Weibel instability on weakly relativistic produced plasma by circular polarization microwave electric field *

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Abstract

Analyzing the production of weakly relativistic plasma produced by microwave fields with circular polarization the electron distribution function is obtained to be non-equilibrium and anisotropic. Furthermore, it is shown that produced plasma is accelerated on the direction of propagation of microwave electric fields. The electron velocity on this direction strongly depends on electron origination phase, electric field phase, and amplitude of microwave electric field. Making use of the dielectric tensor obtained for this plasma, it is shown that the wiebel instability develops due to the anisotrpic property of

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distribution function. The dispersion equation is obtained for this instability and the growth rate of it, is calculated.

I. INTRODUCTION

A microwave produced gas discharge is a rather complicated phenomenon exhibiting a variety of features. Numerous theoretical and experimental studies have been devoted to this phenomenon. The interaction between intense (MW) fields and neutral gas open new possibilities for studying the fundamental property of this produced plasma. In a plasma produced in the interaction of the MW field with a gas, the electron distribution is non-equilibrium and may give rise to various plasma instability.[1] The main features of this interaction is the gas ionization mechanism. Furthermore, the key role in the plasma processes is played by the kinetic effects associated with the specific features of the electron distribution. Due to the short interaction times, these features are governed completely by the pulse parameter of microwave field. In addition the interaction between intense MW fields with an inhomogeneous plasma results in variety of phenomena such as frequency up shift during the propagation of a pulse microwave field through a plasma.[2] In such a strong wave field the electron oscillatory energy ϵ_e is much higher than the ionization energy I_{ioniz} of gas atoms

$$\epsilon_e = \frac{e^2 E_0^2}{2m\omega_0^2} \gg I_{ioniz}, \tag{1}$$

where ω_0 is the radiation frequency, m is electron mass,and E_0 is the electric field strength. When field amplitude is comparable to the atomic field strength $E_a \approx 5.1 \times 10^9\ V cm^{-1}$ the tunneling ionization becomes an important mechanism for direct ionization of the gas atoms. This effect was completely studied in the different papers.[3,4,5] On the other hand relativistic effects come in to play when kinetic energy of electron oscillation in an electromagnetic field is comparable with the electron rest mass energy. Moreover, the microwave fields generated by the present day pulsed duration sources are weaker than the atomic field and are capable of manifesting weakly relativistic behavior of electron produced. On the oder side, the aforementioned effect can be easily manifested by intense laser pulse.[6]

In the previous paper we restricted our study to the non-relativistic regime but here we consider weakly relativistic effects that occur during gas ionization by a strong microwave field.[7] In the present paper we consider the interaction of circularly polarized microwave pulse fields with frequency about $\omega_0 \simeq 2\times10^{10}s^{-1} - 2\times10^{11}s^{-1}$ with a neutral gas taking into weakly relativistic effects. This pulsed radiation source is capable of generating the radiation with an intensity of about $10^8 Wcm^{-2}$, whose electric field being $E_0 \leq 10^6 Vcm^{-1}$ is much weaker than the atomic field E_a. In weakly relativistic case $v_E \ll c$, for example in the weak ionized gas at low plasma density, the electron average drift velocity v_{zav} arises in the wave propagation direction. At $\omega_0 \approx 2\times10^{10}s^{-1}$ and $E_0 \approx 10^4 Vcm^{-1}$ we get $v_E \approx 0.9\times10^9 cms^{-1}$ and $v_{zav} = 2.6\times10^7 cms^{-1}$. If plasma density could get its critical value the drift current in plasma ought be $j_z \approx 0.5 Acm^{-2}$. [8]

In this case, also we study the electron distribution function (EDF) and the stability of the discharge plasma in the aforementioned frequency range at weakly relativistic electron oscillation energy ϵ_e .

This work is organized in four section. In sec. II. we will obtain produced EDF generated by the interaction of the circularly-polarized MW field with a neutral gas. In sec.III. we will obtain the dielectric tensor element for this produced plasma. In sec.IV. we will study the stability of produced plasma and find the wiebel instability growth rate of wiebel instability caused by the anisotropic property of the electron distribution function. Finally, a summary and conclusion is presented.

II. ELECTRON DISTRIBUTION FUNCTION

Under condition (1) the thermal velocity of the electrons in a discharge plasma can be neglected in comparison to the electron oscillation velocity in the MW radiation field. Since the collision frequency is much smaller than the MW field frequency, we can ignore the collisional stochastization of the forced electron oscillation as well. Furthermore, if plasma density $n_e(t)$ produced by the field during gas breakdown, is less than the critical density (that is $\omega_0^2 > \omega_{Le}^2 = 4\pi ne^2/m$) we can neglect the effect of the polarization field. Moreover, the plasma density is assumed to be less than the neutral gas density n_0 so that the latter can be considered constant. We also suppose that the field was adiabatically switched on in the infinite past. Furthermore, we can assume that the MW radiation electric field amplitude $\mathbf{E_0}$ is constant during a single field period. Therefore, the kinetic equation for the plasma electrons produced in the gas breakdown by a strong pulsed field can be written as follows

$$\frac{\partial f_0}{\partial t} + \mathbf{v}.\frac{\partial f_0}{\partial \mathbf{r}} + e[\mathbf{E_0} + \frac{1}{c}(\mathbf{v} \times \mathbf{B_0})].\frac{\partial f_0}{\partial \mathbf{p}} = n_0 \omega_{ioniz} \delta(\mathbf{p}), \qquad (2)$$

where $\mathbf{E_0}(\xi)$ and $\mathbf{B_0}(\xi)$ are electric and magnetic fields of a wave propagating along the Z-axis, respectively; $\xi = \omega_0 t - k_0 r = \omega_0(t - z/c)$, and ω_{ioniz} is the ionization probability of the gas atoms. Here $\delta(\mathbf{p})$ is the delta function of the electron momentum. In the weakly-relativistic limit, we can assuming that the electric field depends only on time. In the case of MW breakdown, electron-impact ionization is governed by the ionization probability ω_{ioniz}. For a MW discharge Eq. (2) for $f_0(\mathbf{p}, t)$ is a homogeneous integro-differential equation whose positive eigenvalue $\gamma(\mathbf{E_0})$ determines the avalanche ionization constant. However, we can neglect the right-hand side of Eq. (2) in the first approximation and calculate the EDF directly by solving the Vlasov equation under the following condition,

$$\omega_0 \gg \gamma(\mathbf{E_0}), \omega_{ioniz} \qquad (3)$$

where the avalanche ionization constant $\gamma(\mathbf{E_0})$ is to be determined.[8] In this case, the condition (3) depends strongly on the neutral gas density and is well satisfied at gas pressures of $p_0 \simeq 10 - 100$ Torr. In this approximation, to calculate the electron energy distribution function, we assume the field components to be circularly polarized

$$\mathbf{E_x} = \mathbf{E_0} \sin \omega_0 t, \qquad \mathbf{E_y} = \mathbf{E_0} \cos \omega_0 t, \qquad \mathbf{E_z} = 0, \tag{4}$$

where, ω_0 is the frequency of MW field. Moreover, electric field amplitude $\mathbf{E_0}$ describes the slowly varying (over the field period) microwave pulsed envelope. consider the condition (3) and solving Eq. (2) by characteristic method the equation of motion for electrons can be obtained as follows

$$\begin{cases} m\dfrac{dv_x}{dt} = eE_x, \\ m\dfrac{dv_y}{dt} = eE_y, \\ m\dfrac{dv_z}{dt} = \dfrac{e}{c}(v_x E_x + v_y E_y), \end{cases} \tag{5}$$

from this we find solution of the vlasov kinetic equation-characteristics

$$\begin{cases} v_x = -v_E(\cos\varphi - \cos\varphi_0), \\ v_y = v_E(\sin\varphi - \sin\varphi_0), \\ v_z = \frac{v_E^2}{c}(1 - \cos(\varphi - \varphi_0)), \end{cases} \tag{6}$$

where, $\varphi = \omega_0 t$ is the MW electric field phase and $\varphi_0 = \omega_0 t_0$ is the MW electric field phase when the electron originates with zero momentum at time t_0, $v_E = eE_0/m\omega_0$ is the electron oscillatory velocity in an alternating electric field. For the solution of the form $f_0(v,t) = n_e(t)\widetilde{f_0}(v)$ we obtain[7]

$$\widetilde{f_0}(v_x, v_y, v_z) = \delta(v_x + v_E(\cos\varphi - \cos\varphi_0))\delta(v_y - v_E(\sin\varphi - \sin\varphi_0)) \qquad (7)$$
$$\times\delta\left(v_z - \frac{v_E^2}{c}[1 - \cos(\varphi - \varphi_0)]\right).$$

The function $\tilde{f}_0(v_x, v_y, v_z)$ satisfies the normalization condition $\int d\,\mathbf{v}\ \tilde{f}_0(\mathbf{v}) = 1$. One can show the product of the two last δ-functions in the distribution function (7) is independent of φ_0. For this reason, by introducing the following notation

$$V_x = -v_y + v_E \sin\omega_0 t\ , \qquad V_y = v_x + v_E \cos\omega_0 t\ ,$$

we obtain

$$\delta(v_x + v_E(\cos\omega_0 t - \cos\varphi_0))\delta(v_y - v_E(\sin\omega_0 t - \sin\varphi_0)) = \frac{1}{2\pi v_E}\ \delta(V_\perp - v_E)\ ,$$

where

$$V_\perp^2 = V_x^2 + V_y^2 = v_\perp^2 + v_E^2 + 2v_E(v_x \cos\omega_0 t - v_y \sin\omega_0 t), \qquad v_\perp^2 = v_x^2 + v_y^2\ .$$

Therefore, Eq. (7) is reduced to

$$\tilde{f}_0(V_\perp, v_z) = \frac{1}{2\pi v_E}\delta(V_\perp - v_E)\delta\left(v_z - \frac{v_E^2}{c}[1 - \cos(\varphi - \varphi_0)]\right). \qquad (8)$$

We can see that, the electron distribution function depends on phase field φ and electron origination phase φ_0. Therefore, we must average of electron distribution function over the phase field period and origination phase. The projection of the phase portrait of the electrons onto the (v_x, v_y) plane is as follows. The electron trajectories uniformly cover a circle of radius v_E, whose center precesses about the origin of the coordinates and describes a circumference of the same radius at a rate equal to the microwave frequency. Consequently, the averaging procedure can be performed separately for the longitudinal and transverse velocity components.

Averaging over the transverse component we obtain

$$\widetilde{f_0}(v_\perp, v_z(\varphi-\varphi_0)) = \frac{\delta\left(v_z - \frac{v_E^2}{c}[1-\cos(\varphi-\varphi_0)]\right)}{2\pi^2 v_\perp\sqrt{4v_E^2 - v_\perp^2}} \qquad (9)$$

III. DIELECTRIC TENSOR

In the previous section we found the distribution function (9) for electrons in a discharge plasma is highly anisotropic with respect to the direction of the MW radiation field, which, first of all, should result in the onset of the well-known wiebel instability.[7] In order to convince ourselves that this conclusion is valid and to find the instability growth rate, we turn to the adiabatic approximation, assuming that the instability grows faster than the plasma density.[5,9] In this approximation, we can use the following dispersion relation for small perturbation

$$\mid k^2\delta_{ij} - k_i k_j - \frac{\omega^2}{c^2}\varepsilon_{ij}(\omega, \mathbf{k}) \mid = 0. \qquad (10)$$

Here $\varepsilon_{ij}(\omega, \mathbf{k})$, the dielectric permitivity tensor of the plasma, is obtained by linearizing Vlasov equation for the electrons

$$\frac{\partial f_e}{\partial t} + \mathbf{v}.\frac{\partial f_e}{\partial \mathbf{r}} + e\left(\mathbf{E_0} + \frac{1}{c}(\mathbf{v}\times \mathbf{B_0})\right).\frac{\partial f_e}{\partial \mathbf{p}} = 0, \qquad (11)$$

and considering self-consistency effect and Maxwell equations and cold ions approximations as follows

$$\varepsilon_{ij}(\omega, \mathbf{k}) = 1 + \delta\varepsilon_{ij}^I + \delta\varepsilon_{ij}^e = 1 - \frac{\omega_{Pi}^2}{\omega^2}\delta_{ij} + \frac{\omega_{Pe}^2}{\omega^2}\int d\,\mathbf{v}\left[v_i\frac{\partial\tilde{f}_0}{\partial v_j} + v_i v_j\frac{\mathbf{k}.\frac{\partial\tilde{f}_0}{\partial\mathbf{v}}}{\omega - \mathbf{k}.\mathbf{v}}\right]. \qquad (12)$$

Here, the second term, i.e., $\delta\varepsilon_{ij}^I$ shows the ion contribution of the plasma dielectric permitivity tensor and the third term, i.e., $\delta\varepsilon_{ij}^e$ is related to the electron contribution of the plasma dielectric permitivity tensor. The latter kinetic equation for the perturbed EDF is valid in the adiabatic approximation when short-wavelength limit is justified, i.e., $\omega_{Pi} \ll kv \leq \omega_{Pe}$. To solve Eq. (12) by using EDF (9) we introduce the following system of coordinate in which $\mathbf{k}$ lies only in the XZ plan

$$\mathbf{k} = (\mathbf{k_\perp}, \mathbf{0}, \mathbf{k_z}) \quad , \qquad\qquad \mathbf{k_\perp} = \sqrt{\mathbf{k_x^2} + \mathbf{k_y^2}}.$$

Under this condition the non zero contributions of the electrons in the dielectric tensor (12) are

$$\delta\epsilon_{11} = -\frac{\omega_{Pe}^2}{\omega^2}\left[1 - \frac{k_z^2}{2k_\perp^2} - \frac{(1-\frac{2k_z^2}{k_\perp^2})s}{2\sqrt{1-\frac{4k_\perp^2 v_E^2}{\beta^2\omega^2}}} + \frac{(1-\frac{k_z^2}{k_\perp^2})s}{2(1-\frac{4k_\perp^2 v_E^2}{\beta^2\omega^2})^{\frac{3}{2}}}\right]$$

$$\delta\epsilon_{33} = -\frac{\omega_{Pe}^2}{2\omega^2}\left[1 - \frac{2ik_\perp v_E^3(1-\cos\phi)^2}{\omega c^2\beta}(1 + \frac{ik_\perp v_E s}{2\omega\beta\sqrt{1-\frac{4k_\perp^2 v_E^2}{\omega^2\beta^2}}}) - \frac{k_\perp^2 v_E^4(1-\cos\phi)^2}{\omega^2 c^2\beta^2(1-\frac{4k_\perp^2 v_E^2}{\omega^2\beta^2})^{\frac{3}{2}}}\right.$$
$$\left. - \frac{2k_z v_E^2(1-\cos\phi)s}{\omega c\beta\sqrt{1-\frac{4k_\perp^2 v_E^2}{\omega^2\beta^2}}} + \frac{k_z^2 v_E^4(1-\cos\phi)^2 s}{\omega^2 c^2\beta^2(1-\frac{4k_\perp^2 v_E^2}{\omega^2\beta^2})^{\frac{3}{2}}}\right] \qquad (13)$$

$$\delta\epsilon_{13} = \frac{\omega_{Pe}^2}{2\omega^2}\left[\frac{k_\perp v_E^2(1-\cos\phi)s}{\omega c\beta}(1+\frac{k_z^2}{k_\perp^2})\left(\frac{1}{\sqrt{(1-\frac{4k_\perp^2 v_E^2}{\omega^2\beta^2})}} - \frac{1}{(1-\frac{4k_\perp^2 v_E^2}{\omega^2\beta^2})^{\frac{3}{2}}}\right)\right.$$
$$\left. + \frac{k_z}{k_\perp}(1 - \frac{s}{\sqrt{1-\frac{4k_\perp^2 v_E^2}{\omega^2\beta^2}}})\right]$$

$$\delta\epsilon_{12} = \delta\epsilon_{21} = 0, \qquad \delta\epsilon_{22} = -\frac{\omega_{Pe}^2}{\omega^2} \qquad \delta\epsilon_{31} = -\frac{i\omega_{Pe}^2 v_E}{2\omega^2 c}(1-\cos\phi) + \delta\epsilon_{13},$$

where

$$s = \begin{cases} \left.\begin{array}{lll} 2 & for & Re\omega \neq 0 \\ 1 & for & Re\omega = 0 \end{array}\right\} when \quad \frac{4k_\perp^2 v_E^2}{\beta^2\omega^2} < 1 \\ \left.\begin{array}{lll} 0 & for & Im\omega = 0 \\ 2 & for & Im\omega \neq 0 \\ 1 & for & Re\omega = 0 \end{array}\right\} when \quad \frac{4k_\perp^2 v_E^2}{\beta^2\omega^2} > 1 \end{cases} \qquad (14)$$

and $\beta = (1 - \frac{k_z v_E^2}{\omega c} - \frac{k_z v_E^2}{\omega c}\cos\phi)$, $\phi = \varphi - \varphi_0$ also, $i = \sqrt{-1}$.

IV. STABILITY OF THE PRODUCED PLASMA

As known the process of the gas breakdown by the MW fields is unstable with respect to the excitation of longitudinal electric fields and transverse magnetic fields. The former is caused due to the positive derivative of the EDF and the latter, treated below, is related to the anisotropy of the EDF.[9] In this step we study the weibel instability, by analyzing electron perturbations propagate across the microwave field. Therefore, by averaging dielectric tensor (13) over the ϕ between $[0, 2\pi]$ and substituting into the dispersion equation (10) by assuming $2kv_E^2/\omega c < 1$ yields the dispersion relation for electron perturbations propagating across the MW radiation field ($k_\perp = 0$)

$$\omega^3 + \frac{\omega_{Pe}^2 k^2 v_E^2 \omega}{(k^2c^2 + \omega_{Pe}^2)} + \frac{2\omega_{Pe}^2 k^3 v_E^4}{c(k^2c^2 + \omega_{Pe}^2)} = 0. \tag{15}$$

With assumption $k^2 v_E^2 \ll \omega_{Pe}^2 \ll k^2c^2$ and Solving Eq. (15) we obtain the following expression, which characterize the growth rate of the weibel instability as follow

$$\omega = \frac{k^2 v_E^2}{\sqrt{k^2c^2 + \omega_{Pe}^2}} + i\frac{kv_E\omega_{Pe}}{\sqrt{k^2c^2 + \omega_{Pe}^2}}\left(1 + \frac{3k^2v_E^2}{2\omega_{Pe}^2}\right) \tag{16}$$

V. CONCLUSION

Using a simplest model, we are calculated the weakly-relativistic EDF for plasma produced by the interaction of an intense microwave pulse field with a neutral gas. The resulting EDF, which can be drive analytically for circularly polarized MW field, is highly anisotropic, which indicates that microwave driven plasmas are subject to weibel instability. In the non-relativistic produced plasma[7] the electrons only oscillate on the MW field direction but on the weakly-relativistic produced plasma the electrons have velocity perpendicular the radiation field. In this case the electrons originating with zero energy are entrained by MW field in a certain phase and are accelerated to weakly-relativistic velocities, so that the accelerated electrons move essentially in phase with the MW field. Then, this process might be repeated for the electrons originating over the next time interval, and so on. Analyzing the dispersion equation and obtaining the growth rate of the instability produced we found that the growth rate modified by an exceed positive term with respect to non-relativistic case. Also the instability frequency has a very small real part that can be represent the entrain of electrons in MW fields.

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