Negative energies and time reversal in Quantum Field Theory and General Relativity, The Dark Side of Gravity

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Abstract

Rehabilitation of negative energies in Quantum Field Theory and General Relativity is undertaken. The proposed link between negative energies, time reversal and the existence of two conjugated metrics opens the way to a new natural gravitational interaction mechanism between positive and negative energy fields. The usual Schwarzschild solution in vacuum gets modified at Post-Post-Newtonian order. A spatially flat universe accelerating expansion phase is obtained naturally without resorting to a cosmological constant. In this more favorable context, classical and quantum instability issues should be reconsidered. Related theoretical issues along with phenomenological consequences are explored.

I) Introduction

With recent cosmological observations related to supernova, CMB and galactic clustering the evidence is growing that our universe is undergoing an accelerated expansion at present. Though the most popular way to account for this unexpected result has been the reintroduction of a cosmological constant or a new kind of dark matter with negative pressure, scalar fields with negative kinetic energy, so-called phantom fields, have recently been proposed [1] as new sources leading to the not excluded possibility that the equation of state parameter be less than minus one. Because such models unavoidably lead to violation of positive energy conditions, catastrophic quantum instability of the vacuum is expected and one has to impose an ultraviolet cutoff to the low energy effective theory in order to keep the instability at unobservable rate. Stability is clearly the challenge for any model trying to incorporate negative energy fields interacting with positive energy fields. But before addressing this crucial issue, it is worth recalling and analyzing how and why Quantum Field Theory discarded negative energy states. We shall find that this was achieved through several not so obvious mathematical choices, often in close relation with the well known pathologies of the theory, vacuum and UV loop divergences. Following another approach starting from the orthogonal alternative mathematical choices, the crucial link between negative energies, time reversal and the existence of conjugated metrics will appear. We shall propose a new kind of gravitational interaction mechanism between classical positive and negative energy fields. In this new context, the instability issue should be reconsidered. At last, we shall explore the impressively long list of promising phenomenological consequences such interaction between positive and negative energy fields offers in high energy physics and cosmology hopefully motivating an important theoretical effort in this direction.

II) Negative energy and classical fields

1) Extremum action principle

Let us first address the stability of paths issue. Consider the path r(t) of a material point of mass m with fixed endpoints at time t_1 and t_2 in the potential U(r,t). The action S is:

$$S = \int_{t_1}^{t_2} (1/2 \;\; mv^2 \;\; -U(r,t))dt$$

The extremum condition (δS=0) is all we need to establish the equation of motion:

$$m\dot{v} = -\frac{\partial U}{\partial r}$$

S has no maximum because of the kinetic term positive sign. The extremum we find is a minimum.

Let us try now a negative kinetic term:

$$S = \int_{t_1}^{t_2} (-1/2\, m\, v^2 - U(r,t))dt$$

The extremum condition (δS=0) is all we need to establish the equation of motion:

$$-m\dot{v} = -\frac{\partial U}{\partial r}$$

S has no minimum because of the kinetic term negative sign. The extremum we find is a maximum.

Eventually, it appears that the fundamental principle is that of stationary (δS=0) action, the extremum being a minimum or a maximum depending on the sign of the kinetic term. In all cases we find stable trajectories.

2) Classical relativistic fields

We can also check that negative kinetic energy terms (ghost terms) in a free field action are not problematic. When we impose the extremum action condition the negative energy field solutions simply maximize the action. Now, in special relativity for a massive or mass-less particle, two energy solutions are always possible:

$$E = \pm\sqrt{p^2 + m^2},\; E = \pm\, |p|$$

In other words, the Lorentz group admits, among others, negative energy representations $E^2 - p^2 = m^2 > 0, E < 0,\;\; E^2 - p^2 = 0,\; E < 0$. Thus, not only can we state that negative energy free field terms are not problematic but also that negative energy field solutions are expected in any relativistic field theory. For instance the Klein-Gordon equation:

$$\left(\partial^\mu \partial_\mu + m^2\right)\overset{(\sim)}{\phi}(x) = 0$$

admits when $m^2 > 0$ (we shall not try to understand here the physical meaning of tachyonic ($m^2 < 0$) and vacuum (E=p=m=0) representations) positive $\phi(x)$ and negative $\tilde{\phi}(x)$ energy free field solutions. Indeed, the same Klein-Gordon equation results from applying the extreme action principle to either the 'positive' scalar action:

$$\int d^4x\,\phi^\dagger(x)\left(\partial^\mu\partial_\mu + m^2\right)\phi(x)$$

or the 'negative' scalar action:

$$-\int d^4x\,\tilde{\phi}^\dagger(x)\left(\partial^\mu\partial_\mu + m^2\right)\tilde{\phi}(x)$$

From the former a positive conserved Hamiltonian is derived through the Noether theorem:

$$\int d^3\boldsymbol{x}\,\frac{\partial\phi^\dagger(\boldsymbol{x},t)}{\partial t}\frac{\partial\phi(\boldsymbol{x},t)}{\partial t} + \sum_{i=1,3}\frac{\partial\phi^\dagger(\boldsymbol{x},t)}{\partial x_i}\frac{\partial\phi(\boldsymbol{x},t)}{\partial x_i} + m^2\phi^\dagger(\boldsymbol{x},t)\phi(\boldsymbol{x},t)$$

while a negative one is derived from the latter:

$$-\int d^3\boldsymbol{x}\,\frac{\partial\tilde{\phi}^\dagger(\boldsymbol{x},t)}{\partial t}\frac{\partial\tilde{\phi}(\boldsymbol{x},t)}{\partial t} + \sum_{i=1,3}\frac{\partial\tilde{\phi}^\dagger(\boldsymbol{x},t)}{\partial x_i}\frac{\partial\tilde{\phi}(\boldsymbol{x},t)}{\partial x_i} + m^2\tilde{\phi}^\dagger(\boldsymbol{x},t)\tilde{\phi}(\boldsymbol{x},t)$$

III) Negative energy in relativistic Quantum Field Theory (QFT)

1) Creating and annihilating negative energy quanta

At first sight it would seem that the negative frequency terms appearing in the plane wave Fourier decomposition of any field naturally stand for the negative energy solutions. But as soon as we decide to work in a self-consistent quantization theoretical framework, that is the second quantization one, the actual meaning of these negative frequency terms is clarified. Operator solutions of field equations in conventional QFT read:

$$\phi(x) = \phi_+(x) + \phi_-(x)$$

with $\phi_+(x)$ a positive frequency term creating **positive** energy quanta and $\phi_-(x)$ a negative frequency term annihilating **positive** energy quanta. So negative energy states are completely avoided thanks to the mathematical choice of creating and annihilating only positive energy quanta and $\phi(x)$ built in this way is just the positive energy solution. This choice would be mathematically justified if one could argue that there are strong reasons to discard the 'negative action' we introduced in the previous section. But there are none and as we already noticed the Klein-Gordon equation is also easily derived from such action and the negative energy field solution:

$$\tilde{\phi}(x) = \tilde{\phi}_+(x) + \tilde{\phi}_-(x)$$

(with $\tilde{\phi}_+(x)$ a positive frequency term annihilating **negative** energy quanta and $\tilde{\phi}_-(x)$ a negative frequency term creating **negative** energy quanta) is only coherent with the negative Hamiltonian derived from the negative action through the Noether theorem (in the same way it is a standard QFT result that the usual positive energy quantum field $\phi(x)$ is only coherent with the above positive Hamiltonian [3][4]). Therefore, it is mathematically unjustified to discard the negative energy solutions. Neglecting them on the basis that negative energy states remain up to now undetected is also very dangerous if we recall that antiparticles predicted by the Dirac equation were considered unphysical before they were eventually observed. If negative (or tachyonic) energy states are given a profound role to play in physics, this must be fully understood otherwise we might be faced with insurmountable difficulties at some later stage.

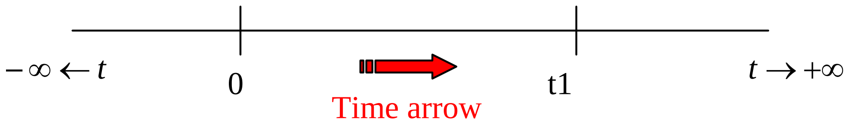
There is a widespread belief that the negative energy issue were once and for all understood in terms of antiparticles. Indeed, because charged fields are required not to mix operators with different charges, the charge conjugated creation and annihilation operators (antiparticles) necessarily enter into the game. Following Feynman's picture, such antiparticles can as well be considered as negative energy particles propagating backward in time. According S.Weinberg [2], it is only in relativistic (Lorentz transformation do not leave invariant the order of events separated by space-like intervals) quantum mechanics (non negligible probability for a particle to get from x_1 to x_2 even if x_1 - x_2 is space-like) that antiparticles are a necessity to avoid the logical paradox of a particle being absorbed before it is emitted. However, these antiparticles have nothing to do with genuine negative energy states propagating forward in time, whose quanta are by construction of the conventional QFT fields never created nor annihilated. Therefore, our deep understanding of the actual meaning of field negative frequency terms in QFT does not "solve" the negative energy issue since the corresponding solutions were actually neglected from the beginning. As we shall see, there is a heavy price to pay for having neglected the negative energy solutions: all those field vacuum divergences that unavoidably arise after quantization and may be an even heavier price are the ideas developed to cancel such infinities without reintroducing negative energy states.

2) A unitary time reversal operator

In a classical relativistic framework, one could not avoid energy reversal under time reversal simply because energy is the time component of a four-vector. But, when one comes to establish in Quantum Field Theory the effect of time reversal on various fields, nobody wants to take this simple picture serious anymore mainly because of the unwanted negative energy spectrum it would unavoidably bring into the theory. It is argued that negative energy states remain undetected and that their existence would necessarily trigger catastrophic decays of particles and vacuum: matter could not be stable. To keep energies positive, the mathematical choice of an anti-unitary time reversal operator comes to the rescue leading to the idea that the time-mirrored system corresponds to 'running the movie backwards' interchanging the roles of initial and final configurations. We shall come back to the stability issue later. But for the time being, let us stress that the running backward movie picture is not self–evident. In particular, the interchange of initial and final state under time reversal is very questionable. To see this, let us first recall that there are two mathematical possibilities for a time reversal operator; either it must be unitary or anti-unitary. These lead to two quite different, both mathematically coherent time reversal conjugated scenarios:

The process $i \to f$ being schematized as:

$$|i\rangle = a^+(E_{i1})...a^+(E_{in})|0\rangle \qquad \langle f| = \langle 0|a(E_{f1})...a(E_{fp})$$

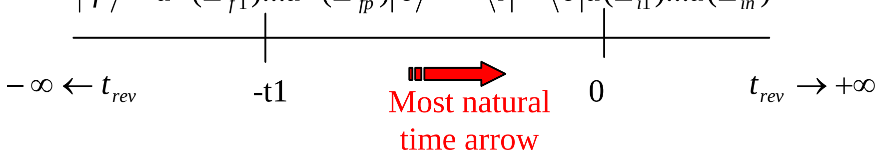


the time reversed coordinate is $t_{rev} = -t$ and:

-The conventional QFT anti-unitary time reversal scenario interchanges initial and final states:

$$i \to f \overset{T}{\Rightarrow} T^A(f) \to T^A(i)$$

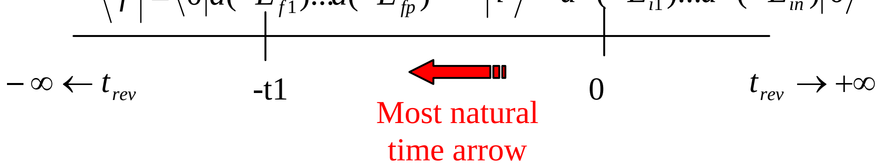
$$|f\rangle = a^+(E_{f1})...a^+(E_{fp})|0\rangle \qquad \langle i| = \langle 0|a(E_{i1})...a(E_{in})$$



-The unitary one does not interchange initial and final state but reverses energies

$$i \to f \overset{T}{\Rightarrow} T^U(i) \to T^U(f)$$

$$\langle \tilde{f}| = \langle 0|a(-E_{f1})...a(-E_{fp}) \qquad |\tilde{i}\rangle = a^+(-E_{i1})...a^+(-E_{in})|0\rangle$$



Our common sense intuition then tells us that the interchange of initial and final state, hence the anti-unitary picture stands to reason. This is because we naively require that in the time reverted picture the initial state (the ket) must come ‘before’ the final state (the bra) i.e for a lower value of t_{rev}. However, paying careful attention to the issue we realize that the time arrow, an underlying concept of time flow which here influences our intuition is linked to a specific property of the time coordinate which is not relevant for a spatial coordinate, namely its irreversibility or causality. But as has been pointed out by many authors, there are many reasons to suspect that such irreversibility and time arrow may only be macroscopic scale (or statistical physics) valid concepts not making sense for a microscopic time, at least before any measurement takes place. We believe that our microscopic time coordinate, before measurement takes place, should be better considered as a spatial one, i.e possessing no property such as an arrow. Then, the unitary picture is the most natural one as a time reversal candidate process simply because it is the usual choice for all other discrete and continuous symmetries.

But if neither t nor t_{rev} actually stand for the genuine flowing time which we experiment and measure, the latter must arise at some stage and it is natural to postulate that its orientation corresponding to the experimented time arrow is simply defined in such a way that, as drawn in the previous pictures, the initial state (the creator) always comes before the final state (the annihilator) in this flowing time. This clearly points toward a theoretical framework where the time will be treated as a quantum object undergoing radical transformations from the microscopic to the macroscopic time we measure. Let us anticipate that the observable velocities will be better understood in term of this new macroscopic flowing time variable which arrow (orientation) keeps the same under reversal of the unflowing space–like t coordinate.

Therefore, the interchange of initial and final states is only justified under the assumption that time coordinate reversal implies time arrow reversal. But this is not at all obvious and thus there is no more strong reason to prefer and adopt the QFT anti-unitary choice. At the contrary, we can now list several strong arguments in favor of the unitary choice:

- The mathematical handling of an anti-unitary operator is less trivial and induces unusual complications when applied for instance to the Dirac field.
- The QFT choice leads to momentum reversal, a very surprising result for a mass-less particle, since in this case it amounts to a genuine wavelength reversal and not frequency reversal, as one would have expected.
-Its anti-unitarity makes T really exceptional in QFT. As a consequence, not all basic four-vectors transform the same way under such operator as the reference space-time four-vector. In our mind, a basic four-vector is an object involving the parameters of a one particle state such as for instance its energy and three momentum components. The one particle state energy is the time component of such an object but does not reverses as the time itself if T is taken anti-unitary. This pseudo-vector behavior under time reversal seems nonsense and leads us to prefer the unitary scenario. At the contrary, we can understand why (and accept that) the usual operator four-vectors, commonly built from the fields, behave under discrete transformations such as unitary parity differently than the reference space-time four-vector. This is simply because, as we shall see, they involve in a nontrivial way the parity-pseudo-scalar 3-volume.
- Time irreversibility at macroscopic scale allows us to define unambiguously our time arrow. But, as we already noticed, the arrow of time at the microscopic scale or before any measurement process takes place may be not so well defined. The statement that the time arrow is only a macroscopic scale (or may be statistical physics) valid concept is not so innovative. We know from Quantum Mechanics that all microscopic quantum observables acquire their macroscopic physical status through the still enigmatic measurement process. Guessing that the time arrow itself only becomes meaningful at macroscopic scale, we could reverse our microscopic time coordinate t as an arrowless spatial coordinate. Reverting the time arrow is more problematic since this certainly raises the well known time reversal and causality paradoxes. But the good new is that reversing the time coordinate does not necessarily imply reversing the arrow of time, i.e interchanging initial and final state. In the unitary picture, you do not actually go backward in time since you just see the same succession (order) of events counting the t_{rev} time “à rebours”, with only the signs of the involved energies being affected and you need not worry anymore about paradoxes. Therefore, in a certain sense, the running backward movie picture was may be just a kind of entropy reversal picture, a confusing and inappropriate macroscopic scale concept which obscured our understanding of the time coordinate reversal and led us to believe that the anti-unitary scenario was obviously the correct one.

- Charge and charge density are invariant while current densities get reversed under a unitary time reversal (see section VI).
- Negative energy fields are natural solutions of all relativistic equations.
- The instability issue might be solved in a modified general relativistic model as we shall show.

IV) Negative energy quantum fields, time reversal and vacuum energies

We shall now explicitly build the QFT neglected solutions, e.g. the usual bosonic and fermionic negative energy fields, show how these are linked to the positive ones through time reversal and how vacuum divergences cancel from the Hamiltonians.

1) The neutral scalar field

The positive energy scalar field solution of the Klein-Gordon equation is:

$$\phi(\boldsymbol{x},t) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}(2E)^{1/2}}\left[a(\boldsymbol{p},E)e^{i(Et-\boldsymbol{px})} + a^\dagger(\boldsymbol{p},E)e^{-i(Et-\boldsymbol{px})}\right]$$

with $E = \sqrt{\boldsymbol{p}^2 + m^2}$.
The negative energy scalar field solution of the same Klein-Gordon equation is:

$$\tilde{\phi}(\boldsymbol{x},t) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}(2E)^{1/2}}\left[\tilde{a}^\dagger(-\boldsymbol{p},-E)e^{i(Et-\boldsymbol{px})} + \tilde{a}(-\boldsymbol{p},-E)e^{-i(Et-\boldsymbol{px})}\right]$$

We just required this field to create and annihilate negative energy quanta. Assuming T is anti-unitary, it is well known that a scalar field is transformed according

$$T\phi(\boldsymbol{x},t)T^{-1} = \phi(\boldsymbol{x},-t)$$

where, for simplicity, an arbitrary phase factor was chosen unity. Then it is straightforward to show that:

$$Ta^\dagger(\boldsymbol{p},E)T^{-1} = a^\dagger(-\boldsymbol{p},E)$$

We do not accept this result because we want time reversal to flip energy, not momentum. If instead, the T operator is chosen unitary like all other discrete transformation operators (P, C) in Quantum Field Theory we cannot require $T\phi(\boldsymbol{x},t)T^{-1} = \phi(\boldsymbol{x},-t)$, but rather:

$$T\phi(\boldsymbol{x},t)T^{-1} = \tilde{\phi}(\boldsymbol{x},-t)$$

The expected result is then obtained as usual through the change in the variable p→-p:

$$Ta^\dagger(\boldsymbol{p},E)T^{-1} = \tilde{a}^\dagger(\boldsymbol{p},-E)$$

This confirms that a unitary T leads to energy reversal of scalar field quanta. Momentum is invariant. For a massive particle this may be interpreted as mass reversal coming along with velocity reversal. But in the unitary time reversal scenario it is not at all obvious that the

velocity is built out of the time coordinate which gets reversed. Instead, as soon as this velocity is measured it seems more natural to build it out of the (as well measured) flowing time which never gets reversed. In this case, neither velocity nor mass get reversed.
The Hamiltonian for our free neutral scalar field reads:

$$H = +\frac{1}{2}\int d^3x \;\; [\;\; (\frac{\partial\phi(\boldsymbol{x},t)}{\partial t})^2 + (\frac{\partial\phi(\boldsymbol{x},t)}{\partial x})^2 + m^2\phi^2(\boldsymbol{x},t)]$$

The Hamiltonian for the corresponding negative energy field is:

$$\tilde{H} = \tilde{P}^0 = -\frac{1}{2}\int d^3\boldsymbol{x} \;\; [\;\; (\frac{\partial\tilde{\phi}(\boldsymbol{x},t)}{\partial t})^2 + (\frac{\partial\tilde{\phi}(\boldsymbol{x},t)}{\partial x})^2 + m^2\tilde{\phi}^2(\boldsymbol{x},t)]$$

The origin of the minus sign under time reversal of H will be investigated in sections VI and VIII. After replacing the scalar fields by their expressions, the computation then follows the same line as in all QFT books, leading to:

$$H = \frac{1}{2}\int d^3\boldsymbol{p} \quad p^0(a^\dagger(\boldsymbol{p},E)a(\boldsymbol{p},E) + a(\boldsymbol{p},E)a^\dagger(\boldsymbol{p},E))$$
$$\tilde{H} = -\frac{1}{2}\int d^3\boldsymbol{p} \quad p^0(\tilde{a}^\dagger(-\boldsymbol{p},-E)\tilde{a}(-\boldsymbol{p},-E) + \tilde{a}(-\boldsymbol{p},-E)\tilde{a}^\dagger(-\boldsymbol{p},-E))$$

With $p^0 = \sqrt{\boldsymbol{p}^2 + m^2}$ and the usual commutation relations,

$$[a_p^\dagger, a_{p'}] = \delta^4(p-p')\,,[\tilde{a}_p^\dagger, \tilde{a}_{p'}] = \delta^4(p-p')$$

vacuum divergences cancel (as we shall see, in a general relativistic framework, these only cancel as gravitational sources), and for the total Hamiltonian we get:

$$H_{total} = \int d^3\boldsymbol{p} \quad p^0\left\{a^\dagger(\boldsymbol{p},E)a(\boldsymbol{p},E) - \tilde{a}^\dagger(-\boldsymbol{p},-E)\tilde{a}(-\boldsymbol{p},-E)\right\}$$

It is straightforward to check that the energy eigenvalue for a positive (resp negative) energy ket is positive (resp negative), as it should. For a vector field, the infinities would cancel in the same way assuming as well the usual commutation relations.

2) The Dirac field

Let us investigate the more involved case of the Dirac field. The Dirac field is solution of the free equation of motion:

$$(i\gamma^\mu\partial_\mu - m)\psi(\boldsymbol{x},t) = 0$$

When multiplying this Dirac equation by the unitary T operator from the left, we get:

$$(iT\gamma^\mu T^{-1}\partial_\mu - TmT^{-1})T\psi(\boldsymbol{x},t) = (iT\gamma^\mu T^{-1}\partial_\mu - TmT^{-1})\tilde{\psi}(\boldsymbol{x},-t) = 0$$

If the rest energy term m is related to the Higgs field value at its minimum (or another dynamical field) its transformation under time reversal is more involved than that of a pure number. Rather, we have:

$$m = g\phi_0(\boldsymbol{x},t) \rightarrow \tilde{m} = TmT^{-1} = g\tilde{\phi}_0(\boldsymbol{x},-t)$$

Making the replacement, $\partial_0 = -\partial^0, \partial_i = \partial^i$ and requiring that the T conjugated Dirac and scalar fields at its minimum $\tilde{\psi}(\boldsymbol{x},-t) = T\,\psi(\boldsymbol{x},t)T^{-1}, \tilde{\phi}_0(\boldsymbol{x},-t) = T\,\phi_0(\boldsymbol{x},t)T^{-1}$ together should obey the same equation, e.g. $(i\gamma^\mu\partial^\mu - g\tilde{\phi}_0(\boldsymbol{x},-t))\tilde{\psi}(\boldsymbol{x},-t) = 0$ as $\psi(\boldsymbol{x},t)$ and $\phi_0(\boldsymbol{x},t)$, leads to:

$$T\gamma^i T^{-1} = \gamma^i,\ \ T\gamma^0 T^{-1} = -\gamma^0$$

The T operator is then determined to be $T = \gamma^1\gamma^2\gamma^3$. Now assuming also that $\tilde{\phi}_0(\boldsymbol{x},t) = -\phi_0(\boldsymbol{x},t)$, the Dirac equation satisfied by $\tilde{\psi}(x,t)$ reads:

$$(i\gamma^\mu\partial_\mu + m)\tilde{\psi}(\boldsymbol{x},t) = 0$$

γ^0,γ^i being a particular gamma matrices representation used in equation $(i\gamma^\mu\partial_\mu - m)\psi(\boldsymbol{x},t) = 0$, then $(i\gamma^\mu\partial_\mu + m)\tilde{\psi}(\boldsymbol{x},t) = 0$ can simply be obtained from the latter by switching to the new gamma matrices representation $-\gamma^0,-\gamma^i$ and the negative energy Dirac field $\tilde{\psi}(x,t)$. As is well known, all gamma matrices representations are unitary equivalent and here γ^5 is the unitary matrix transforming the set γ^0,γ^i into $-\gamma^0,-\gamma^i$ ($\gamma^5\gamma^\mu\left(\gamma^5\right)^{-1} = -\gamma^\mu$). Thus $\tilde{\psi}(\boldsymbol{x},t)$ satisfies the same Dirac equation as $\gamma^5\psi(\boldsymbol{x},t)$. The physical consequences will be now clarified. Let us write down the positive (resp negative) energy Dirac field solutions of their respective equations.

$$\psi(\boldsymbol{x},t) = \frac{1}{(2\pi)^{3/2}}\sum_{\sigma=\pm 1/2}\int_p \frac{d^3\boldsymbol{p}}{(2E)^{1/2}}\{u(-E,m,-\boldsymbol{p},-\sigma)a_c(E,m,\boldsymbol{p},\sigma)e^{i(Et-\boldsymbol{px})} + u(E,m,\boldsymbol{p},\sigma)a^\dagger(E,m,\boldsymbol{p},\sigma)e^{-i(Et-\boldsymbol{px})}\}$$

$$\tilde{\psi}(\boldsymbol{x},t) = \frac{1}{(2\pi)^{3/2}}\sum_{\sigma=\pm 1/2}\int_p \frac{d^3\boldsymbol{p}}{(2E)^{1/2}}\{u(-E,-m,-\boldsymbol{p},-\sigma)\tilde{a}^\dagger(-E,-m,-\boldsymbol{p},-\sigma)e^{i(Et-\boldsymbol{px})} + u(E,-m,\boldsymbol{p},\sigma)\tilde{a}_c(-E,-m,-\boldsymbol{p},-\sigma)e^{-i(Et-\boldsymbol{px})}\}$$

with $E = \sqrt{\boldsymbol{p}^2 + m^2}$.

Classifying the free Dirac waves propagating in the x direction we have as usual for the positive energy field spinors:

$$u(E,m,p_x,+1/2) = \begin{bmatrix} 1 \\ 0 \\ \dfrac{\sigma_x p_x}{m+E} \\ 0 \end{bmatrix},\ \ u(-E,m,-p_x,-1/2) = \begin{bmatrix} 1 \\ 0 \\ \dfrac{\sigma_x p_x}{m-E} \\ 0 \end{bmatrix}$$

$$u(E,m,p_x,-1/2) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \dfrac{\sigma_x p_x}{m+E} \end{bmatrix},\ \ u(-E,m,-p_x,+1/2) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \dfrac{\sigma_x p_x}{m-E} \end{bmatrix}$$

The negative energy field spinors are also easily obtained through the replacement m → -m

$$u(-E,-m,-p_x,-1/2)=\begin{bmatrix}1\\0\\ \frac{\sigma_x p_x}{-m-E}\\0\end{bmatrix},\quad u(E,-m,p_x,1/2)=\begin{bmatrix}1\\0\\ \frac{\sigma_x p_x}{-m+E}\\0\end{bmatrix}$$

$$u(-E,-m,-p_x,+1/2)=\begin{bmatrix}0\\1\\0\\ \frac{\sigma_x p_x}{-m-E}\end{bmatrix},\quad u(E,-m,p_x,-1/2)=\begin{bmatrix}0\\1\\0\\ \frac{\sigma_x p_x}{-m+E}\end{bmatrix}$$

We demand that:

$$T\psi(\boldsymbol{x},t)T^{-1}=\tilde{\psi}(\boldsymbol{x},-t)$$

This implies:

$$Ta^{\dagger}(E,m,\boldsymbol{p},\sigma)T^{-1}u(E,m,\boldsymbol{p},\sigma)=u(-E,-m,-\boldsymbol{p}\to\boldsymbol{p},-\sigma)\tilde{a}^{\dagger}(-E,-m,\boldsymbol{p},-\sigma)$$

Hence:

$$Ta^{\dagger}(E,m,\boldsymbol{p},\sigma)T^{-1}=\tilde{a}^{\dagger}(-E,-m,\boldsymbol{p},-\sigma)$$

Thus, upon time reversal, energy, rest energy and spin are reversed. Because momentum is invariant helicity also flips its sign. Without having reverted the rest energy term in the negative energy Dirac field equation we could not have obtained this simple link through time reversal between the positive and negative energy creation operators. The rest energy reversal in the spinor expressions also reveals the difference between a true negative energy spinor $u(-E,-m,.,.)$ and a negative frequency spinor $u(-E,m,.,.)$.

The Hamiltonian for $\psi(\boldsymbol{x},t)$ is:

$$H=P^0=\int d^3x[\bar{\psi}(\boldsymbol{x},t)(-i\gamma^i.\partial_i+m)\psi(\boldsymbol{x},t)]\quad+\quad h.c$$

The negative energy field Hamiltonian will be built out of negative energy fields explicitly different from those entering in *H*. Hence, it is hopeless trying to obtain such kind of simple transformation relations such as $P^0\Rightarrow\pm P^0$. On the other hand we can build the negative energy Hamiltonian and check that it provides the correct answer when applied to a given negative energy ket.

We know that $T\gamma^iT^{-1}=\gamma^i$, $T\gamma^0T^{-1}=-\gamma^0$, so that:

$$T\bar{\psi}(\boldsymbol{x},t)T^{-1}=T\psi^{\dagger}(\boldsymbol{x},t)\gamma^0T^{-1}=-T\psi^{\dagger}(\boldsymbol{x},t)T^{-1}\gamma^0=-(T\psi(\boldsymbol{x},t)T^{-1})^{\dagger}\gamma^0=-\bar{\tilde{\psi}}(\boldsymbol{x},-t)$$

This will produce an extra minus sign in the negative energy Dirac field Hamiltonian. The origin of the other minus sign is the same as for the scalar field Hamiltonian and will be clarified later. The Hamiltonian for $\tilde{\psi}(\boldsymbol{x},t)$ is then:

$$\tilde{H}=\tilde{P}^0=--\int d^3x[\bar{\tilde{\psi}}(\boldsymbol{x},t)(-i\gamma^i\partial_i-m)\tilde{\psi}(\boldsymbol{x},t)]\quad+\quad h.c$$

Because the positive (resp negative) energy spinor satisfies $(i\gamma^{\mu}\partial_{\mu}-m)\psi(\boldsymbol{x},t)=0$ (resp $(i\gamma^{\mu}\partial_{\mu}+m)\tilde{\psi}(\boldsymbol{x},t)=0$), we have $(-i\gamma^{i}\partial_{i}+m)\psi(\boldsymbol{x},t)=i\gamma^{0}\partial_{0}\psi(\boldsymbol{x},t)$, (resp $(-i\gamma^{i}\partial_{i}-m)\tilde{\psi}(\boldsymbol{x},t)=i\gamma^{0}\partial_{0}\tilde{\psi}(\boldsymbol{x},t)$).
The Hamiltonians then read:

$$H=P^{0}=i\int d^{3}x[\psi^{\dagger}(\boldsymbol{x},t)\partial_{0}\psi(\boldsymbol{x},t)] \ + \ h.c$$
$$\tilde{H}=\tilde{P}^{0}=i\int d^{3}x[\tilde{\psi}^{\dagger}(\boldsymbol{x},t)\partial_{0}\tilde{\psi}(\boldsymbol{x},t)] \ + \ h.c$$

Assuming for simplicity that we are dealing with a neutral field, the computation proceeds as usual for the positive energy Hamiltonian. With $p^{0}=\sqrt{\boldsymbol{p}^{2}+m^{2}}$:

$$H=\frac{1}{2}\sum_{\sigma=\pm1/2}\int d^{3}\boldsymbol{p} \quad p^{0}(a^{\dagger}(E,\boldsymbol{p},\sigma)a(E,\boldsymbol{p},\sigma)-a(E,\boldsymbol{p},\sigma)a^{\dagger}(E,\boldsymbol{p},\sigma))$$

Negative energy spinors possessing the same orthogonality properties as positive energy spinors, the negative energy Hamiltonian is then obtained by the simple replacements $a^{\dagger}(E,\boldsymbol{p},\sigma)\rightarrow\tilde{a}(-E,-\boldsymbol{p},-\sigma)$; $a(E,\boldsymbol{p},\sigma)\rightarrow\tilde{a}^{\dagger}(-E,-\boldsymbol{p},-\sigma)$:

$$\tilde{H}=\frac{1}{2}\sum_{\sigma=\pm1/2}-\int d^{3}\boldsymbol{p} \quad p^{0}(\tilde{a}^{\dagger}(-E,-\boldsymbol{p},-\sigma)\tilde{a}(-E,-\boldsymbol{p},-\sigma)-\tilde{a}(-E,-\boldsymbol{p},-\sigma)\tilde{a}^{\dagger}(-E,-\boldsymbol{p},-\sigma))$$

Infinities cancel as for the boson fields when we apply the fermionic anti-commutation relations $\left\{a^{\dagger}_{p,\sigma},a_{p',\sigma'}\right\}=\delta^{4}(p-p') \ \ \delta_{\sigma,\sigma'}$, $\left\{\tilde{a}^{\dagger}_{p,\sigma},\tilde{a}_{p',\sigma'}\right\}=\delta^{4}(p-p') \ \ \delta_{\sigma,\sigma'}$, leading to:

$$H_{total}=\sum_{\sigma=\pm1/2}\int d^{3}\boldsymbol{p} \quad p^{0}\left\{a^{\dagger}(\boldsymbol{p},E,\sigma)a(\boldsymbol{p},E,\sigma)-\tilde{a}^{\dagger}(-\boldsymbol{p},-E,-\sigma)\tilde{a}(-\boldsymbol{p},-E,-\sigma)\right\}$$

It is also easily checked that the energy eigenvalue for a positive (resp negative) energy ket is positive (resp negative), as it should. When we realize how straightforward are the cancellation of vacuum divergences for all fields it is very tempting to state that such infinities appeared only because half of the field solutions were neglected! We shall soon show that actually, in a general relativity context, our vacuum divergences only vanish as a source for gravitation. But the Casimir effect should still survive.

V) Phenomenology of the uncoupled positive and negative energy worlds

We shall now show that the uncoupled positive and negative energy worlds are both perfectly viable: no stability issue arises and in both worlds the behavior of matter and radiation is completely similar so that the negative signs may just appear as a matter of convention [8][9].

Consider a gas made with negative energy matter particles (fermions) and negative energy photons. The interaction between two negative energy fermions is going on through negative energy photons exchange. Because the main result will only depend on the bosonic nature of the considered interaction field, let us compute and compare the simpler propagator of the positive and negative energy scalar fields.

-For a positive energy scalar field:

$$\phi(x) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}(2p^0)^{1/2}}\left[a(p)e^{ipx} + a_c^{\dagger}(p)e^{-ipx}\right]$$

we get as usual:

$$\left\langle 0\right|T(\phi(x)\phi^{\dagger}(y))\left|0\right\rangle = \left\langle 0\right|\phi(x)\phi^{\dagger}(y)\left|0\right\rangle \ \ \theta(x_0-y_0) + \left\langle 0\right|\phi^{\dagger}(y)\phi(x)\left|0\right\rangle \ \ \theta(y_0-x_0)$$

$$= \left\langle 0\right|\int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2p^0}a(p)a^{\dagger}(p)e^{ip(x-y)}\left|0\right\rangle\theta(x_0-y_0) + \left\langle 0\right|\int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2p^0}a_c(p)a_c^{\dagger}(p)e^{-ip(x-y)}\left|0\right\rangle\theta(y_0-x_0)$$

$$= \int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2p^0}e^{ip(x-y)}\theta(x_0-y_0) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2p^0}e^{-ip(x-y)}\theta(y_0-x_0)$$

$$= \Delta(y-x)\theta(x_0-y_0) + \Delta(x-y)\theta(y_0-x_0)$$

-For a negative energy scalar field:

$$\tilde{\phi}(x) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}(2p^0)^{1/2}}\left[\tilde{a}^{\dagger}(p)e^{ipx} + \tilde{a}_c(p)e^{-ipx}\right]$$

we obtain:

$$\left\langle 0\right|T(\tilde{\phi}(x)\tilde{\phi}^{\dagger}(y))\left|0\right\rangle = \left\langle 0\right|\tilde{\phi}(x)\tilde{\phi}^{\dagger}(y)\left|0\right\rangle \ \ \theta(x_0-y_0) + \left\langle 0\right|\tilde{\phi}^{\dagger}(y)\tilde{\phi}(x)\left|0\right\rangle \ \ \theta(y_0-x_0)$$

$$= \left\langle 0\right|\int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2p^0}\tilde{a}_c(p)\tilde{a}_c^{\dagger}(p)e^{-ip(x-y)}\left|0\right\rangle\theta(x_0-y_0) + \left\langle 0\right|\int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2p^0}\tilde{a}(p)\tilde{a}^{\dagger}(p)e^{ip(x-y)}\left|0\right\rangle\theta(y_0-x_0)$$

$$= \int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2p^0}e^{-ip(x-y)}\theta(x_0-y_0) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2p^0}e^{ip(x-y)}\theta(y_0-x_0)$$

$$= \Delta(x-y)\theta(x_0-y_0) + \Delta(y-x) \ \ \theta(y_0-x_0)$$

Summing the two propagators, the theta functions cancel:

$$\left\langle 0\right|T(\tilde{\phi}(x)\tilde{\phi}^{\dagger}(y))\left|0\right\rangle + \left\langle 0\right|T(\phi(x)\phi^{\dagger}(y))\left|0\right\rangle = (\Delta(x-y)+\Delta(y-x))(\theta(x_0-y_0)+\theta(y_0-x_0) \ )$$

$$= \Delta(x-y)+\Delta(y-x) \propto \int (\delta\left(E-p^0\right)+\delta\left(E+p^0\right))e^{-iE(x_0-y_0)}dE$$

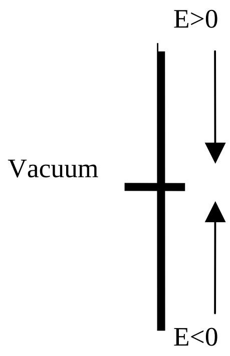
Therefore, if the two propagators could contribute with the same coupling to the interaction between two currents, the virtual particle terms would cancel each other. Only on-shell particles could still be exchanged between the two currents provided energy momentum conservation does not forbid it. For a photon field as well the two off-shell parts of the propagators would be found opposite. Hence the coulomb potential derived from the negative energy photon field propagator would be exactly opposite to the coulomb potential derived from the positive energy photon field propagator: as a consequence, the 1/r Coulomb potential and electromagnetic interactions would simply disappear. The interesting point is that in our negative energy gas, where we assume that only the exchange of negative energy virtual photons takes place, the coulomb potential is reversed compared to the usual coulomb potential generated by positive energy virtual photons exchange. However in this repulsive potential between oppositely charged fermions, these still attract each other, as in the positive energy world, because of their negative inertial terms in the equation of motion (as deduced from their negative terms in the action). The equation of motion for a given negative energy matter particle in this Coulomb potential is:

$$-m\dot{v} = --\frac{\partial U_c}{\partial r}$$

or

$$m\dot{v} = -\frac{\partial U_c}{\partial r}$$

We find ourselves in the same situation as that of a positive energy particles gas interacting in the usual way e.g through positive energy photons exchange. Hence negative energy atoms will form and the main results of statistical physics apply: following Boltzman law, our particles will occupy with the greatest probabilities states with minimum $\frac{1}{2}m\dot{v}^2$, thus with maximum energy $-\frac{1}{2}m\dot{v}^2$. Entropy and temperatures are negative, entropy decreases (just as if time were reversed!), etc…This result can be extended to all interactions propagated by bosons as are all known interactions. The conclusion is that the non-coupled positive and negative energy worlds are perfectly stable, with positive and negative energy particles minimizing the absolute value of their energies:



VI) Actions and Hamiltonians under Time reversal and Parity

1) Negative integration volumes?

Starting from the expression of the Hamiltonian density for a positive energy neutral scalar field:

$$T^{00}(x,t) = \left(\frac{\partial\phi(\boldsymbol{x},t)}{\partial t}\right)^2 + \sum_{i=1,3}\left(\frac{\partial\phi(\boldsymbol{x},t)}{\partial x_i}\right)^2 + m^2\phi^2(\boldsymbol{x},t)$$

and applying time reversal we get:

$$\left(\frac{\partial\tilde{\phi}(\boldsymbol{x},-t)}{\partial t}\right)^2 + \sum_{i=1,3}\left(\frac{\partial\tilde{\phi}(\boldsymbol{x},-t)}{\partial x_i}\right)^2 + m^2\tilde{\phi}^2(\boldsymbol{x},-t)$$

with $T\phi(\boldsymbol{x},t)T^{-1} \equiv \tilde{\phi}(\boldsymbol{x},-t)$

From such expression, a naive free Hamiltonian density for the scalar field $\tilde{\phi}(x,t)$ may be proposed:

$$\tilde{T}^{00}(\boldsymbol{x},t) = \left(\frac{\partial\tilde{\phi}(\boldsymbol{x},t)}{\partial t}\right)^2 + \sum_{i=1,3}\left(\frac{\partial\tilde{\phi}(\boldsymbol{x},t)}{\partial x_i}\right)^2 + m^2\tilde{\phi}^2(\boldsymbol{x},t)$$

It thus happens that $\tilde{T}^{00}(\boldsymbol{x},t)$ is manifestly positive since it is a sum of squared terms. We of course cannot accommodate negative energy fields with positive Hamiltonian densities so following the procedure used to obtain negative kinetic energy terms for a phantom field, we just assumed in the previous sections a minus sign in front of this expression. But how could we justify this trick if time reversal does not provide us with this desired minus sign? One possible solution appears when we realize that according to general relativity, actually T^{00} is not a spatial energy density but rather $\sqrt{g}T^{00}$ where $g \equiv -Det\, g_{\mu\nu}$. This is also expected to still remain positive because of a rather strange mathematical choice in general relativity: integration volumes such as dt, d^4x, $d^3\boldsymbol{x}$ are not signed and should not flip sign under time reversal or parity transformations.

Let us try the more natural opposite way: $t \to -t \Rightarrow dt \to -dt$ and $x \to -x \Rightarrow dx \to -dx$, natural in the sense that this is naively the straightforward mathematical way to proceed and let us audaciously imagine that for instance a negative 3-dimentional volume is nothing else but the image of a 3-dimentional positive volume in a mirror. Then, the direct consequence of working with signed volumes is that the general relativistic integration element $d^4x\sqrt{g}$ is not invariant anymore under coordinate transformations (such as P or T) with negative Jacobian (it is often stated that the absolute value of the Jacobian is imposed by a fundamental theorem of integral calculus[2]. But should not this apply only to change of variables and not general coordinate transformations?). We are then led to choose an invariant integration element under any coordinate transformations: this is $d^4x\left|\frac{\partial\xi}{\partial x}\right|$, where $\left|\frac{\partial\xi}{\partial x}\right|$ stands for the Jacobian of the transformation from the inertial coordinate system ξ^α to x^μ. Because $\left|\frac{\partial \xi}{\partial x}\right|$ is not necessarily positive as is $\sqrt{g}$ in general relativity, it will get reversed under P or T transformations affecting Lorentz indices only so that **spatial** charge density $\left|\frac{\partial \xi}{\partial x}\right| J^0$, scalar charge $Q = \int \left|\frac{\partial \xi}{\partial x}\right| J^0 d^3\boldsymbol{x}$, **spatial** energy-momentum densities $\left|\frac{\partial \xi}{\partial x}\right| T^{\mu 0}$ and energy-momentum four-vector $P^\mu = \int \left|\frac{\partial \xi}{\partial x}\right| T^{\mu 0} d^3\boldsymbol{x}$ should transform accordingly. For instance, it is often stated that a unitary time reversal operator is not allowed because it would produce the not acceptable charge reversal. This analysis is no more valid if the Jacobi determinant flips its sign. Indeed, though J^0, as all four-vector time components, becomes negative, the **spatial** charge density $\left|\frac{\partial \xi}{\partial x}\right| J^0$ and scalar charge $Q = \int \left|\frac{\partial \xi}{\partial x}\right| J^0 d^3\boldsymbol{x}$ remain positive under unitary time reversal.

It is also worth checking what is now the effect of unitary space inversion:

$P^\mu = \int \left|\frac{\partial \xi}{\partial x}\right| T^{\mu 0} d^3\boldsymbol{x}$ transforms under Parity as $T^{\mu 0}$ times the pseudo-scalar Jacobi determinant $\left|\frac{\partial \xi}{\partial x}\right|$, so that:

$$P^0 \Rightarrow -P^0, \;\; P^i \Rightarrow P^i$$

$Q = \int \left|\frac{\partial \xi}{\partial x}\right| J^0 d^3\boldsymbol{x}$ also transforms under Parity as J^0 times the pseudo-scalar Jacobi determinant $\left|\frac{\partial \xi}{\partial x}\right|$, so that:

$$Q \Rightarrow -Q$$

So, if unitary Parity has the same effect on various fields, currents and energy densities as in conventional quantum field theory, it now produces a flip in the energy and charge signs but does not affect momentum!

Anyway, we see that the signed Jacobi determinant could do the good job for providing us with the desired minus signs. However, working with negative integration volumes amounts to give up the usual definition of the integral which insures that it is positive definite. If we are not willing to give up this definition, another mechanism should be found to provide us with the necessary minus sign. The issue will be reexamined and a more satisfactory solution described at section VIII.

2) Interactions between positive and negative energy fields ?

Postulate the existence of a new inertial coordinate system $\tilde{\xi}$ such that $\left|\frac{\partial \tilde{\xi}}{\partial x}\right|$ is negative. This can be achieved simply by considering the two time reversal conjugated (with opposite proper times) inertial coordinate systems ξ and $\tilde{\xi}$. We may then define the positive energy quantum $F(x)$ fields (resp negative energy $\tilde{F}(x)$ fields) as the fields entering in the action with positive $\left|\frac{\partial\xi}{\partial x}\right|$ (resp negative $\left|\frac{\partial\tilde{\xi}}{\partial x}\right|$) entering in the integration volume so that the energy $P^0 = \int \left|\frac{\partial\xi}{\partial x}\right| T^{00} d^3\boldsymbol{x}$ (resp $\tilde{P}^0 = \int \left|\frac{\partial\tilde{\xi}}{\partial x}\right| \tilde{T}^{00} d^3\boldsymbol{x}$) is positive (resp negative). The action for positive energy matter and radiation is then as usual:

$$S = \int d^4x \left|\frac{\partial\xi}{\partial x}\right| \left\{ L(\Psi(x), \frac{\partial\xi^\alpha}{\partial x^\mu}(x)) + L(A_\mu(x), \frac{\partial\xi^\alpha}{\partial x^\mu}(x)) + J_\mu(x)A^\mu(x) \right\}$$

Similarly, the action for negative energy matter and radiation is:

$$\tilde{S} = \int d^4x \left|\frac{\partial\tilde{\xi}}{\partial x}\right| \left\{ L(\tilde{\Psi}(x), \frac{\partial\tilde{\xi}^\alpha}{\partial x^\mu}(x)) + L(\tilde{A}_\mu(x), \frac{\partial\tilde{\xi}^\alpha}{\partial x^\mu}(x)) + \tilde{J}_\mu(x)\tilde{A}^\mu(x) \right\}$$

Hence positive energy fields move under the influence of the gravitational field $\frac{\partial\xi^\alpha}{\partial x^\mu}$, while negative energy fields move under the influence of the gravitational field $\frac{\partial\tilde{\xi}^\alpha}{\partial x^\mu}$. Then, the mixed coupling in the form $J_\mu(x)\tilde{A}^\mu(x)$ that we might have naively hoped is not possible just because the integration volume must be $d^4x\left|\frac{\partial\xi}{\partial x}\right|$ for $F(x)$ type fields and $d^4x\left|\frac{\partial\tilde{\xi}}{\partial x}\right|$ for $\tilde{F}(x)$ type fields. Indeed coherence requires that in the action the negative Jacobian be associated with negative energy fields $\tilde{F}(x)$ involving negative energy quanta creation and annihilation operators. This is a good new since it is well known that couplings between positive and negative energy fields lead to an unavoidable stability problem due to the fact that energy conservation keeps open an infinite phase space for the decay of positive energy particles into positive and negative energy particles. A scenario with positive and negative energy fields living in different metrics also provides a good way to account for the undiscovered negative energy states.

However the two metrics should not be independent if we want to introduce a connection at least gravitational between positive and negative energy worlds, mandatory to make our divergences actually cancel. We shall soon explicit this dependency between the two conjugated metrics and the mechanism that gives rise through the extremum action principle to the negative source terms in the Einstein equation. It will be clear that this mechanism only works properly if, as in general relativity, we keep working with Jacobi determinants absolute values and do not give up the usual definition of integrals.

VII) **Intermediate synthesis: negative energies in special relativity**

It's time to gather the information we learned from our preliminary investigation of negative energies in special relativity:

- All relativistic field equations admit negative energy field solutions.
- Time reversal is the most natural symmetry to provide the link between positive and negative energy fields.
- Positive and negative energy fields vacuum divergences we encounter after second quantization are exactly opposite.
- The non coupled positive and negative energy worlds are both perfectly stable and viable.

However,

- If positive and negative energy fields are time reversal conjugated, their Hamiltonian densities (or the actions from which they are derived) must also be so. For a scalar field with an Hamiltonian density which is just a squared terms sum and with a positive-definite integration it seems that there is no way for any symmetry to generate a negative Hamiltonian density from a positive definite Hamiltonian density. The only remaining way is to investigate how the metric field transforms upon time reversal and let us hope that this can generate the sign flipping of the general relativistic Hamiltonian densities.

- As is well known, vacuum divergences are only a concern when gravity comes into the game. This is because any energy density source must affect the metric through the Einstein equations. Thus, the positive and negative energy worlds must be maximally gravitationally coupled in such a way as to produce exact cancellation if we really want all vacuum divergences gravitational effects to disappear from the theory.

- It was shown for the electromagnetic interaction that allowing both positive and negative energy photon propagators to propagate the interaction simply makes the interaction disappear. We are going to build a model where the positive and negative energy worlds only interact gravitationally. In such model the gravitational interaction should of course not vanish.

- If both positive and negative energy worlds are separately stable, it is well known that a generic catastrophic instability arises whenever the positive and negative energy fields are allowed to interact [8][9]. This is because energy conservation does not forbid a positive energy object to absorb an unlimited amount of positive energy from a negative energy object which simultaneously falls into unbounded from below more and more negative energy states. In the quantum version, one says that because the phase space of the final states involving positive and negative energy particles is infinite, the vacuum decay rate into such states is also infinite, leading to a catastrophic instability. By allowing only gravity to propagate the interaction, we restrict the stability issue to gravity. But we will have to show that it is possible to avoid the above disastrous scenario thanks to an enriched model of gravitation properly taking into account discrete symmetries, resulting in a completely new picture of the graviton.

The inverse metric (time reversal conjugated metric) mechanism and extremum action procedure we shall now describe will give rise to negative energy sources in a modified Einstein equation in such a way that vacuum divergences cancellation actually takes place. The Schwarzschild solution, neglecting possible effects due to the cosmological background, will get modified at PPN order. A spatially flat universe accelerated expansion phase will also result as a natural behavior of our new cosmological equations solution.

VIII) **Positive and negative energy worlds gravitational coupling**

As is well known, general coordinate transformations do not leave the integration four volume invariant so a compensating Jacobi determinant modulus must be introduced to build a scalar integration volume. Then, if Parity and Time reversal transform the general coordinates, this will not affect our scalar actions however if the inertial coordinates ξ^α are also transformed in a non trivial way:

$$\xi^\alpha \xrightarrow{T} \tilde{\xi}^\alpha$$
$$\xi^\alpha \xrightarrow{P} -\tilde{\xi}^\alpha$$

non trivial in the sense that in general $\tilde{\xi}^\alpha \neq \xi_\alpha$, our metric terms will be affected and our action is not expected to be invariant under P or T . Then it appears that for a given general coordinate system there exists not one but two time reversal conjugated inertial coordinate systems allowing to build, following the usual procedure, two time reversal conjugated metric tensors:

$$g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\beta}{\partial x^\nu}$$
$$\tilde{g}_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \tilde{\xi}^\alpha}{\partial x^\mu} \frac{\partial \tilde{\xi}^\beta}{\partial x^\nu}$$

Then, a new set of fields will couple to our new $\tilde{g}_{\mu\nu}$ metric field and we shall see how such fields will eventually acquire a negative energy from the point of view of our world**.** We also notice that in such picture, since our P and T symmetries jump from one set of inertial coordinates to another set of inertial coordinates, they keep discrete even in a general relativity theoretical framework.

Now we have all we need in hands to deduce the modified Einstein equation. Let us first consider I_M, the usual action for positive energy matter and radiation in the external gravitational field $g_{\mu\nu}$ and $\tilde{I}_M$ the action for negative energy matter and radiation in the external gravitational field $\tilde{g}_{\mu\nu}$. Infinitesimal arbitrary variation of the metric fields produces a variation of our actions which takes the form:

$$\delta I_M + \delta \tilde{I}_M = \frac{1}{2}\int d^4x \sqrt{g(x)} T^{\mu\nu}(x) \delta g_{\mu\nu}(x) + \frac{1}{2}\int d^4x \sqrt{\tilde{g}(x)} \tilde{T}^{\mu\nu}(x) \delta \tilde{g}_{\mu\nu}(x)$$

where we recognize the familiar energy-momentum tensor $T^{\mu\nu}(x)$ for positive energy fields (for example $A_\mu(x)$) along with $\tilde{T}^{\mu\nu}(x)$ for negative energy fields (for example $\tilde{A}_\mu(x)$). The latter is obtained by varying $\tilde{g}_{\mu\nu}(x)$ with negative energy field $\tilde{A}_\mu(x)$ held fixed. Notice that because $A_\mu(x)$ and $\tilde{A}_\mu(x)$ don't live in the same metric there is no way for them to interact. The two actions are built in such a way that they are separately general coordinate scalars. But the metric terms are non trivially affected under improper transformations so adding the two pieces is just necessary to obtain a time reversal and parity invariant total action. Also notice that in both actions the invariant volume is PT invariant since $\xi^\alpha \to -\xi^\alpha$ neither affects the Jacobi determinant modulus nor the metric fields. Hence fields live in the same metric as their PT conjugated and may interact. Both P symmetric and T symmetric of our world positive energy fields will at the end of the story acquire a negative energy field status from our world point of view.

Though there is no coupling in the usual sense between fields involved in the two time reversal conjugated actions, $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are necessarily linked since these are time reversal conjugated objects explicitly built out of space-time coordinates. Now, let us postulate that there exists at each space-time point a general coordinate system (we will discuss later whether this should be assumed local or global) such that $\tilde{g}_{\mu\nu}$ identifies with $g^{\mu\nu}$. Because the equality is not one between tensors of the same kind it obviously does not apply in arbitrary general coordinate systems. A link is then established between our two metrics which had remained up to now completely independent. But though time reversal conjugated, $A_\mu(x)$ and $\tilde{A}^\mu(x)$ remain completely dynamically independent fields contrary to $g_{\mu\nu}(x)$ and $\tilde{g}_{\mu\nu}(x)$ because they are not explicitly built out of space-time coordinates. In the privileged coordinate system our action variation becomes:

$$\delta I_M + \delta\tilde{I}_M = \frac{1}{2}\int d^4x\sqrt{g(x)}T^{\mu\nu}(x)\delta g_{\mu\nu}(x) + \frac{1}{2}\int d^4x\sqrt{g^{-1}(x)}\tilde{T}^{\mu\nu}(x)\delta g^{\mu\nu}(x)$$

Then, using the relation $\delta g^{\rho\kappa}(x) = -g^{\rho\mu}(x)g^{\nu\kappa}(x)\delta g_{\mu\nu}(x)$ one gets:

$$\delta I_M + \delta\tilde{I}_M = \frac{1}{2}\int d^4x\sqrt{g(x)}T^{\mu\nu}(x)\delta g_{\mu\nu}(x) - \frac{1}{2}\int d^4x\sqrt{g^{-1}(x)}\tilde{T}^{\rho\kappa}(x)g^{\rho\mu}(x)g^{\nu\kappa}(x)\delta g_{\mu\nu}(x)$$

When we eventually ask that the total action variation should vanish under arbitrary field variations, the previous matter and radiation action will produce in Einstein gravitational equation positive and negative energy conjugated source terms of the form:

$$\frac{1}{2}\sqrt{g(x)}T_{\mu\nu}(x) - \frac{1}{2}\sqrt{g^{-1}(x)}\tilde{T}^{\mu\nu}(x)$$

For a cosmological constant source term:

$$T_{\mu\nu}(x) = \Lambda g_{\mu\nu}(x), \tilde{T}^{\mu\nu}(x) = \tilde{\Lambda} g_{\mu\nu}(x)$$

we get :

$$\frac{1}{2}g_{\mu\nu}(x)\left[\Lambda\sqrt{g(x)} - \tilde{\Lambda}\sqrt{g^{-1}(x)}\right]$$

The desired cancellation between vacuum energy terms will take place as in flat space time if everywhere:

$$\Lambda = \frac{\sqrt{g^{-1}(x)}}{\sqrt{g(x)}}\tilde{\Lambda} = \frac{1}{g(x)}\tilde{\Lambda}$$

But we can write :

$$\sqrt{g(x)} = \frac{d^4\xi(x)}{d^4x}; \sqrt{g^{-1}(x)} = \frac{d^4\tilde{\xi}(x)}{d^4x}$$

$$\Rightarrow g(x) = \frac{d^4\xi}{d^4\tilde{\xi}}(x)$$

so that the previous condition reads :

$$d^4\xi(x)\Lambda = d^4\tilde{\xi}(x)\tilde{\Lambda}$$

This relation would simply follow from the time reversal invariance of a pure constant if the time reversal conjugated inertial four volumes are supposed to be equal. Not only such cosmological constant terms cancel but also, as can be checked easily for a perfect fluid, all field second quantization vacuum energies vanish as expected from special relativity if and only if:

$$\rho_{vac}(x) = \frac{1}{g(x)}\tilde{\rho}_{vac}(x),\ p_{vac}(x) = \frac{1}{g(x)}\tilde{p}_{vac}(x)$$

or

$$d^4\xi(x)\rho_{vac}(x) = d^4\tilde{\xi}(x)\tilde{\rho}_{vac}(x),\ d^4\xi(x)p_{vac}(x) = d^4\tilde{\xi}(x)\tilde{p}_{vac}(x)$$

(If the time reversal conjugated inertial four volumes are again supposed to be equal, these relations would also follow from the equality of the time conjugated scalar energy densities and pressures as expected from our previous analysis in a Special Relativistic framework.)
Indeed, with

$$T_{\mu\nu} = p_{vac}g_{\mu\nu} + (p_{vac} + \rho_{vac})U_\mu U_\nu,\ \tilde{T}^{\mu\nu} = \tilde{p}_{vac}\tilde{g}^{\mu\nu} + (\tilde{p}_{vac} + \tilde{\rho}_{vac})\tilde{U}^\mu\tilde{U}^\nu$$

and $U^\mu, \tilde{U}^\mu$ the velocity four-vectors defined so that:

$$g^{\mu\nu}U_\mu U_\nu = -1$$

$$\tilde{g}^{\mu\nu}\tilde{U}_\mu\tilde{U}_\nu = -1$$

we then have :

$$\tilde{T}^{\mu\nu} = \tilde{p}_{vac}g_{\mu\nu} + (\tilde{p}_{vac} + \tilde{\rho}_{vac})U_\mu U_\nu$$

because $U_\mu = \tilde{U}^\mu$ follows from the fact that the above relations uniquely define the velocity four-vectors. Then the condition for the divergences to vanish is again satisfied :

$$T_{\mu\nu} = \frac{\tilde{T}^{\mu\nu}}{g}$$

or

$$d^4\xi T_{\mu\nu} = d^4\tilde{\xi}\tilde{T}^{\mu\nu}$$

Now the action $I_G + \tilde{I}_G$ for the gravitational field alone reads:

$$I_G + \tilde{I}_G \equiv -\frac{1}{16\pi G}\int d^4x\sqrt{g(x)}R(x) - \frac{1}{16\pi G}\int d^4x\sqrt{\tilde{g}(x)}\tilde{R}(x)$$

where $\tilde{R}(x)$ can be obtained from $R(x)$ through the replacements $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g^{\mu\nu}$. This is manifestly a scalar under general coordinate transformations as well as discrete P and T transformations. Under arbitrary variations of our metrics we get:

$$\delta I_G + \delta\tilde{I}_G \equiv \frac{1}{16\pi G}\int d^4x\sqrt{g(x)}\left[R^{\mu\nu}(x) - \frac{1}{2}g^{\mu\nu}(x)R(x)\right]\delta g_{\mu\nu}(x) + \frac{1}{16\pi G}\int d^4x\sqrt{\tilde{g}(x)}\left[\tilde{R}^{\mu\nu}(x) - \frac{1}{2}\tilde{g}^{\mu\nu}(x)\tilde{R}(x)\right]\delta\tilde{g}_{\mu\nu}(x)$$

But, working again in our privileged coordinate system, we get:

$$\delta I_G + \delta\tilde{I}_G \equiv \frac{1}{16\pi G}\int d^4x\sqrt{g(x)}\left[R^{\mu\nu}(x) - \frac{1}{2}g^{\mu\nu}(x)R(x)\right]\delta g_{\mu\nu}(x) + \frac{1}{16\pi G}\int d^4x\sqrt{g^{-1}(x)}\left[\tilde{R}^{\mu\nu}(x) - \frac{1}{2}\tilde{g}^{\mu\nu}(x)\tilde{R}(x)\right]\delta g^{\mu\nu}(x)$$

It's now time to require the total action to be stationary with respect to arbitrary variations in the metric field. We again make use of the simplifying relation $\delta g^{\rho\kappa}(x) = -g^{\rho\mu}(x)g^{\nu\kappa}(x)\delta g_{\mu\nu}(x)$ to obtain our new gravitational equation:

$$-8\pi G\left(\sqrt{g(x)}T_{\rho\sigma} - \sqrt{g^{-1}(x)}\tilde{T}^{\rho\sigma}\right) = \sqrt{g(x)}\left(R_{\rho\sigma} - \frac{1}{2}g_{\rho\sigma}R\right) - \sqrt{g^{-1}(x)}\left(R^{\rho\sigma} - \frac{1}{2}g^{\rho\sigma}R\right)_{g^{\rho\sigma}\to g_{\rho\sigma},\, g_{\rho\sigma}\to g^{\rho\sigma}}$$

This is of course not a general covariant equation. For example, through the replacement $g^{\mu\nu}(x) \to g_{\mu\nu}(x),\, g_{\mu\nu}(x) \to g^{\mu\nu}(x)$ the affine connection becomes a quite strange object

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\rho}\left(\frac{\partial g_{\rho\mu}}{\partial x^{\nu}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\rho}}\right) \to \frac{1}{2}g_{\lambda\rho}\left(\frac{\partial g^{\rho\mu}}{\partial x^{\nu}} + \frac{\partial g^{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g^{\mu\nu}}{\partial x^{\rho}}\right)$$

which obviously violates the familiar 'rules' for summing or multiplying tensor-like objects. But it should be kept in mind that the above equation is only valid in our privileged working coordinate system so it is not intended to be generally covariant. However, once our metric solutions $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ have been determined in our privileged coordinate system, these can still be exported to any arbitrary general coordinate system in a straightforward way since these are tensors. Our point of view is that general covariance only needs to be satisfied as a symmetry property of the action we start with. The same for Parity and Time reversal transformations which can still be considered in general relativity as "discrete" symmetries since these are really jumping from one system of inertial coordinates to its conjugated. This is why we required our action to be a scalar under general coordinate transformations plus Parity and time reversal transformations. But our final field equations then follow from both a least action principle and a nontrivial relation between one metric and its time reversal conjugated. This relation involves a privileged coordinate system so that general covariance is not anymore a property of our modified gravitational field equations.

Explaining how a negative energy can be generated through time reversal, we have now completed the program initiated in section VI and by the way reinforced various results obtained there in the naïve signed volumes approach. We have also achieved the important task of introducing the negative energy sources in the gravitational equation. The corresponding fields live in the conjugated metric which prevent them from interacting with our world fields except through gravitation. In the presence of a gravitational field at a given space-time point, there exists a locally inertial coordinate system ξ^α where our usual metric identifies with the Minkowski metric and its first derivatives vanish. There is also another $\tilde{\xi}^\alpha$ locally inertial coordinate system where the conjugated metric identifies with the Minkowski metric and its first derivatives vanish. At last, considering that we have two metric fields, it is not very surprising that there exists at least locally a third coordinate system where $g^{\mu\nu} = \tilde{g}_{\mu\nu}$. Because the two metrics are not independent the model is not actually bi-metric and we shall check that the linearized gravitational side of our new Einstein equation is the same as in General Relativity so that working in the harmonic gauge, the number of physical degrees of freedom of the gravitational waves polarization tensor is still two. We may alternatively say that the gravitational field is a two-sided object, one side $g_{\mu\nu}$ where we live, and the other side, its inverse $g^{\mu\nu}$ where, from our metric point of view, the negative energy fields live.

It may also be that we could exceptionally find locally inertial coordinate systems satisfying $\tilde{\xi}^\alpha = \xi_\alpha$ where $g_{\mu\nu} = \tilde{g}_{\mu\nu} = \eta_{\alpha\beta} = \eta^{\alpha\beta}$, the first part of this relation being an equality between tensors must remain locally valid in any general coordinate system. In such case, which does not necessarily follow from a flat metric situation, we may say that there is no gravitational field in an absolute way. This opens the interesting perspective that then fields (particles) may be able to jump from one metric to its conjugated identical metric.

The minus signs needed to interpret our new energy-momentum sources as negative sources naturally emerge from the extremum action and inverse metric mechanism, starting from a fully general coordinate, time reversal and parity scalar action. Therefore fields living in the reversed time world are just seen from our world as negative energy fields (we shall soon make this statement more precise). Last but not least, divergences cancel provided vacuum action terms are equal in the time conjugated inertial four-volumes. The solutions will be now derived and their physical consequences explored for the most important physical cases. The Schwarzschild solution will be now derived and its physical outcomes explored. This will help us to further clarify in the next paragraph the meaning of the model.

IX) The Schwarzschild solution

We now want to solve our gravitational equation in the very important physical case of a static isotropic gravitational field. For simplicity we neglect possible background effects which might arise if the local privileged system appropriate to determine the Schwarzschild solution and the global coordinate system for doing cosmology do not identify. We assume that in our privileged coordinate system where our fundamental relation $g^{\mu\nu} = \tilde{g}_{\mu\nu}$ applies, the solution is isotropic and static. We start from the proper time interval written in the so-called isotropic form:

$$d\tau^2 = B(r)dt^2 - A(r)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

whose Jacobian modulus for our metric tensor is:

$$\sqrt{g(x)} = A(r)\sqrt{A(r)B(r)}\, r^2 \sin\theta$$

Obviously, in the polar coordinate system, $g^{\mu\nu}$ cannot be identified with a metric $\tilde{g}_{\mu\nu}$ because $g_{\mu\nu}$ is not dimensionless. Moreover, it is natural to require that in our privileged coordinate system, both conjugated metrics should in general possess the same isometries so here be static and isotropic. Thus the polar coordinates are not appropriate. An acceptable system is the Cartesian one where we have:

$$d\tau^2 = B(r)dt^2 - A(r)\left(d\mathbf{x}^2\right)$$
$$\left|\frac{\partial \xi^\alpha}{\partial x^\mu}\right| = A(r)\sqrt{A(r)B(r)}$$
$$d\tilde{\tau}^2 = \frac{1}{B(r)}dt^2 - \frac{1}{A(r)}\left(d\mathbf{x}^2\right)$$
$$\left|\frac{\partial \xi^\alpha}{\partial x^\mu}\right|^{-1} = \frac{1}{A(r)\sqrt{A(r)B(r)}}$$

and the gravitation equation reads (keeping only diagonal terms is enough for our needs):

$$\begin{bmatrix} A\sqrt{AB}G_{xx} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{xx}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{yy} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{yy}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{zz} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{zz}\right]_{A\to 1/A;B\to 1/B} \\ A\sqrt{AB}G_{tt} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{B^2}G_{tt}\right]_{A\to 1/A;B\to 1/B} \end{bmatrix} = A\sqrt{AB}\begin{bmatrix} T_{xx} \\ T_{yy} \\ T_{zz} \\ T_{tt}\end{bmatrix} - \frac{1}{A\sqrt{AB}}\begin{bmatrix} \widehat{T}_{xx} \\ \widehat{T}_{yy} \\ \widehat{T}_{zz} \\ \widehat{T}_{tt}\end{bmatrix}$$

Where we define the covariant tensor $\widehat{T}_{\mu\nu} = \tilde{T}^{\mu\nu}$, which existence follows from $\tilde{g}^{\mu\nu} = g_{\mu\nu}$ in case of a perfect fluid. Then, because G, T, and $\widehat{\mathrm{T}}$ are tensors:

$$\begin{bmatrix} G_{xx} \\ G_{yy} \\ G_{zz} \\ G_{tt}\end{bmatrix} = C\begin{bmatrix} G_{rr} \\ G_{\theta\theta} \\ G_{\phi\phi} \\ G_{tt}\end{bmatrix}, \begin{bmatrix} T_{xx} \\ T_{yy} \\ T_{zz} \\ T_{tt}\end{bmatrix} = C\begin{bmatrix} T_{rr} \\ T_{\theta\theta} \\ T_{\phi\phi} \\ T_{tt}\end{bmatrix}, \begin{bmatrix} \widehat{T}_{xx} \\ \widehat{T}_{yy} \\ \widehat{T}_{zz} \\ \widehat{T}_{tt}\end{bmatrix} = C\begin{bmatrix} \widehat{T}_{rr} \\ \widehat{T}_{\theta\theta} \\ \widehat{T}_{\phi\phi} \\ \widehat{T}_{tt}\end{bmatrix} \text{ with } C = \begin{bmatrix} \left(\frac{\partial r}{\partial x}\right)^2 & \left(\frac{\partial \theta}{\partial x}\right)^2 & \left(\frac{\partial \phi}{\partial x}\right)^2 & \\ \left(\frac{\partial r}{\partial y}\right)^2 & \left(\frac{\partial \theta}{\partial y}\right)^2 & \left(\frac{\partial \phi}{\partial y}\right)^2 & \\ \left(\frac{\partial r}{\partial z}\right)^2 & \left(\frac{\partial \theta}{\partial z}\right)^2 & \left(\frac{\partial \phi}{\partial z}\right)^2 & \\ & & & 1 \end{bmatrix}$$

Substituting this in the above gravitational equation, the C matrix drops out and one obtains in the polar coordinate system:

$$
\begin{bmatrix}
A\sqrt{AB}G_{rr} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{rr}\right]_{A\to 1/A;B\to 1/B} \\
A\sqrt{AB}G_{\theta\theta} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{\theta\theta}\right]_{A\to 1/A;B\to 1/B} \\
A\sqrt{AB}G_{\phi\phi} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{A^2}G_{\phi\phi}\right]_{A\to 1/A;B\to 1/B} \\
A\sqrt{AB}G_{tt} - \frac{1}{A\sqrt{AB}}\left[\frac{1}{B^2}G_{tt}\right]_{A\to 1/A;B\to 1/B}
\end{bmatrix}
= A\sqrt{AB}\begin{bmatrix} T_{rr} \\ T_{\theta\theta} \\ T_{\phi\phi} \\ T_{tt} \end{bmatrix} - \frac{1}{A\sqrt{AB}}\begin{bmatrix} \widehat{T}_{rr} \\ \widehat{T}_{\theta\theta} \\ \widehat{T}_{\phi\phi} \\ \widehat{T}_{tt} \end{bmatrix}
$$

So we can compute $R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}(x)R(x)$ in the more convenient polar coordinate system, starting from the non vanishing metric tensor components:

$$
g_{rr} = A(r),\ g_{\theta\theta} = r^2 A(r),\ g_{\phi\phi} = r^2\sin^2\theta A(r),\ g_{tt} = -B(r)
$$
$$
g^{rr} = A^{-1}(r),\ g^{\theta\theta} = r^{-2}A^{-1}(r),\ g^{\phi\phi} = r^{-2}\sin^{-2}\theta A^{-1}(r),\ g^{tt} = -B^{-1}(r)
$$

then get $\left(R^{\rho\sigma} - \frac{1}{2}g^{\rho\sigma}R\right)_{g^{\rho\sigma}\to g_{\rho\sigma},\, g_{\rho\sigma}\to g^{\rho\sigma}}$ readily in the polar coordinate system where the interpretation of the final solutions is easier. We first compute the non-vanishing components of the affine connection and insert them in the four non-vanishing components of the diagonal Ricci tensor. This yields:

$$
R_{rr} = \frac{A''}{A} + \frac{B''}{2B} - \frac{B'^2}{4B^2} - \left(\frac{A'}{A}\right)^2 + \frac{A'}{Ar} - \frac{1}{4}\frac{A'}{A}\left(\frac{B'}{B}\right)
$$
$$
R_{\theta\theta} = \frac{3}{2}\frac{A'r}{A} + \frac{A''r^2}{2A} - \frac{A'^2r^2}{4A^2} + \frac{1}{2}\frac{B'}{B}\left(\frac{A'r^2}{2A} + r\right)
$$
$$
R_{\phi\phi} = \sin^2\theta R_{\theta\theta}
$$
$$
R_{tt} = -\frac{1}{2}\frac{B''}{A} - \frac{1}{4}\frac{B'A'}{A^2} + \frac{1}{4}\frac{B'^2}{BA} - \frac{1}{A}\frac{B'}{r}
$$

from which the curvature scalar readily follows:

$$
R = 2\frac{A''}{A^2} + \frac{B''}{AB} - \frac{B'^2}{2AB^2} - \frac{3}{2A}\left(\frac{A'}{A}\right)^2 + 4\frac{A'}{A^2r} + \frac{1}{2}\frac{A'}{A^2}\left(\frac{B'}{B}\right) + 2\frac{B'}{AB}\left(\frac{1}{r}\right)
$$

as well as the needed gravitational terms:

$$
R_{rr} - \frac{1}{2}g_{rr}R = -\frac{1}{4}\left(\frac{A'}{A}\right)^2 - \frac{A'}{Ar} - \frac{B'}{Br} - \frac{1}{2}\frac{A'}{A}\left(\frac{B'}{B}\right)
$$
$$
\frac{R_{\phi\phi} - \frac{1}{2}g_{\phi\phi}R}{\sin^2\theta} = R_{\theta\theta} - \frac{1}{2}g_{\theta\theta}R = -\frac{1}{2}\frac{A'r}{A} - \frac{A''r^2}{2A} + \frac{A'^2r^2}{2A^2} - \frac{1}{2}\frac{B'}{B}r - \frac{B''r^2}{2B} + \frac{B'^2r^2}{4B^2}
$$
$$
R_{tt} - \frac{1}{2}g_{tt}R = \frac{BA''}{A^2} - \frac{3B}{4A}\left(\frac{A'}{A}\right)^2 + 2\frac{BA'}{A^2r}
$$

The terms $\left(R^{\rho\sigma}-\frac{1}{2}g^{\rho\sigma}R\right)_{g^{\rho\sigma}\to g_{\rho\sigma},g_{\rho\sigma}\to g^{\rho\sigma}}$ are then simply obtained through $A(r)\to A^{-1}(r),\ B(r)\to B^{-1}(r)$ starting from $R^{\mu\nu}(x)-\frac{1}{2}g^{\mu\nu}(x)R(x)$ where tensor indices were raised using the Cartesian metric components.

$$\left(R^{rr}-\frac{1}{2}g^{rr}R\right)_{A\to 1/A,B\to 1/B}=A^2\left\{-\frac{1}{4}\left(\frac{A'}{A}\right)^2+\frac{A'}{Ar}+\frac{B'}{Br}-\frac{1}{2}\frac{A'}{A}\left(\frac{B'}{B}\right)\right\}$$

$$\frac{\left(R^{\phi\phi}-\frac{1}{2}g^{\phi\phi}R\right)_{A\to 1/A,B\to 1/B}}{\sin^2\theta}=\left(R^{\theta\theta}-\frac{1}{2}g^{\theta\theta}R\right)_{A\to 1/A,B\to 1/B}=A^2\left\{\frac{1}{2}\frac{A'r}{A}+\frac{A''r^2}{2A}-\frac{A'^2r^2}{2A^2}+\frac{1}{2}\frac{B'}{B}r+\frac{B''r^2}{2B}-\frac{3B'^2r^2}{4B^2}\right\}$$

$$\left(R^{tt}-\frac{1}{2}g^{tt}R\right)_{A\to 1/A,B\to 1/B}=B^2\left\{\frac{A}{B}\left(-\frac{A''}{A}+2\frac{A'^2}{A^2}\right)-\frac{3A}{4B}\left(\frac{A'}{A}\right)^2-2\frac{A'}{Br}\right\}$$

In the approximation $\sqrt{AB}\approx 1$ (valid for not so weak fields) we then obtain:

$$G_{rr}^{total}=A\sqrt{AB}\left(R_{rr}-\frac{1}{2}g_{rr}R\right)-\frac{1}{A\sqrt{AB}}\left(R^{rr}-\frac{1}{2}g^{rr}R\right)_{A\to 1/A,B\to 1/B}=-2\frac{A}{r}\left(\frac{A'}{A}+\frac{B'}{B}\right)$$

$$G_{\theta\theta}^{total}=A\sqrt{AB}\left(R_{\theta\theta}-\frac{1}{2}g_{\theta\theta}R\right)-\frac{1}{A\sqrt{AB}}\left(R^{\theta\theta}-\frac{1}{2}g^{\theta\theta}R\right)_{A\to 1/A,B\to 1/B}=A\left\{-\frac{A'r}{A}-\frac{A''r^2}{A}+\frac{A'^2r^2}{A^2}-\frac{B'}{B}r-\frac{B''r^2}{B}+\frac{B'^2r^2}{B^2}\right\}$$

$$G_{tt}^{total}=A\sqrt{AB}\left(R_{tt}-\frac{1}{2}g_{tt}R\right)-\frac{1}{A\sqrt{AB}}\left(R^{tt}-\frac{1}{2}g^{tt}R\right)_{A\to 1/A,B\to 1/B}=2B\left\{\frac{A''}{A}+2\frac{A'}{Ar}-\frac{A'^2}{A^2}\right\}$$

Substituting $A=e^a$ and $B=e^b$, many derivatives quadratic terms simplify in these expressions thanks to the difference between the two conjugated metric contributions.

$$G_{rr}^{total}=-2\frac{e^a}{r}\left(a'+b'\right)$$
$$G_{\theta\theta}^{total}=e^a\left\{-a'r-a''r^2-b'r-b''r^2\right\}$$
$$G_{tt}^{total}=2e^b\left\{a''+2\frac{a'}{r}\right\}$$

In vacuum we find new **exact** Schwarzschild solutions

$$0 = \{ra'' + 2a'\} = (a'r + a)' \Rightarrow a = \frac{2MG}{r}$$

$$\Rightarrow A = e^{\frac{2MG}{r}} \approx 1 + 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2}$$

$$\Rightarrow B = \frac{1}{A} = e^{-\frac{2MG}{r}} \approx 1 - 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} - \frac{4}{3}\frac{M^3G^3}{r^3}$$

The metric solutions we get are different from the exact usual ones though in good agreement up to Post-Newtonian order:

$$A = \left(1 + \frac{MG}{2r}\right)^4 \approx 1 + 2\frac{MG}{r} + \frac{3}{2}\frac{M^2G^2}{r^2}$$

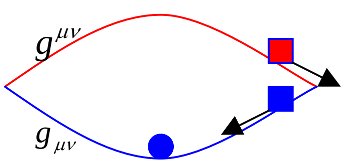
$$B = \frac{\left(1 - \frac{MG}{2r}\right)^2}{\left(1 + \frac{MG}{2r}\right)^2} \approx 1 - 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} - \frac{3}{2}\frac{M^3G^3}{r^3}$$

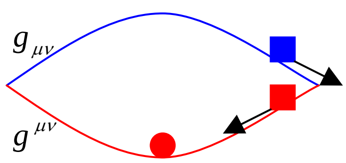
Notice that our new solutions do not share the pathological behavior at finite radius B=0. The singularities at r=0 are normal since the source mass M confined within an asymptotically null volume becomes an infinite density source. No finite radius (black hole) singularity arises in the Cartesian isotropic metric. Singularities will again show up if we export our metric solutions to the "standard" form of the metric but we have now good reasons to consider that this is due to a choice of coordinate system which does not respect the isotropy of our configuration for both conjugated metrics. Notice that in standard general relativity, if we are willing to allow the world an unusual topology, it is also possible to find a singularity free coordinate system but this is completely artificial.

The Newtonian potential $\phi(r) = -MG/r$ generated by the central mass with rest energy M living in the metric whose components are A and B is attractive for another object living in the same metric. But an object living in the conjugated metric which components are 1/A, 1/B will feel the repulsive Newtonian potential $\tilde{\phi}(r) = MG/r$. In the same way, it is straightforward to derive the conjugated potentials created by a source mass $\tilde{M}$ in $\tilde{g}_{\mu\nu}$ from the point of view of each metric :

$$\tilde{\phi}(r) = -\tilde{M}G/r \quad \phi(r) = \tilde{M}G/r$$

Objects living in the same metric attract each other. Objects living in different metrics repel each other. This may be clarified by the following picture where the colored circles stand for the mass sources and the colored squared for the test masses.





X) In a quasi-Minkowskian coordinate system

1) Energy, momentum, mass and velocity

It is easy to show that fields following the geodesics of a given metric are seen from other fields living in the same metric as positive energy fields while fields living in a given metric are seen as negative energy fields from the point of view of fields living in the conjugated metric. Let us postulate (as we did to study the Schwazschild solution) that locally our privileged coordinate system is a quasi-Minkowskian one where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with the perturbation $h_{\mu\nu}$ vanishing at infinity. The linear term in the geometric side of our new Einstein equation becomes:

$$G^{(1)}{}_{\mu\nu} = \left(R^{(1)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}R_{\lambda\rho}{}^{(1)} \right) - \left(R^{(1)\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\eta^{\lambda\rho}R_{\lambda\rho}{}^{(1)} \right)_{g^{\mu\nu}\to g_{\mu\nu};g_{\mu\nu}\to g^{\mu\nu}}$$

$$= 2\left(R^{(1)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}R_{\lambda\rho}{}^{(1)} \right)$$

so that this equation becomes simply :

$$-8\pi G\tau_{\mu\nu} = -8\pi G(t_{\mu\nu} + \sqrt{g}T_{\mu\nu} - \frac{1}{\sqrt{g}}\tilde{T}^{\mu\nu}) = G^{(1)}{}_{\mu\nu} = 2\left(R^{(1)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}R_{\lambda\rho}{}^{(1)} \right)$$

As in general relativity, the term on the right hand side satisfies the linearised Bianchi identity and is a Lorentz tensor. Therefore, the left hand side term can still be interpreted as a total radiation, matter and gravitation energy-momentum pseudo-(general and Lorentz)-tensor conserved in the usual sense. In the weak field approximation, this is just a conservation law of matter and radiation only which simplifies to:

$$\partial_\mu G^{\mu\nu(1)} = \partial_\mu(-\tilde{T}^{\rho\sigma}\eta_{\rho\mu}\eta_{\nu\sigma} + T^{\mu\nu}) = 0$$

Notice however that neither $\tau^{\mu\nu}$ nor $t^{\mu\nu}$, nor even $-\tilde{T}^{\rho\sigma}\eta_{\rho\mu}\eta_{\nu\sigma} + T^{\mu\nu}$ are Lorentz tensors as was the case in general relativity which reminds us that our new Einstein equation is only valid as it stands in the privileged coordinate system (a local rest frame?) but not in any other Lorentz transformed coordinate system where it would develop frame velocity dependent terms! However a total energy and momentum four component object can still be built out of $\tau^{\mu\nu}$ and such components, though not Lorentz covariant, are invariant under any space-time translation or coordinate transformation that reduces at infinity to the identity since the demonstration as in GR only makes use of the good properties of $G^{\mu\nu(1)}$.

The latter equation and all the previous ones follow from the Einstein equation of the model which we obtained by requiring that the variation of our action induced by the metric variation $\delta g_{\mu\nu}$ should vanish. Hence the conserved tensors we finally obtained are defined from the point of view of the $g_{\mu\nu}$ metric. We would obviously have obtained a similar result from the point of view of the conjugated metric, deriving in a similar way our equations expressed versus this metric. Now, what this equation tells us is that we have a pseudo-tensor, conserved in the usual sense which energy and momentum components read:

$$T^{00} - \tilde{T}^{00} \ , \quad T^{0i} + \tilde{T}^{0i}$$

.

Hence the matter and radiation fields have their energy reversed from the other metric point of view while their momentum keeps the same. This is perfectly coherent with what the Unitary time reversal scenario led us to expect: energy reverses but momentum is invariant under time reversal. We learned in the previous paragraph that point masses living in different metric repel each other. Hence not only their momenta but their velocities as well should be opposite in the system where the total momentum vanishes. Then, because as usual the velocity and momentum of a mass in our metric point toward the same direction, it is easily seen that the velocity and momentum of the mass living in the conjugated metric also point in the same direction from our same point of view. Thus mass does not reverse under time reversal implying velocity time reversal invariance too. This is again in perfect agreement with what we learned from the Unitary time reversal scenario though in a completely different (quantum) context.

2) Gravitational energy

i. Energy-momentum of the gravitational wave

We now carry out the computation of the average energy-momentum tensor $t_{\mu\nu}^{(2)}$ of the gravitational wave following the method described in [2] (p 259). Working in the harmonic gauge, the gravitational plane wave propagating in vacuum satisfies as in General Relativity the equations of motion $R^{(1)}{}_{\mu\nu} = \Box^2 h_{\mu\nu} = 0$. The same helicity degrees of freedom (+/-2 components) propagate as in General Relativity. Because the metric already satisfies the linear equation of motion $R^{(1)}{}_{\mu\nu} = 0$, the expression of $t_{\mu\nu}^{(2)}$ simplifies:

$$8\pi G t^{(2)}{}_{\mu\nu} = \left(R^{(2)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho} R_{\lambda\rho}{}^{(2)} \right) - \left(R^{(2)\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\eta^{\lambda\rho} R_{\lambda\rho}{}^{(2)} \right)_{g^{\mu\nu} \to g_{\mu\nu};\, g_{\mu\nu} \to g^{\mu\nu}}$$
$$= \left(R^{(2)}{}_{\mu\nu} - \eta^{\mu\rho}\eta^{\nu\sigma} R^{(2)}{}_{\rho\sigma} \right)$$

We get :

$$t^{(2)}{}_{\mu\nu \neq 0i} = 0$$
$$\left\langle t^{(2)}{}_{0i} \right\rangle = \frac{2}{8\pi G} \left\langle R^{(2)}{}_{0i} \right\rangle = \frac{1}{8\pi G} k_i k_0 (|e_+|^2 + |e_-|^2)$$

expressed versus the helicity +/-2 amplitudes. In particular we have $t^{(2)}{}_{00} = 0$ i.e the gravitational wave carries momentum but no energy as we might have anticipated for an object (the graviton) which lies at the boundary (interface) between positive energy and negative energy worlds, from either metric point of view.

ii. Gravitational energy in the Schwarzschild solution

For the above Schwarzschild solution we may compute the energy of the gravitational field by performing the difference between the total energy of matter plus gravitation in our privileged coordinate system minus the energy of matter only. According [II] p 302 this is:

$$\int_0^R 4\pi r^2 (1-\sqrt{A(r)B(r)})\rho(r)dr$$

where R stands for the radius of the source object. If this source has zero pressure, $B = \frac{1}{A}$ holds everywhere in the volume, and we find that the energy of gravitation vanishes.

But what about a nonzero pressure source? We just obtained that in our privileged coordinate system which is also the rest frame of our zero pressure source (where all its subcomponents are also at rest), gravitational energy but obviously also gravitational momentum cancel. But we expect preferred frame effects such as local Lorentz invariance violation effects in our model though not at the Post–Newtonian level as we shall check. So, performing a Lorentz transformation on this gravitational energy-momentum pseudo-Lorentz-vector, it will develop new frame velocity dependent terms in such a way that its components will not vanish anymore. This is not however a real concern since the components of the object we get in this new system cannot anymore be interpreted as representing the energy and momentum of gravity, such interpretation being only valid in the privileged rest frame of the source. We then understand why, for a source with non-vanishing pressure, the above computation that seems to imply that the gravitational energy would not vanish should not be taken serious. This is of course because we cannot anymore consider that such source subcomponents are all at rest. Therefore a single privileged coordinate system cannot be valid as we implicitly assumed and we just have no right to derive in the way we did the Schwarzschild solution in this case.

The gravitational energy associated with a point source mass is well defined and vanishes in its privileged rest frame and all translated frames. For an extended source with all its subcomponents at rest there is still no problem to define and find again a total vanishing energy and momentum thanks to this translation invariance of the theory. But when the pressure does not vanish we are in trouble, because the theory lacks Lorentz invariance and no energy and momentum can be defined for a source with subcomponents moving relative to each other.

XI) **Stability of the model**

In general relativity, the energy of gravitational radiation is always positive. Therefore negative energy sources are problematic for a negative energy particle would catastrophically fall to an infinite negative energy by radiating a positive infinite gravitational energy [8][9]. The instability is also clearly seen in the phenomenology of a positive energy mass interacting with a negative energy mass through an usual interaction propagated by positive energy virtual interaction particles. The negative energy object is being attracted by the positive energy object, the latter being repulsed by the former. They then accelerate together for ever this resulting in an obviously instable picture!

But in our model the gravitational interaction of two masses living in different metric exhibits no such instability since they just repel each other. Yet, from the point of view of one of the two metrics, this is really the interaction between a positive energy mass and a negative energy mass.

How could we describe in quantum worlds or Feynman diagrams such a stable situation, the full answer would certainly have to be formulated in a complete quantum theory of the graviton. But we can anticipate that, if we analyse the problem from the point of view of one metric, the fields living there need to be quantified as positive energy quantum fields while fields living in the conjugated metric need to be quantified as negative energy quantum fields. Such fields do not couple to each other but interact with the gravitational field. It then appears clearly that the graviton can neither be quantified as a positive energy field nor a negative energy field if one wants to reproduce the above phenomenology. This is because in either cases the interacting masses would manifest the instability mentioned above, in contradiction with what we learnt from the classical picture. Moreover, we also computed that the energy of the gravitational wave is zero. We therefore have to consider the graviton as an object of a completely new kind. The most intuitive procedure would be to quantify our graviton as a mixed positive and negative energy object able to create, annihilate and propagate simultaneously both signs of the energy. If such object is seen by a positive (resp negative) energy field as a positive (resp negative) energy graviton then no instability should arise at each vertex where all objects carry the same sign of the energy. Moreover because the two interacting masses do not see the graviton with the same energy sign, the repulsion can be obtained. At last, either there is no self –interaction of the graviton, or this self interaction occurs as one between objects of the same energy sign which also avoids any instability. Anyway, the quantum instability issue must be readdressed in a new and quite unusual context.

XII) Einstein Equivalence Principle

The weak equivalence principle is obviously not menaced by the proposed model since once the metric field solution is established the behavior of matter and radiation living in the metric is described by the same action as in GR. But, because of the non covariance of our modified equation of motion, a violation of the strong equivalence principle seems unavoidable, especially a local Lorentz invariance violation. Indeed, our non covariant Einstein equation is only valid as it stands in a privileged local coordinate system, a locally rest frame. Exported by a Lorentz transformation it is expected to develop frame velocity dependent terms. The alpha Post Newtonian parameters constrain with tremendous accuracies the amount of local Lorentz invariance violation induced but such terms. However, as we shall now show, in our model the local Lorentz violation might only arise at the Post-Post Newtonian level of accuracy. All Post Newtonian parameters are found to be identical to the ones expected from GR, and in good agreement with observation. We follow the same notations as in [2] (chapter 9). Assuming there is no source term in the conjugated metric our equation reads:

$$-8\pi G T_{\rho\sigma} = \left(R_{\rho\sigma} - \frac{1}{2} g_{\rho\sigma} R \right) - \frac{1}{g}\left(g_{\rho\mu} g_{\nu\sigma} R_{\mu\nu\, g\to g^{-1}} - \frac{1}{2} g_{\rho\sigma} R_{g\to g^{-1}} \right)$$

Let us transform the equation into a similar form as the one in [2] 9.1.41 p217. It is straightforward to obtain :

$$-8\pi G \left(T_{\rho\sigma} - \frac{1}{2} g_{\rho\sigma} T \right) = R_{\rho\sigma} - \frac{1}{g} g_{\rho\mu} g_{\nu\sigma} R_{\mu\nu\, g\to g^{-1}}$$

In the Parametrised Post-Newtonian formalism, the metric solution are expanded in powers of GM/r :

$$g_{00} = -1 + \overset{2}{g}_{00} + \overset{4}{g}_{00} \quad , \quad g^{00} = -1 + \overset{2}{g}{}^{00} + \overset{4}{g}{}^{00}$$

$$g_{ij} = \delta_{ij} + \overset{2}{g}_{ij} \quad , \quad g^{ij} = \delta^{ij} + \overset{2}{g}{}^{ij}$$

$$g_{i0} = \overset{3}{g}_{i0} \quad , \quad g^{i0} = \overset{3}{g}{}^{i0}$$

From the inverse metric relation we can derive:

$$\overset{2}{g}{}^{00} = -\overset{2}{g}_{00}$$

$$\overset{2}{g}{}^{ij} = -\overset{2}{g}_{ij}$$

$$\overset{3}{g}{}^{i0} = \overset{3}{g}_{i0}$$

To this order, we get the same equations (redefining G→G/2) as in general relativity since :

$$\overset{2}{R}_{ij} - \overset{2}{R}_{ij\,g\to g^{-1}} = 2\overset{2}{R}_{ij}$$

$$\overset{3}{R}_{i0} + \overset{3}{R}_{i0\,g\to g^{-1}} = 2\overset{3}{R}_{i0}$$

$$\overset{2}{R}_{00} - \overset{2}{R}_{00\,g\to g^{-1}} = 2\overset{2}{R}_{00}$$

The Post-Newtonian approximation stops here for g_{ij} et g_{i0} so that the Post Newtonian parameters are identical to those in General relativity for such metric element solutions.
We also have :

$$\overset{2}{g}_{ij} = \delta_{ij}\overset{2}{g}_{00}$$

this yields

$$\sqrt{\overset{2}{g}} = \frac{1}{2}\overset{2}{g}_{\mu\nu}\eta^{\mu\nu} = -\frac{1}{2}\overset{2}{g}_{00} + \frac{1}{2}\overset{2}{g}_{ii} = -\frac{1}{2}\overset{2}{g}_{00} + \frac{3}{2}\overset{2}{g}_{00} = \overset{2}{g}_{00}$$

thus

$$1/g = -2\overset{2}{g}_{00}$$

To the next order in g_{00} the following equation has to be satisfied:

$$\overset{4}{R}_{00} - \overset{4}{R}_{00\,g\to g^{-1}} + \overset{2}{\left(\frac{1}{g}\right)}\overset{2}{R}_{00} + 2\overset{2}{g}_{00}\overset{2}{R}_{00\,g\to g^{-1}} = \frac{1}{2}\left(-8\pi G\left[\overset{2}{T}{}^{00} + \overset{2}{T}{}^{ii}\right] + 16\pi G\overset{2}{g}_{00}\overset{0}{T}{}^{00}\right)$$

where substituting $4\overset{2}{R}_{00} = -8\pi G\overset{0}{T}{}^{00}$ implies :

$$\overset{4}{R}_{00} - \overset{4}{R}_{00\,g\to g^{-1}} = -8\pi G\frac{1}{2}\left[\overset{2}{T}{}^{00} + \overset{2}{T}{}^{ii}\right]$$

but ([2] p217),

$$\overset{4}{R}_{00}=\frac{1}{2}\nabla^2\overset{4}{g}_{00}-\frac{1}{2}\frac{\partial^2\overset{2}{g}_{00}}{\partial t^2}-\frac{1}{2}\overset{2}{g}_{ij}\frac{\partial^2\overset{2}{g}_{00}}{\partial x_i\partial x_j}+\frac{1}{2}\left(\nabla^2\overset{2}{g}_{00}\right)^2$$

now using $\overset{4}{g}{}^{00}=-\overset{4}{g}_{00}-\left(\overset{2}{g}_{00}\right)^2$ allows to obtain:

$$\overset{4}{R}_{00}-\overset{4}{R}_{00\,g\to g^{-1}}=\frac{1}{2}\nabla^2\overset{4}{g}_{00}-\frac{1}{2}\nabla^2\left(-\overset{4}{g}_{00}-\left(\overset{2}{g}_{00}\right)^2\right)-\frac{\partial^2\overset{2}{g}_{00}}{\partial t^2}=\nabla^2\overset{4}{g}_{00}+\frac{1}{2}\nabla^2\left(\left(\overset{2}{g}_{00}\right)^2\right)-\frac{\partial^2\overset{2}{g}_{00}}{\partial t^2}$$

and finally

$$\nabla^2\overset{4}{g}_{00}=\frac{\partial^2\overset{2}{g}_{00}}{\partial t^2}-\frac{1}{2}\nabla^2\left(\left(\overset{2}{g}_{00}\right)^2\right)-8\pi G\frac{1}{2}\left[\overset{2}{T}{}^{00}+\overset{2}{T}{}^{ii}\right]$$

which is exactly the result we had in RG ([2]9.1.63) provided the G constant is redefined as G→G/2 . This is the ultimate proof that all PN parameters are identical to those in RG and that Local Lorentz invariance violation does not arise at the Post-Newtonian order in the proposed model (though it might well appear at the Post-Post-Newtonian order!).

XIII) Cosmology

1) The metric

Negative energy density is of course a good candidate to produce the universe acceleration that is actually observed. But most of this effect is believed to be produced by a negative pressure source, the so called dark vacuum energy. However, the analysis of cosmological data favoring this picture needs to be revised since it is not clear that in our model the observed expansion $\left(H_0\right)$ of our universe will give the universe such a high expansion gravitational energy as $H_0{}^2$. Indeed, we could show that some quadratic terms in the metric field derivatives are strongly suppressed for weak fields in the Schwarzschild configuration and this may also happen to gravitational kinetic energy terms in the FRW metric.

Our first task is to find a coordinate system where both metrics would appear in the spatially homogeneous and isotropic form appropriate for cosmology. Again the most general suitable working privileged system is here the Cartesian one:

$$d\tau^2=c^2(t)dt^2-a^2(t)f(k,r)d\mathbf{x}^2$$

where c(t) and a(t) are dimensionless scale factors. Notice that the global privileged coordinate system we need for cosmology is of course different from the local privileged coordinate system where the metric was in a quasi-Minkowskian form! The FRW metric in polar coordinates and standard form reads:

$$d\tau^2=c^2(t)dt^2-a^2(t)\left\{\frac{dr'^2}{1-kr'^2}+r'^2d\theta^2+r'^2\sin^2\theta d\phi^2\right\}$$

The corresponding isotropic form reads:

$$d\tau^2 = c^2(t)dt^2 - a^2(t)f(k,r)\left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right\}$$

The change of variable from r to $r' : r'^2 = f(k,r)r^2$ is such that :

$$\frac{1}{1-kr'^2} = \frac{1}{1-kf(k,r)r^2} = \frac{1}{\left(1+\frac{r}{2f(k,r)}\frac{df(k,r)}{dr}\right)^2}$$

Thus $f(k,r)$ is solution of the differential equation :

$$-kf(k,r)r^2 = \frac{r^2}{4}\left(\frac{f'(k,r)}{f(k,r)}\right)^2 + r\frac{f'(k,r)}{f(k,r)}$$

The only solution such that $f^{-1}(k,r) = f(k',r)$ (both metrics are required to be isotropic and homogeneous) with $k = \pm 1, 0; k' = \pm 1, 0$ is obtained for $k = k' = 0$ and $f(k,r) = 1$ so that in our coordinate system we get:

$$d\tau^2 = c^2(t)dt^2 - a^2(t)d\mathbf{x}^2$$

We see that our model predicts without resorting to inflation that our 3-dimensionnal space is flat. We then follow the same method as for the Schwarzschild solution to get in the polar coordinate system:

$$R_{rr} - \frac{1}{2}g_{rr}R = -2a\dot{a}\frac{\dot{c}}{c^3} + 2a\ddot{a}\frac{1}{c^2} + \frac{\dot{a}^2}{c^2}$$

$$R_{tt} - \frac{1}{2}g_{tt}R = 3\left(-\frac{\dot{a}^2}{a^2}\right)$$

and our modified cosmological Einstein equations with $\tilde{a} = \frac{1}{a}$, $\tilde{c} = \frac{1}{c}$:

$$-8\pi G\left(a^3 c\rho c^2 - \frac{1}{a^3 c}\tilde{\rho}c^2\right) = 3a^3 c\left(-\frac{\dot{a}^2}{a^2}\right) - \frac{3}{a^3 c}c^4\left(-\frac{\dot{\tilde{a}}^2}{\tilde{a}^2}\right)$$

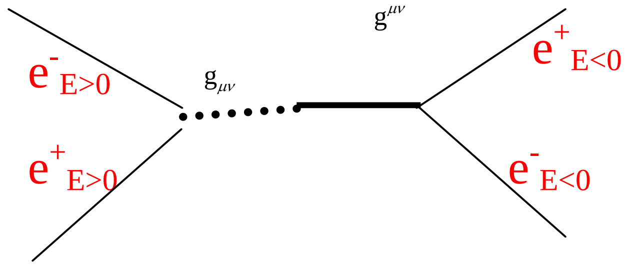
$$-8\pi G\left(R^3 cpR^2 - \frac{1}{R^3 c}\tilde{p}R^2\right) = R^3 c\left\{-2a\dot{a}\frac{\dot{c}}{c^3} + 2a\ddot{a}\frac{1}{c^2} + \frac{\dot{a}^2}{c^2}\right\} - \frac{a^4}{a^3 c}\left\{-2\tilde{a}\dot{\tilde{a}}\frac{\dot{\tilde{c}}}{\tilde{c}^3} + 2\tilde{a}\ddot{\tilde{a}}\frac{1}{\tilde{c}^2} + \frac{\dot{\tilde{a}}^2}{\tilde{c}^2}\right\}$$

Furthermore, we postulate that our privileged coordinate system is such that $\tilde{c} = \tilde{a}, c = a$ because we suspect another kind of symmetry linking space and time coordinates to provide strong reasons for such choice. Our cosmological equations then read:

$$\frac{8\pi G}{3}\left(\rho a^6 - \tilde{\rho}\frac{1}{a^2}\right) = \left(a^4 - 1\right)\frac{\dot{a}^2}{a^2}$$

$$\frac{8\pi G}{3}\left(p a^6 - \tilde{p}\frac{1}{a^2}\right) = a^4\left(-\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a}\right) - \left(-2\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2}\right)$$

However, at high pressures (which occurs usually at the center of stars or galaxies or in the radiative era) we expect matter exchanges between the two metrics to become non-negligible through diagrams such as:



So, it is not a trivial task to predict the evolution of the various density and pressure sources. However, in the cold-cold era, the source terms vanish in the space-space equation and this allows us to simply determine the evolution of the scale factor which remarkably is only driven by the gravitational energy in both metrics, independently of their actual matter and radiation content.

2) An exact cosmological solution in the cold-cold era

Our space-space equation becomes:

$$0 = a^4\left\{-\left(\frac{\dot{a}}{a}\right)^2 + 2\ddot{a}\frac{1}{a}\right\} - \left\{-2\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2}\right\}$$

$$\Rightarrow \qquad \frac{3+a^4}{1+a^4} = 2\frac{a\ddot{a}}{\dot{a}^2}$$

Hence, in our working conformal time coordinate system our universe is always accelerating during the non relativistic matter dominated era but not in the proper time coordinate system as we shall see. By the way, it is easy to compute that the space-time curvature of our universe is negative:

$$R_{space-time} = -6\frac{\ddot{a}}{a^3} < 0$$

At last, to express our solution in the most usual cosmological coordinate system we need $a'(t')$ in the proper time t' coordinate system where $c'(t') = 1$. Because the coordinate transformation only affects the time coordinate, the space-space metric element is left unchanged: $a'(t') = a(t)$. So we only need to express $a(t)$ in term of t' using :

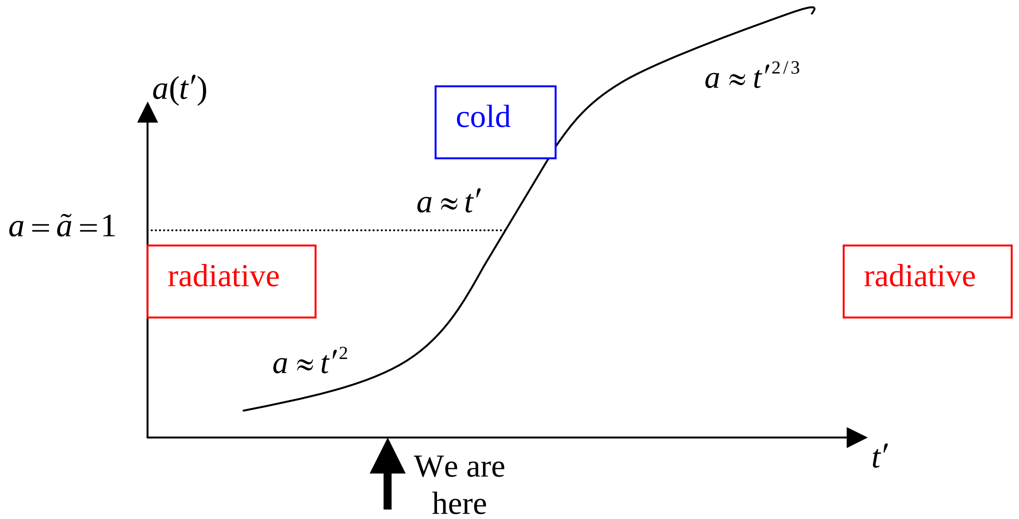
$$dt' = a(t)dt \Rightarrow t' = \int a(t)dt \equiv f(t)$$

$a(f^{-1}(t'))$ is the searched solution in the proper time coordinate system. During the non relativistic matter dominated era we find that our spatially flat universe undergoes a constant acceleration (present) phase followed by a deceleration phase with in between a no acceleration radius where our universe and its conjugate will temporarily identify:

$$a << 1 \Rightarrow \ddot{a} \approx \frac{3}{2}\frac{\dot{a}^2}{a} \Rightarrow a \approx \frac{1}{(t-1)^2} \approx t'^2 \text{ where } t \propto 1 - \frac{1}{t'} < 1$$

$$a \approx 1 \Rightarrow \ddot{a} \approx \frac{\dot{a}^2}{a} \Rightarrow a \approx e^t \approx t'$$

$$a >> 1 \Rightarrow \ddot{a} \approx \frac{1}{2}\frac{\dot{a}^2}{a} \Rightarrow a \approx t^2 \approx t'^{2/3}$$



3) An approached cosmological solution in the cold-cold era

Assuming no matter exchanges between the conjugated metrics, let us approximate: $\rho \approx \frac{M}{a^3}, \tilde{\rho} \approx a^3 M$. Our time-time equation then becomes:

$$\frac{8\pi GM}{3} a^2 \left(a - \frac{1}{a}\right) = \left(a^4 - 1\right)\left(\frac{\dot{a}^2}{a^2}\right)$$

$$\Rightarrow \quad \frac{8\pi GM}{3} \frac{a^3}{\left(a^2+1\right)} = \dot{a}^2 \quad (1)$$

Taking the first derivative of (1) and dividing by (1) the constant factor cancels and we obtain:

$$\frac{3+a^2}{1+a^2} = 2\frac{a\ddot{a}}{\dot{a}^2}$$

Thus our space-space and time-time equations are in good agreement which at posteriori justifies our primary assumptions $\rho \approx \frac{M}{a^3}, \tilde{\rho} \approx a^3 M$ when $p << \rho, \tilde{p} << \tilde{\rho}$ in the various ranges:

$$a << 1 \Rightarrow \ddot{a} \approx \frac{3}{2}\frac{\dot{a}^2}{a}$$

$$a \approx 1 \Rightarrow \ddot{a} \approx \frac{\dot{a}^2}{a}$$

$$a >> 1 \Rightarrow \ddot{a} \approx \frac{1}{2}\frac{\dot{a}^2}{a}$$

The fact that space-space and time-time equations do not identify is again a manifestation of matter exchanges between our conjugated metrics even in the non-relativistic matter dominated era.

4) Ages and luminosity distances

Let us compute what luminosity distance the model predicts at the beginning of the first accelerating phase during the non relativistic matter dominated (cold-cold) era. For the comoving radial coordinate of a light source at redshift z we have (k=0):

$$r_1 = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{t_1}^{t_0} \frac{dt}{t^2} = \frac{1}{t_1} - \frac{1}{t_0} = \frac{1}{\sqrt{a_1}} - \frac{1}{\sqrt{a_0}} = \frac{1}{\sqrt{a_0}}\left(\sqrt{1+z} - 1\right)$$

this yields for the luminosity distance

$$d_L = a_0 r_1 (1+z) = \sqrt{a_0}\left(\sqrt{1+z} - 1\right)(1+z) = \frac{2}{H_0}\left(\sqrt{1+z} - 1\right)(1+z)$$

This relation should approximately fit the observed magnitude versus red-shift supernovae plot as long as we are far away from the a=1 zone where the two universes will cross and acceleration will temporarily vanish. A more accurate study should give access to the time and radius at which the crossing will happen.

In this model the age of our universe is roughly $\frac{2}{H_0} \approx 28.10^9$ years and the age of the oldest galaxy (z=5) is $\approx 17.10^9$ years in quite a good agreement with the oldest stars ages while providing more time for galaxy formations. The expansion rate was also slower in the past.

XIV) **Outlooks: phenomenological aspects and theoretical outcomes**

The existence of negative energy fields interacting gravitationally with positive energy fields has many interesting consequences.

1) Astrophysics, galaxies and large scale structures

First of all, missing mass effects and gravitational anomalies could all be reinterpreted as being produced by negative energy invisible matter and no dark matter may be needed anymore. For instance, wide empty areas in the universe at very large scale may be due to the presence of large negative energy structures repulsing normal matter at the frontier of what would appear to us as empty bubbles.

By the way, we were also led to a surprising and unexpected result: finite radius singularity has disappeared from our final Schwarzschild metric solutions expressed in the most natural coordinate system. So what about black holes in this new context?

2)Theoretical issues

- UV divergences

We have demonstrated how vacuum divergences vanish but what about UV loop divergences? [5] As we have seen, negative energy virtual propagators would completely cancel positive energy ones if both were allowed to propagate the interaction with identical couplings. Suppose that such a connection between the two worlds is actually fully reestablished above a given energy threshold and that then loop divergences naturally get cancelled. Such reconnection could take place through a new transformation process allowing quantum particles to jump from one metric to the conjugated one [5]: such particles would just disappear from our world, maximally violating charge and energy conservation: an easy detectable signature for a particle tracker if it is not filtered out by a low level trigger! If the process takes place at the spontaneous symmetry breaking scale it may be possible to discover it in the forthcoming high energy accelerators. But a more realistic possibility as we already noticed is that such a transformation be triggered where our two conjugated metrics are exactly equal. Indeed, here our positive and negative energy fields live in the same metric. At last, as we already noticed the described model already allows for matter exchanges between the two metrics and such kind of processes may also play a significant role in the canceling of UV loop divergences.

- Maximal C, P and baryonic asymmetries

One of the most painful concerns in High Energy Physics is related to our seemingly inability to provide a satisfactory explanation for the maximal Parity violation observed in the weak interactions. The most popular model that may well account, through the seesaw mechanism, for the smallness of neutrino masses is quite disappointing from this point of view since parity violation is just put in by hand, as it is in the standard model, in the form of different spontaneous symmetry breaking scalar patterns in the left and right sectors. The issue is just postponed, and we are still waiting for a convincing explanation for this trick. Actually, one gets soon convinced that the difficulty comes from the fact that Parity violation apparently only exists in the weak interaction. Much more easy would be the task to search for its origin if this violation was universal. And yet, quite interestingly, it seems possible to extend parity violation to all interactions, just exploiting the fundamental structure of fermion fields and at the same time explain why this is only detectable and apparent in the weak interactions.

There exists four basic degrees of freedom, solutions of the Dirac field equations: these are $\psi_L(x), \psi_R(x), \psi_{cL}(x), \psi_{cR}(x)$ but two of them suffice to create and annihilate quanta of both charges and helicities: for instance the usual $\psi(x) = \psi_L(x) + \psi_R(x)$ may be considered as the most general Dirac solution:

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int_{p,\sigma} u(p,\sigma) a_c(p,\sigma).e^{i(px)} + v(p,\sigma) a^{\dagger}(p,\sigma) e^{-i(px)} \ d^3\boldsymbol{p}$$

But another satisfactory base, as far as our concern is just to build kinetic_interaction terms and not mass terms, could be the pure left handed $\psi_L(x) + \psi_{cL}(x)$ field making use of the charge conjugated field.

$$\psi_c(x) = \frac{1}{(2\pi)^{3/2}} \int_{p,\sigma} u(p,\sigma) a(p,\sigma) \ e^{i(px)} + v(p,\sigma) a_c^{\dagger}(p,\sigma) e^{-i(px)} \ d^3\boldsymbol{p}$$

Indeed, from a special relativistic mass-less Hamiltonian such as

$$H_L^0 = \int d^3x [\psi_L^{\dagger}(-i\alpha.\nabla)\psi_L] \ + \int d^3x [\psi_{cL}^{\dagger}(-i\alpha.\nabla)\psi_{cL}]$$

the same normal ordered current and physics as the usual one are derived when requiring various global symmetries to become local (this is checked in the Annexe).

$$\overline{\Psi}\gamma_\mu \frac{1-\gamma_5}{2}\Psi(x) - \overline{\Psi}_c\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) = :\overline{\Psi}\gamma_\mu\Psi(x):$$

Assume now that the corresponding general Dirac field built out of only right handed components is not redundant with the previous (as is generally believed because except for a Majorana particle, both create and annihilate quanta of all charges and helicities) but lives in the conjugated metric, an assumption which we shall later justify. This then would be from our world point of view a negative energy density field. This manifestly maximal parity violating framework would not allow to detect any parity violating behavior in those interactions involving only charged Dirac particles in their multiplets, because the charge conjugated left handed field $\psi_{cL}(x)$ can successfully mimic the right handed field $\psi_R(x)$. However, in any interaction involving a completely neutral e.g Majorana fermion, $\psi_{cL}(x)$ could not play this role anymore resulting as in the weak interaction in visible maximal parity and charge violation (we claim that though no symmetry forbids it, the one degree of freedom Majorana field for a neutrino cannot be associated simultaneously with the two degrees of freedom of the Dirac charge field, since this amounts to duplicate the Majorana kinetic term and appears as an awkward manipulation, therefore one has to choose which electron/positron charge is associated with the neutral particle (neutrino) in the multiplet, this resulting in maximal charge violation and making the already present parity violation manifest). Even neutral-less fermion multiplets as in the quark sector of the weak interactions could then have inherited this parity and charge violation provided their particles lived together with neutral fermion particles in higher dimensional groups before symmetry breaking occurred producing their separation into distinct multiplets.

Now what about mass terms? For charged fields, coupling with a positive energy right handed field must take place to produce the chirality flipping mass term. But the right handed field is not there. It may be that no bare mass term is explicitly allowed to appear in an action and that a new mechanism should be found to produce interaction generated massive propagators starting from a completely mass-less action. Let us guess that such scenario is not far from the one which is actually realized in nature, because maximal Charge and Parity violation, and the related bayonic asymmetry of the universe has otherwise all of the characteristics of a not solvable issue.

But why should right handed chiral fields be negative energy fields? Because, as we already noticed, they live in the same metric $\tilde{g}_{\mu\nu}$ as the time reversal conjugated fields. Therefore, they just in the same way both acquire through the inverse metric and extremum action mechanism described above a negative energy from our world perspective. The pseudo-vector behavior under Parity of the operator four-momentum was also foreseen using the naïve Jacobi determinant approach in section VI.1. Remember however that the unitary parity conjugated field creates positive energy point-like quanta and can be viewed as a positive point-like energy field (this is a standard QFT result). This four-vector behavior under parity of the one particle state four-momentum (an object we called a basic four-vector in section III.2) seems to be in contradiction with the pseudo-vector behavior of the four-momentum field operator. Actually the measured energy of the particle is obtained by acting with the energy operator on the one particle state ket. So there is no contradiction because this measurement is of course performed on a non zero three dimensional volume and we have to admit that the measured energy of the particle we get is negative from our world point of view as a result of the particle being living in an enantiomorphic 3-dimensionnal space. In other words, from our world point of view, the parity conjugated field has a negative energy **density,** which we may consider as a positive energy per negative inertial 3-volume, so that it leads to a negative energy when integrated on a general coordinate 3-volume (as if it was a parity scalar, the behavior of this 3-volume under a parity transformation plays no role in our discussion since this is just one of the general coordinate transformations).

Then the PT fields are again positive integrated energy (energy measured in a finite volume) fields but oppositely charged (charge measured in a finite volume), i.e describing anti-particles (see VI.1) living in our world metric and interacting with their PT symmetric fields describing particles.

In short, we believe that recognizing the universality of Parity violation, i.e the fact that we are living in a left chiral world, is also an interesting approach to the issue. It then suffices to introduce the right chiral parity conjugated world (its action) to plainly restore Parity invariance of the total action. Eventually it may be, as already Sakharov suggested in 1967 [6], that we are living in a left chiral positive energy world with its particles and antiparticles while the conjugated world is from our world point of view a right chiral negative energy world with its particles and antiparticles.

XV) Conclusion

Of course, negative energy matter remains undiscovered at present and the stability issue strongly suggests that making it interact with normal matter requires new non standard interaction mechanisms. However, considering the seemingly many related theoretical and phenomenological issues and recalling the famous historical examples of equation solutions that were considered unphysical for a long time before they were eventually observed, we believe it is worth trying to understand how negative energy solutions should be handled in general relativity. We proposed a special treatment for discrete parity and time reversal transformations in General Relativity. A new gravitational picture is then derived which opens rich phenomenological and theoretical perspectives and makes us confident that the approach is on the right way.

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Appendix

For a purely left-handed kinetic lagrangien,

$$L_{kin} = -\overline{\Psi}_L \gamma^\mu \partial_\mu \Psi_L - \overline{\Psi}_{Lc} \gamma^\mu \partial_\mu \Psi_{Lc}$$

Gauge invariance yields interaction terms :

$$L_{kin} + L_{\text{int}} = -\overline{\Psi}_L (\gamma^\mu [\partial_\mu + ieA_\mu]) \Psi_L - \overline{\Psi}_{Lc} (\gamma^\mu [\partial_\mu - ieA_\mu]) \Psi_{Lc}$$

from which follows the purely left-handed QED current :

$$[\overline{\Psi} \gamma_\mu (\frac{1-\gamma_5}{2}) \Psi(x) - \overline{\Psi}_c \gamma_\mu (\frac{1-\gamma_5}{2}) \Psi_c(x)]$$

To compute this current we will first need to establish some useful formulae

Useful formulae ([3] p219&225)

$$u^+(q,\sigma) = (u^*(q,\sigma))^T = \left(-\beta C v(q,\sigma)\right)^T = -v(q,\sigma)^T C^T \beta^T$$
$$\Rightarrow u^+(q,\sigma) = v(q,\sigma)^T C \ \beta$$

$$v^+(q,\sigma) = (v^*(q,\sigma))^T = \left(-\beta C u(q,\sigma)\right)^T = -u(q,\sigma)^T C^T \beta^T$$
$$\Rightarrow v^+(q,\sigma) = u(q,\sigma)^T C \ \beta \ (1)$$

then

$$u^+(q',\sigma')\beta\gamma_\mu \frac{1-\gamma_5}{2} u(q,\sigma) = v(q',\sigma')^T C \gamma_\mu \frac{1-\gamma_5}{2} u(q,\sigma)$$
$$v^+(q,\sigma)\beta\gamma_\mu \frac{1+\gamma_5}{2} v(q',\sigma') = u(q,\sigma)^T C \gamma_\mu \frac{1+\gamma_5}{2} v(q',\sigma')$$

using

$$(C\gamma_\mu \frac{1-\gamma_5}{2})^T = \frac{1-\gamma_5{}^T}{2} \gamma_\mu{}^T C^T = -\frac{1-\gamma_5{}^T}{2} \gamma_\mu{}^T C$$
$$= \frac{1-\gamma_5{}^T}{2} C \gamma_\mu = C \frac{1-\gamma_5}{2} \gamma_\mu = C \gamma_\mu \frac{1+\gamma_5}{2}$$

we obtain the first useful formula

$$u^{+}(q',\sigma')\beta\gamma_{\mu}\frac{1-\gamma_5}{2}u(q,\sigma)=v^{+}(q,\sigma)\beta\gamma_{\mu}\frac{1+\gamma_5}{2}v(q',\sigma')$$

From (1) we get

$$v^{+}(q,\sigma)\beta\gamma_{\mu}\frac{1-\gamma_5}{2}u(q',\sigma')=u(q,\sigma)^{T}C\gamma_{\mu}\frac{1-\gamma_5}{2}u(q',\sigma')$$
$$=u(q',\sigma')^{T}C\gamma_{\mu}\frac{1+\gamma_5}{2}u(q,\sigma)$$

but

$$v^{+}(q',\sigma')\beta=u(q',\sigma')^{T}C$$

which leads to the second useful formula

$$v^{+}(q,\sigma)\beta\gamma_{\mu}\frac{1-\gamma_5}{2}u(q',\sigma')=v^{+}(q',\sigma')\beta\gamma_{\mu}\frac{1+\gamma_5}{2}u(q,\sigma)$$

Calculation of the left-handed current

$$\overline{\Psi}\gamma_{\mu}\frac{1-\gamma_5}{2}\Psi(x)=$$
$$\frac{1}{(2\pi)^3}\int_{p,p',\sigma,\sigma'}u^{*}(p',\sigma')\beta\gamma_{\mu}\frac{1-\gamma_5}{2}u(p,\sigma).e^{i(-p'x+px)}a^{\dagger}(p',\sigma')a(p,\sigma)d^3pd^3p'$$
$$+\frac{1}{(2\pi)^3}\int_{p,p',\sigma,\sigma'}v^{*}(p',\sigma')\beta\gamma_{\mu}\frac{1-\gamma_5}{2}v(p,\sigma).e^{i(p'x-px)}a_c(p',\sigma')a_c^{\dagger}(p,\sigma)d^3pd^3p'$$
$$+\frac{1}{(2\pi)^3}\int_{p,p',\sigma,\sigma'}v^{*}(p',\sigma')\beta\gamma_{\mu}\frac{1-\gamma_5}{2}u(p,\sigma).e^{i(p'x+px)}a_c(p',\sigma')a(p,\sigma)d^3pd^3p'$$
$$+\frac{1}{(2\pi)^3}\int_{p,p',\sigma,\sigma'}u^{*}(p',\sigma')\beta\gamma_{\mu}\frac{1-\gamma_5}{2}v(p,\sigma).e^{i(-p'x-px)}a^{\dagger}(p',\sigma')a_c^{\dagger}(p,\sigma)d^3pd^3p'$$

$$\overline{\Psi_c}\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) =$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2} v(p',\sigma').e^{i(px-p'x)} a(p,\sigma)a^\dagger(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2} u(p',\sigma').e^{i(-px+p'x)} a_c{}^\dagger(p,\sigma)a_c(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2} v(p',\sigma').e^{i(-px-p'x)} a_c{}^\dagger(p,\sigma)a^\dagger(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2} u(p',\sigma').e^{i(px+p'x)} a(p,\sigma)a_c(p',\sigma')d^3pd^3p'$$

➔

$$\overline{\Psi_c}\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) =$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2} u(p,\sigma).e^{i(px-p'x)} a(p,\sigma)a^\dagger(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2} v(p,\sigma).e^{i(-px+p'x)} a_c{}^\dagger(p,\sigma)a_c(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2} v(p,\sigma).e^{i(-px-p'x)} a_c{}^\dagger(p,\sigma)a^\dagger(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2} u(p,\sigma).e^{i(px+p'x)} a(p,\sigma)a_c(p',\sigma')d^3pd^3p'$$

➔

$$\overline{\Psi_c}\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) =$$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2} u(p,\sigma).e^{i(px-p'x)} a^\dagger(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2} v(p,\sigma).e^{i(-px+p'x)} a_c(p',\sigma')a_c{}^\dagger(p,\sigma)d^3pd^3p'$$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2} v(p,\sigma).e^{i(-px-p'x)} a^\dagger(p',\sigma')a_c{}^\dagger(p,\sigma)d^3pd^3p'$$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2} u(p,\sigma).e^{i(px+p'x)} a_c(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,\sigma} u^*(p,\sigma)\beta\gamma_\mu \frac{1+\gamma_5}{2} u(p,\sigma).d^3p$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,\sigma} v^*(p,\sigma)\beta\gamma_\mu \frac{1+\gamma_5}{2} v(p,\sigma).d^3p$$

➔

$$\overline{\Psi}\gamma_\mu \frac{1-\gamma_5}{2}\Psi(x) - \overline{\Psi_c}\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) =$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu u(p,\sigma).e^{i(px-p'x)}a^\dagger(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu v(p,\sigma).e^{i(-px+p'x)}a_c(p',\sigma')a_c^\dagger(p,\sigma)d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu v(p,\sigma).e^{i(-px-p'x)}a^\dagger(p',\sigma')a_c^\dagger(p,\sigma)d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu u(p,\sigma).e^{i(px+p'x)}a_c(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,\sigma} v^*(p,\sigma)\beta\gamma_\mu v(p,\sigma).d^3p$$

At last:

$$\overline{\Psi}\gamma_\mu \frac{1-\gamma_5}{2}\Psi(x) - \overline{\Psi_c}\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) = :\overline{\Psi}\gamma_\mu\Psi(x):$$

For a Majorana field, $\Psi_c(x)$ is not there and we are left only with a chiral kinetic term:

$$\overline{\Psi}\gamma_\mu \frac{1-\gamma_5}{2}\Psi(x)$$

We believe that such term cannot be duplicated to be found associated in multiplets with both $\Psi(x)$ and $\Psi_c(x)$ of a Dirac field, so that the above chiral kinetic term will necessarily result in a chiral interaction term in which parity and charge violation explicitly manifest themselves.