Quasi-1D Bose-Einstein condensates in the dimensional crossover regime

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PACS. 03.75.Hh – Static properties of condensates; thermodynamical, statistical and structural properties.
PACS. 03.75.Gg – Entanglement and decoherence in Bose-Einstein condensates .

Abstract. – We study theoretically the dimensional crossover from a three-dimensional elongated condensate to a one-dimensional condensate as the transverse degrees of freedom get frozen by tight confinement, in the limit of small density fluctuations, *i.e.* for a strongly degenerate gas. We compute analytically the radially integrated density profile at low temperatures using a local density approximation, and study the behavior of phase fluctuations with the transverse confinement. Previous studies of phase fluctuations in trapped gases have either focused on the 3D elongated regimes or on the 1D regime. The present approach recovers these previous results and is able to interpolate between them. We show in particular that in this strongly degenerate limit the shape of the spatial correlation function is insensitive to the transverse regime of confinement, pointing out to an almost universal behavior of phase fluctuations in elongated traps.

In the recent years, one-dimensional (1D) ultracold atomic gases have been produced in very elongated magnetic [1, 2] or optical traps [3, 4], with such tight confinement in two transverse directions that the atomic motion"freezes" to radial zero-point oscillations. The equilibrium phase diagram of these trapped 1D gases shows a rich behavior [5]: at low densities the cloud behaves as a gas of impenetrable bosons ("Tonks-Girardeau gas" [6]), and for higher densities (corresponding to most current experimental setups), the cloud is a "quasi-condensate [5, 7], characterized by suppressed density fluctuations and the same kind of local correlations as in a true condensate, but also by the lack of long-range phase coherence due to significant phase fluctuations. The latter die out continuously around a characteristic temperature T_ϕ, below which the quasicondensate turns into a true condensate. It was realized in [8] that such quasicondensates could also exist in very elongated, but three-dimensional (3D) traps: although atomic motion is possible in all directions, the lowest-lying excitations of these systems, which dominate the long range decay of the phase correlations, are 1D in character [9]. These 3D quasicondensates were observed in equilibrium [10] and non-equilibrium samples [11], and their coherence properties studied through Bragg spectroscopy [12, 13] or matter-wave interferometry [14].

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Currently, although the limiting 3D or 1D cases are well understood theoretically, the kinematic crossover from the "3D elongated" to the truly 1D regime of transverse confinement have been less studied, despite being relevant to several recent experiments with "conventional traps" [1, 12], or to ongoing research on the manipulation of coherent atomic ensembles on micro-fabricated substrates [15, 16]. For the case of an infinitely long waveguide with uniform axial density, a many-body theory has been developed in [17] to describe this crossover (see also [18]). The influence of an axial trapping potential on the density profile and collective excitations was addressed in [18] by numerical integration of the transverse dynamics, in [19] through a separability *ansatz*, which is correct in the 1D limit but not in the 3D case, and in [20], where solvable hydrodynamic models that reproduce the 3D and 1D TF limits are introduced.

In the present paper, we follow a different route and introduce an accurate analytical approximation for the "equation of state" of the quasi-1D gas, that connects smoothly to the 3D to the 1D mean-field regimes. In the local density approximation (LDA), we are then able to work out analytically the 1D (integrated over radial degrees of freedom) density profile $n_1(z)$, from which many properties of the trapped quasi-1D gas can be calculated. As an application, we investigate the influence of the transverse confinement on phase-fluctuations in a quasi-1D geometry. Ref. [21] considers the related problem at zero temperature case in a box geometry, and investigate the 3D to 1D crossover as the box cross-section is reduced. In the trapped case considered in this paper, quantum fluctuations of the phase are small away from the Tonks regime [5], so that we only consider thermal fluctuations. We calculate the spatial correlation function of the radially averaged atomic field operator $\hat{\Psi}$,

$$\mathcal{C}^{(1)}(s) = \int d\overline{z}\ \langle \hat{\Psi}^\dagger(\overline{z} + s/2)\hat{\Psi}(\overline{z} - s/2)\rangle \tag{1}$$

at finite (but low) temperatures. This important quantity gives a global measure of phase coherence across the atomic cloud, and is also the Fourier transform of the momentum distribution (see for instance [22]). The local density approach in this context is nothing else than a slowly-varying-envelope approximation, applied to the long-wavelength excitations responsible for the fluctuations of the phase. We show that this turns out to be a good approximation even if $T \gtrsim T_\phi$, provided density fluctuations are small. More importantly, we find the remarkable property that the shape of the spatial correlation function (or equivalently the momentum distribution) of the quasi-condensate is almost insensitive to the precise regime of transverse confinement, pointing out to the universal character of phase fluctuations in very elongated traps.

We follow the general method introduced in Ref. [5], assuming a weakly interacting gas well below the degeneracy temperature (see [23] and related discussion at the end of the paper). Then, density fluctuations are small and the equilibrium 3D density profile n_0 in the trapping potential $V_{\rm trap}(\mathbf{r}) = m\omega_\perp^2\rho^2/2 + V(z)$ is given by the solution of the usual Gross-Pitaevskii equation, even if the cloud is a quasi-condensate. Without loss of generality, we introduce the 1D density $n_1(z) = \int d^{(2)}\rho\ n_0(\rho, z)$ obtained by integration over the transverse plane, and the radial mode $f_\perp$ through $n_0(\rho, z) =\mid f_\perp(\rho, z) \mid^2 n_1(z)$. Assuming a sufficiently shallow axial confinement and neglecting the density derivatives of n_1, $f_\perp$ with respect to z obtains

$$-\frac{\hbar^2}{2M}\frac{\Delta_\perp f_\perp}{f_\perp} + \frac{1}{2}M\omega_\perp^2\rho^2 + Un_1(z) \mid f_\perp \mid^2 = \mu_{\rm l.e.}[n_1(z)]. \tag{2}$$

Here, we have used cylindrical coordinates, denoting the transverse radius as ρ and the longitudinal coordinate as z. The 3D coupling constant U is related to the s-wave scattering length a through $U = 4\pi\hbar^2 a/M$, where M is the atomic mass. The local equilibrium chemical potential $\mu_{\rm l.e.}$ depends on z through

$$\mu_{\rm l.e.}[n_1(z)] + V(z) = \mu, \qquad (3)$$

with μ the global chemical potential of the cloud in the trap. The relation between n_1 and $\mu_{\rm l.e.}$, that includes the effect of transverse confinement, can be seen as an effective "equation of state" for the 1D gas [18].

To obtain this equation of state, it is sufficient to solve (2) locally, *i.e.* for each value of $n_1(z)$, or equivalently by considering a geometry with radial harmonic trapping, but homogeneous 1d density $n_1 = N/2L$, where $2L$ is the axial length of the system ("cylinder model", see inset in Fig. 1a). The equation of state in this model was found in [18] by numerical integration. Here we follow an alternative route, and derive the equilibrium properties by taking for $f_\perp[n_1]$ a Gaussian trial wavefunction, whose width $w_\perp[n_1]$ is a variational parameter, and by minimizing the *chemical potential* $\mu_{\rm l.e.}$. This simple calculation yields the optimized width $w_\perp[n_1] = a_\perp(1+4an_1)^{1/4}$, where $a_\perp = \sqrt{\hbar/M\omega_\perp}$ is the radial oscillator length, and the local chemical potential

$$\mu_{\rm l.e.}[n_1] = \hbar\omega_\perp\sqrt{1+4an_1}. \qquad (4)$$

The very good agreement of Eq. (4) with the available numerical results [18], illustrated in Fig. 1a, has been pointed out in [20] on a phenomenological basis. Here we show that this expression follows from the condition that $\mu_{\rm l.e.}$ is the lowest eigenvalue of the Gross-Pitaevskii equation. Note that this approximation does not correspond to a separability *ansatz*, as n_1 is in general a function of the axial coordinate z. In this respect, the method used here differs from the work reported in [17].

We now reintroduce the axial trapping potential, assuming for definiteness a harmonic form, $V(z) = \frac{1}{2}M\omega_z^2 z^2$. From the local equilibrium condition (3) and the equation of state (4), one finds the density profile of the trapped gas,

$$n_1(z) = \frac{1}{16a}\frac{V(L)-V(z)}{\hbar\omega_\perp}\left[\frac{V(L)-V(z)}{\hbar\omega_\perp}+1\right] = \frac{\alpha}{4a}(1-\tilde{z}^2)\left[\alpha(1-\tilde{z}^2)+4\right]. \qquad (5)$$

Here we have introduced the parameter $\alpha = 2(\mu/\hbar\omega_\perp - 1)$ and $\tilde{z} = z/L$. The condensate length L is defined by the relation $\mu_{\rm l.e.}[n_1(L)=0] + \frac{1}{2}M\omega_z^2 L^2 = \mu$, and is given explicitly by

$$L = \frac{a_z^2}{a_\perp}\sqrt{\alpha}, \qquad (6)$$

where $a_z = \sqrt{\hbar/M\omega_z}$ is the axial oscillator length. Using $\int_{-L}^{L} n_1(z)dz = N$, and the density profile (5), we obtain the equation for the key quantity α,

$$\alpha^3(\alpha+5)^2 = (15\chi)^2. \qquad (7)$$

The only parameter of the calculation, $\chi = Naa_\perp/a_z^2$, roughly gives the ratio of the interaction energy to the radial zero-point energy [18]. Numerical solution of Eq. (7) is straightforward, and obtains the static properties of the condensate at any confinement strength. In the limit $\chi \gg 5$, the mean-field interaction dominate over the transverse confinement, and one recovers the well-known 3D TF result, $\alpha \approx \alpha_{\rm 3D} = (15\chi)^{2/5}$. Conversely, if $\chi \ll 5$, the transverse motion is almost frozen and one finds $\alpha \approx \alpha_{\rm 1D} = (3\chi)^{2/3}$. The crossover between the two regimes occurs approximately for $\alpha_{\rm 1D} = \alpha_{\rm 3D}$, giving a cross-over value $\chi_{\rm cross} = 5^{3/2}/3 \approx 3.73$.

Our analytical results are very accurate even in the crossover region, as shown in Fig. 1b where we compare Eq. (5) to a direct numerical solution of the GP equation.

Equations (5,6,7) are the key results of this paper. The calculation of the density profile for any strength of the confinement in the transverse direction allows to deduce a number of interesting quantities. As an example, we focus in the remainder of the paper on phase fluctuations, a key phenomenon to understand the physics of 1D gases. In general, phase fluctuations originate from the very large population of the 1D excited states of the system, *i.e.* those in the low-energy energy range $\hbar\omega_z < \epsilon \ll \min\{\mu, \hbar\omega_\perp\}$ (referred to in the following as the "axial branch" of excitations). Keeping with the local density approach, we analyze first these excitations in the cylinder geometry. The axial branch corresponds to excitations characterized by an axial wavenumber k, and no radial nodes [9]. Well below the degeneracy temperature [23], the elementary excitations of the phase-fluctuating ensemble obey the same Bogoliubov-De Gennes equations than in the usual coherent ensemble [5]. We introduce the operators $\hat{\phi}$ and $\delta\hat{n}$ describing fluctuations of the phase and of the density, and their expansion on the set of axial plane waves, $\delta\hat{n}(z) = \sum_k \delta n_k \mathcal{A}_k(\rho) e^{ikz}\hat{b}_k/\sqrt{2n_1L}$ + h.c., and $\hat{\phi}(z) = -i\sum_k \phi_k e^{ikz}\hat{b}_k/\sqrt{2n_1L}$ + h.c.. The function $\mathcal{A}_k(\rho)$ describes the radial dependance of density fluctuations, and the operators $\hat{b}_k$, $\hat{b}_k^\dagger$ destroy and create one quasi-particle with wavevector k. The Fourier components of $\hat{\phi}$ and $\delta\hat{n}$ read

$$\hbar\omega_k^{\mathrm{B}}\delta n_k\mathcal{A}_k(\rho) = \frac{\hbar k^2}{M} n_0(\rho)\phi_k, \tag{8}$$

$$\hbar\omega_k^{\mathrm{B}}\phi_k n_0(\rho) = \left(\frac{\hbar^2k^2}{4M} + Un_0(\rho)\right)\delta n_k\mathcal{A}_k(\rho) - \frac{\hbar^2\delta n_k}{4M}\nabla_\perp\left[n_0(\rho)\nabla_\perp\left(\frac{\mathcal{A}_k(\rho)}{n_0(\rho)}\right)\right]. \tag{9}$$

For the axial branch of interest, the transverse envelope changes continuously from a flat profile in the 3D regime to the radial ground state in the 1D regime [9]. In any case, the quantity $\mathcal{A}_k/n_0$ is almost flat; The corresponding spatial derivatives in (9) are strictly zero in the 1D limit, and of order $(\hbar\omega_\perp/\mu)^2 \ll 1$ compared to the first term of the right hand side of (9) in the 3D limit. We thus neglect their contribution, and average over transverse degrees of freedom to get rid of the remaining radial dependance [9]. The equations obtained in this way are of the usual Bogoliubov form, with an excitation spectrum given by spectrum $\omega_k^{\mathrm{B}} = \sqrt{(\hbar k^2/2M)^2 + c_{\mathrm{1D}}^2k^2}$, and the amplitudes $\delta n_k = n_1(\omega_k/\omega_k^{\mathrm{B}})^{1/2}$ and $\phi_k = (\omega_k^{\mathrm{B}}/4\omega_k)^{1/2}$. The longitudinal speed of sound is $c_{\mathrm{1D}}^2(k) = U\overline{n_0\mathcal{A}_k}/M$, and may depend on k through $\overline{n_0\mathcal{A}_k} = \int d^{(2)}\rho\; n_0\mathcal{A}_k$ for $\omega_k^{\mathrm{B}} \lesssim \omega_\perp$. This has been suggested in [24] as a possible mechanism contributing to the decrease of the critical velocity in 3D elongated gases. The phase coherence properties are however not affected, since they are determined the phonon-like, lowest-energy modes with energy $\lesssim (T/T_\phi)^{1/2}\hbar\omega_z \ll \hbar\omega_\perp$ [5,8], for which $\mathcal{A}_k$ is k-independent regardless of the transverse regime [9].

We now include the effect of the trapping potential by introducing a local density profile $n_1(z)$ according to (3) and (5). In the above expressions, one than has to replace everywhere the chemical potential and density by their local values (see [13] for further details). Using $\langle\hat{b}_k^\dagger\hat{b}_k\rangle \approx k_{\mathrm{B}}T/\hbar\omega_k^{\mathrm{B}}$, the variance of phase fluctuations then reads

$$\Delta\phi^2(\overline{z}, s) = \langle[\hat{\phi}(z) - \hat{\phi}(z')]^2\rangle \approx \frac{T}{T_\phi}\frac{n_1(0)}{n_1(\overline{z})}\frac{\mid s\mid}{L} = \frac{n_1(0)}{n_1(\overline{z})}\frac{\mid s\mid}{d_\phi}, \tag{10}$$

with the relative distance $s = z - z'$ and the mean coordinate $\overline{z} = (z + z')/2$. The phase temperature in (10) is $k_{\mathrm{B}}T_\phi = \hbar^2 n_1(0)/ML \propto N(\hbar\omega_z)^2/\mu$ [5,8], and the phase coherence length is $d_\phi = LT_\phi/T = \hbar^2 n_1(0)/Mk_{\mathrm{B}}T$. Together with the density envelope (5), the expression (10) is sufficient to find the long wavelength behavior of the spatial correlation function [5],

$$\mathcal{C}^{(1)}\left(\frac{s}{L}\right) = \frac{1}{N}\int_{-L}^{L} d\overline{z}\ \sqrt{n_1(\overline{z}+s/2)n_1(\overline{z}-s/2)}\exp(-\frac{1}{2}\Delta\phi^2(\overline{z},s)). \tag{11}$$

This expression differs slightly from the one used in [13] in the treatment of the overlap term, defined here as $\sqrt{n_1(\overline{z}+s/2)n_1(\overline{z}-s/2)}$. The way we write it here yields better agreement with the numerical calculation of the correlation function in the 3D case (see Fig.2b).

Several comments can be made on this expression. First, already for $T = 4T_\phi$, the LDA expression (11) agrees well with the numerical calculation of the correlation function based on Refs. [5, 8]. This is shown in Fig.2a for the 3D case, and Fig.2b for the 1D case. Second, the expression (11), which depends on the dimensionless space variable s/L, is a universal function completely determined by two dimensionless parameters, χ, which controls the regime of transverse confinement and the functional form of the density profile, and T/T_ϕ, which controls the magnitude of phase fluctuations. The third and most important conclusion is that the resulting correlation function is insensitive to a large extent to the transverse regime of confinement (in other words, to the value of χ). This is illustrated in Fig.2b, where we plot the correlation function for $T = 4T_\phi$ and $\chi = 100$ (dotted), $\chi = 1$ (solid) and $\chi = 10^{-2}$ (dashed). Despite the dissimilar transverse profiles, which correspond to very different experimental systems (see Table I), the axial correlation function are almost identical, showing almost exponential decay on a $1/e$ length scale $\approx 1.54 d_\phi$. This very weak dependance on χ points out to the almost universal nature of thermal phase fluctuations in ultracold, very elongated trapped gases. Modifications of the functional profile $n_1(\overline{z})/n_1(0)$, explicitly present in Eq. (11), are not significant. Rather, the effects of transverse confinement are almost entirely contained in the scaling variable $d_\phi \propto n_1(0)$.

The approximate scaling identified here relies on the zero-temperature equation of state, Eq. (4). Its validity requires that (i) axial density fluctuations are negligible, and (ii) that when going to the 3D regime, the presence of a normal cloud, mostly composed of 3D excited states with energy $\gg \hbar\omega_\perp$, do not modify significantly the density profile of the quasicondensate. For (i) to be true, one requires that $T \ll T_{\rm d}$, where $T_{\rm d} = N\hbar\omega_z$ is the 1D degeneracy temperature [5, 8]. Note this is always the case in the 3D case since $T_{\rm d} \gg T_{\rm c}^{(3D)}$ [23]. To check point (ii), we note that although the quasicondensate is significantly depopulated for temperature $T \gtrsim 0.5\ T_{\rm c}^{(3D)}$, the mechanical effect on the density profile is only noticeable for $T \gtrsim 0.8\ T_{\rm c}^{(3D)}$ [25]. Coherence properties of the strongly degenerate part of the cloud are still described by Eq. (11), and provided the correction to $n_1(0)$ are taken into account, we expect that the scaling behavior still holds to a good approximation, since the correlation function is largely insensitive to the precise functional form of $n_1(z)$.

In conclusion, we have investigated in this paper the crossover from a very elongated, 3D Bose gas to a 1D situation where transverse motion is frozen in the limit of vanishing density fluctuations, *i.e.* for a gas strongly in the degenerate regime. By relying on a local density approximation, we have been able to compute the radially integrated density profile for any transverse confinement; we believe these results are simple enough to prove useful for the analysis of time of flight images of very elongated samples, with $\epsilon_{\rm int} \sim \hbar\omega_\perp$. We have applied the method to the problem of phase fluctuations arising in such geometry at finite temperatures, and have found an almost "universal" behavior of quasicondensates in an elongated geometry, related to the essentially classical nature of thermal phase fluctuations.

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We acknowledge stimulating discussions with the members of the Atom Optics group at Institut d'Optique, in particular with Simon Richard and Isabelle Bouchoule, and thank Joseph H. Thywissen and Gora Shlyapnikov for critical comments on this manuscript.

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[23] The degeneracy temperature in the 3D case is just the critical temperature, $T_c^{(3D)} \approx 0.94\hbar\overline{\omega}N^{1/3}$, where the mean trapping frequency is $\overline{\omega} = (\omega_\perp\omega_z)^{1/3}$. In 1D, it is given instead by $T_d = N\hbar\omega_z$. The parameter $\eta = N\omega_z/\omega_\perp$ fixes the ratio $T_c/\hbar\omega_\perp \sim \eta^{1/3}$ and $T_c^{(3D)}/T_d \sim \eta^{-2/3}$.
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Table I – *Realistic experimental parameters illustrating the various regimes considered in the paper. Notations are explained in the text.*

	Atom number	$\omega_\perp/2\pi$	$\omega_z/2\pi$	χ	T_ϕ	$T_c^{(3D)}$	T_d
3D regime: ^{23}Na, optical trap	10^6	1 kHz	20 Hz	90	120 nK	1.2 μK	1 mK
Crossover regime: ^{87}Rb, magnetic trap [12]	5×10^4	760 Hz	5 Hz	3	35 nK	250 nK	120 μK
1D regime: ^{87}Rb, 2D optical lattice [3]	200	20 kHz	50 Hz	0.02	4 nK	720 nK	480 nK

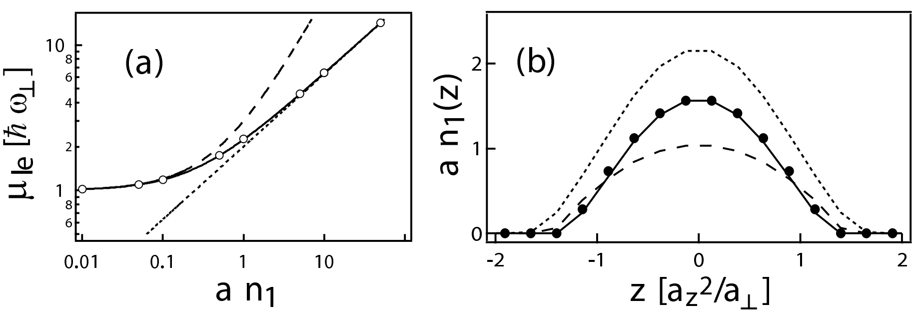

Fig. 1 – Accuracy of the local density approximation. **(a)** Local chemical potential near the 3D-1D crossover, as a function of the local 1D density n_1. The circles show the results of a numerical calculation [18], undistinguishable at the scale of the figure from Eq. (4) [solid line]. The dotted and dashed lines show the 3D and 1D Thomas-Fermi limiting cases. **(b)** Integrated density profiles taking the (harmonic) axial trapping potential into account. To highlight the crossover, the parameter $\chi = 1$ has been chosen (corresponding to $\mu \approx 1.85\hbar\omega_\perp$). The circles, resulting from a numerical solution of the Gross-Pitaevskii equation, are indistinguishable from the LDA result (solid line) at the scale of the figure. The dotted and dashed lines give the 3D and 1D Thomas-Fermi profiles, extrapolated to $\chi = 1$ for comparison.

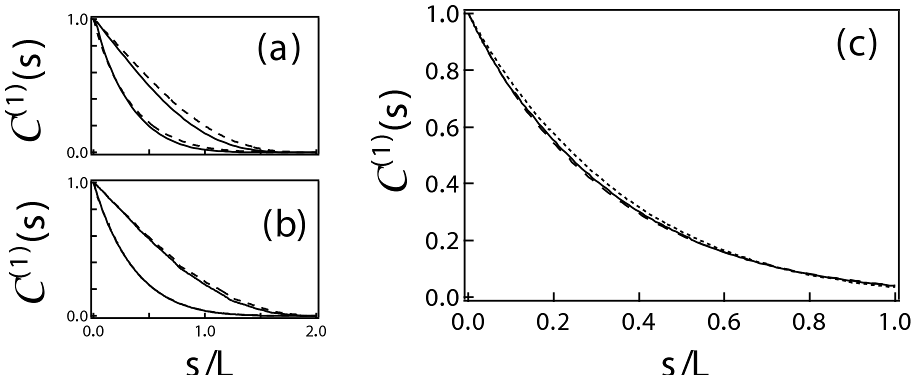

Fig. 2 – Spatial correlation function of an elongated, phase-fluctuating condensate for various confinement regimes. In **(a)**, the spatial correlation function is drawn in the 3D case as a function of the reduced distance s/L, for $T = T_\phi$ (upper curve) and for $T = 4T_\phi$ (lower curve). The solid line follows from a numerical calculation based on the results in [8], and the dashed line is the LDA. Figure **(b)** shows the corresponding curves for the 1D case [5]. In **(c)**, the correlation function of an elongated condensate is plotted for $T = 4T_\phi$ and different regimes of transverse confinement: $\chi = 100$ (3D case, dotted), $\chi = 1$(intermediate case, solid) and $\chi = 10^{-2}$ (1D case, dashed). Despite 4 orders of magnitude of variations in χ, the functional form of the spatial correlation function is almost unchanged.