the ground states.

The authors want to thank R. Lombard for discussions and J. Kiener, J. Sauvage, and D. Lunney for attentive readings.

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