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The rotation of extra-solar planets

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Abstract. Although observations of the rotations rates of the extra-solar planets will probably not be obtained soon, one can make predictions for these rates for all planets that should have tidally evolved, despite the fact that it is very probable that none of these planets are strictly in a 1/1 spin orbit resonance state.

1. Introduction

Even in the Solar System, the determination of the rotational periods of the main planets has only be achieved recently for Mercury and Venus, when it became possible to use radar ranging on the planets (Pettengill and Dyce, 1965, Goldstein, 1964, Carpenter, 1964). We do not thus expect that it will be immediately possible to observe the rotation of the newly discovered extra-solar planets. Nevertheless, as was done by Schiaparelli for Mercury and Venus (1889, 1890), it is possible to make predictions for the rotation periods of the extra-solar planets that should be tidally evolved through tidal friction. Although the predictions of Schiaparelli were wrong for both Mercury and Venus, one can nevertheless consider that his theoretical estimates, based on Darwin' work (1880), were much closer to the true values of the rotation periods than most observations made in the two previous centuries. As Schiaparelli, we will dare to make predictions for some of the rotation periods of the known extra-solar planets, hoping that the additional knowledge that we gained from a better understanding of the rotation of Mercury and Venus, will prevent us from being as much in error as he was.

2. Tidal friction

The theory of tidal friction was initiated by Darwin (1880), with more recent contributions of (Love, 1927, Munk & MacDonald, 1960, Kaula, 1964, Goldre-

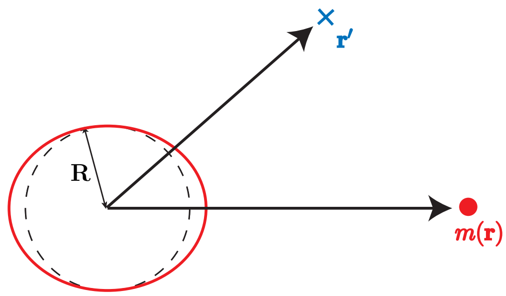

Figure 1.

ich and Peale, 1966, 1968, Mignard, 1979, 1980). We consider the potential generated by the action on a planet by a perturbating body $m(\mathbf{r})$ (Fig.1). The potential in any point in $\mathbf{r}'$, generated by $m(\mathbf{r})$ is expressed in term of Legendre polynomials as

$$V(\mathbf{r}') = -\frac{Gm}{r}\sum_{l=2}^{+\infty}\left(\frac{r'}{r}\right)^l P_l(\cos(\mathbf{r},\mathbf{r}')) \ . \tag{1}$$

The tidal response at the planet surface (with radius R) is then

$$\mathcal{V}(\mathbf{R}) = -\frac{Gm}{R}\sum_{l=2}^{+\infty}k_l\left(\frac{R}{r}\right)^{l+1} P_l(\cos(\mathbf{r},\mathbf{R})) = F(\mathbf{r},\mathbf{R}) \ , \tag{2}$$

where G is the gravitational constant and k_l are the usual Love numbers. The potential in $\mathbf{r}'$ generated by this deformation (limited to $l \leq 2$) is then

$$\tilde{V}(\mathbf{r}') = -\frac{Gm}{R}k_2\left(\frac{R}{r}\right)^3\left(\frac{R}{r'}\right)^3 P_2(\cos(\mathbf{r},\mathbf{r}')) \tag{3}$$

In a viscous model that is adapted to giant gaseous planets, we can consider that the non elasticity of the planet induces a small constant delay Δt between the excitation of the planet and the planet response (Mignard, 1979, 1980). The tidal response at time t in the direction $\mathbf{R}$ is thus $\mathcal{V}_d(\mathbf{R},t) = F(\mathbf{r}(t-\Delta t),\mathbf{R}(t-\Delta t))$, and the potential at $\mathbf{r}$ at time t can be written on the form

$$\tilde{V}_d(\mathbf{r}) = -\frac{Gm}{R}k_2\left(\frac{R}{r^*}\right)^3\left(\frac{R}{r}\right)^3 P_2(\cos(\mathcal{R}_\omega(\Delta t)\mathbf{r}^*,\mathbf{r})) \tag{4}$$

where $\mathbf{r}^* = \mathbf{r}(t-\Delta t)$, and where $\mathcal{R}_\omega(\Delta t)$ is a rotation along the axis of rotation of angle $\omega\Delta t$ (Fig.2).

For an obliquity $\varepsilon = 0$, that can be assumed for tidally evolved planets (see for example Correia *et al.*, 2003), the tidal frequencies that arise in (4) are $\sigma = 2\omega + kn$, where k is an integer. As the main tidal term is associated with

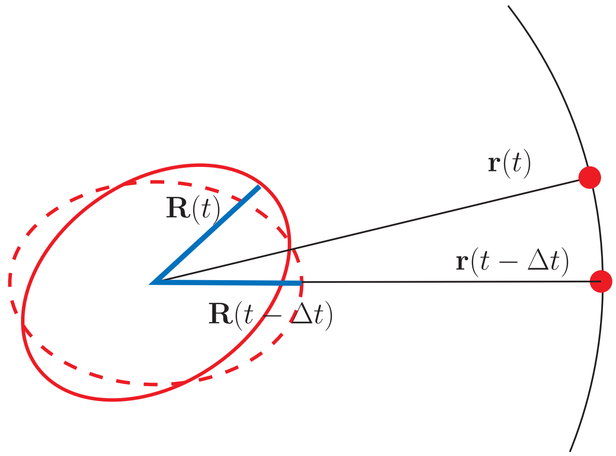

Figure 2.

the tidal frequency 2ω, the constant time lag Δt for our viscous model can then be related to the usual dissipative factor Q as

$$\Delta t = \frac{1}{2\omega_0 Q} \tag{5}$$

where ω_0 is the initial angular velocity of the planet. The rotational evolution of the planet can then be computed and leads, for a constant eccentricity e and semi major axis a, to (Goldreich and Peale, 1966, Hut, 1980, Correia *et al.*, 2003, Correia and Laskar, 2004)

$$\frac{\omega}{n} = E(e) + \left(\frac{\omega_0}{n} - E(e)\right) e^{-\Omega(e)Kt} \ , \tag{6}$$

where

$$K = \frac{9Gm_*^2}{4\pi\rho\omega_0 a^6}\frac{k_2}{\xi Q} \ , \tag{7}$$

where m_* is the mass of the central star, ρ the density of the planet, $\xi = C/MR^2$ its structure constant, and

$$\Omega(e) = \frac{1 + 3e^2 + 3e^4/8}{(1-e^2)^{9/2}} \ . \tag{8}$$

The angular velocity of the planet ω will evolve toward the asymptotic value ω_f that can be computed easily at $\varepsilon = 0$, for all values of the eccentricity (Goldreich and Peale, 1966, Hut, 1980) as[1]

$$\frac{P_{orb}}{P_f} = \frac{\omega_f}{n} = E(e) = \frac{1 + 15e^2/2 + 45e^4/8 + 5e^6/16}{(1+3e^2+3e^4/8)(1-e^2)^{3/2}} \tag{9}$$

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[1]There is a misprint in the e^6 term of the equation of Goldreich and Peale (1966)

where P_{orb} is the orbital period of the planet, P_f its final rotation period. One should note that in the presence of significant planetary perturbations, the rotation period of the planet will not follow strictly $E(e(t))$ (correia and Laskar, 2004).

2.1. Tidally evolved planetary rotations

We can thus expect that if the rotation of a planet, with non zero eccentricity and close to its central star, is tidally evolved, its rotation period will not be synchronized with its orbital period, but will be given by relation (9).

The time required for dampening the rotation of the planet depends on their dissipation factor k_2/Q (7). As we can assume that these planets are similar to the Solar system outer planets (Table 1), we have assumed that $k_2 = 0.4$ for all planets, and a range for Q from 10^4 to 10^5.

quantity	Jupiter	Saturn	Uranus	Neptune
P_0 (h)	9.92	10.66	17.24	16.11
ρ ($\mathrm{g\,cm^{-3}}$)	1.33	0.69	1.32	1.64
$\xi = C/MR^2$	0.25	0.21	0.23	0.24
k_2	0.49	0.32	0.36	0.41
Q ($\times 10^4$)	~ 3	≥ 2	$1-4$	?

Table 1. Constants for the Solar System outer planets (Yoder, 1995, Veeder *et. al.*, 1994, Dermott *et. al.*, 1988, Tittemore and Wisdom, 1990).

We have then plotted in figures 3 and 4, all known extra-solar planets, taken from the catalog of J. Schneider (2004), that could have been tidally evolved over their history. We consider that they are tidally evolved if their rotation period, starting with an initial period of 10 hours, is dampened to a value such that $|(\omega - \omega_f)/\omega_f| < 0.01$. The curves represent the planets that have tidally evolved in a given time interval ranging from 0.001 Gyr to 10 Gyr. In Table 2, the estimated ages of the central stars are given, when available. This allows to check whether the planet should be fully tidally evolved. In absence of this data, for a Solar type star, we can expect that all planets that are above the 1 Gyr curve are tidally evolved. On the other hand, planets that are below the 10 Gyr curve are probably not yet tidally evolved.

As expected, all planets in circular orbits with $a < 0.05$ AU are tidally evolved, but we will concentrate more on the planets further from the star with non zero eccentricity. We can obtain for all these planets that are tidally evolved an estimate of their rotation period, through formula (9). It is thus possible to predict the rotation period of planets that are far from their central star if they have a large eccentricity, as around HD80606. In this extreme case, the orbital period is 111.7 days, but the rotation period should be 1.9 days. One should note that contrarily to the terrestrial planets (see Goldreich and Peale, 1966, Correia and Laskar, 2004), it is very difficult for the gaseous planets to be trapped into a spin-orbit resonance, as these planets will have a symmetry of revolution around their rotation axis, and their momentum of inertia A, B around axis orthogonal to their rotation axis will be practically identical.

Planet	a (AU)	e	m_* $(m_\odot)$	age[a] (Gyr)	age[b] (Gyr)	P_{orb} (d)	$P_l(e)$ (d)
51 Peg	0.05	0.0	1.07	4.3	7.1	4.2	4.2
HD 162020	0.072	0.277	0.75	-	-	8.4	5.7
HD 130322	0.088	0.048	0.91	< 1	0.4	10.7	10.5
HD 108147	0.104	0.498	1.23	< 1	2.0	10.9	3.9
55 Cnc b	0.11	0.02	0.89	11.0	5.0	14.6	14.6
Gliese 86	0.11	0.046	0.79	-	-	15.7	15.5
HD 38529 b	0.129	0.29	1.42	3.0	-	14.3	9.4
Gliese 876 c	0.13	0.12	0.32	-	-	30.1	27.7
HD 195019	0.14	0.05	1.04	8.7	3.2	18.3	18.0
HD 6434	0.15	0.30	0.80	> 17	3.7	22.0	14.1
Gliese 876 b	0.21	0.27	0.32	-	-	61.0	42.1
rho CrB	0.23	0.028	0.90	12.6	-	39.6	39.4
HD 74156 b	0.276	0.649	1.27	-	-	51.6	10.2
HD 3651	0.284	0.63	0.79	-	-	62.2	13.4
HD 168443 b	0.29	0.529	1.03	9.3	7.4	58.1	18.7
HD 114762	0.3	0.334	0.91	14.0	-	84.0	49.4
HD 121504	0.32	0.13	1.11	4.0	2.8	64.6	58.6
HD 178911	0.32	0.1243	0.87	-	-	71.4	65.4
HD 16141	0.35	0.28	1.18	3.2	6.7	75.8	51.1
70 Vir	0.43	0.4	1.05	7.9	-	116.6	57.0
HD 80606	0.439	0.927	0.90	-	-	111.7	1.9
HD 52265	0.49	0.29	1.25	0.5	4.0	118.9	78.3
GJ 3021	0.49	0.505	0.99	0.7	< 0.1	133.8	46.8

Table 2. Exra-solar planets that could have a tidally evolved rotation (Figs. 3,4). The hot Jupiters in circular orbits at about 0.05 AU are not listed, except for 51 Peg. For each planet, the semi major axis a, eccentricity e, mass of the central star m_*, and orbital period (P_{orb}) are given, as well as the limit period ($P_l(e)$). When available, the age of the central star is also given. These data are collected from Laws *et al.*(2003) and on the Web sites of Mayor *et al.*(2004), Marcy *et al.*(2004), and Schneider (2004). The determination of the ages of the central stars are model dependent; ages[a] are derived from Padua stellar isochrones (Salasnich *et al.*, 2000), while ages[b] are computed from Henry *et al.*(1996) and Laws *et al.*(2003).

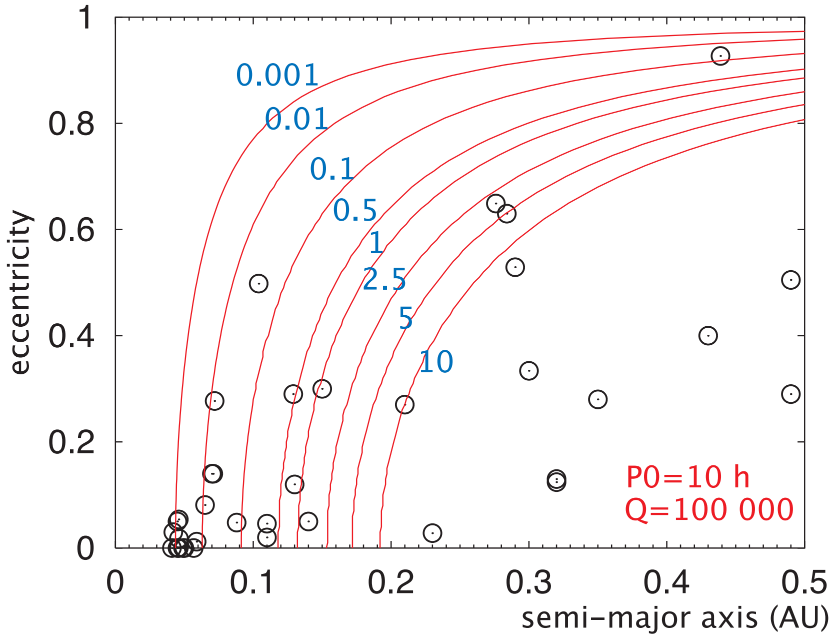

Figure 3. Tidally evolved planets with $Q = 10^5$ and initial period $P_0 = 10$ h. The labelled curves denotes (in Gyr) the time needed to reach the equilibrium.

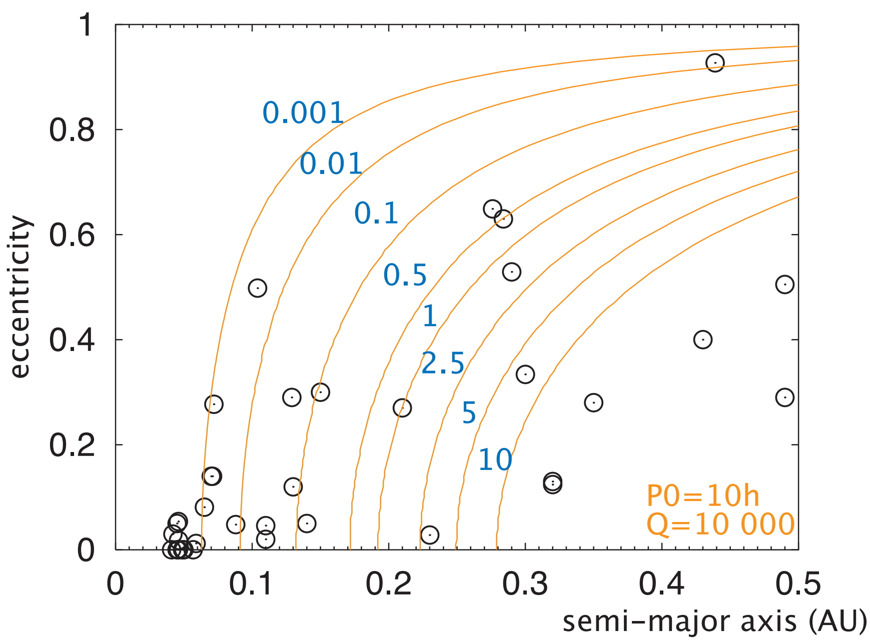

Figure 4. Tidally evolved planets with $Q = 10^4$ and initial period $P_0 = 10$ h.

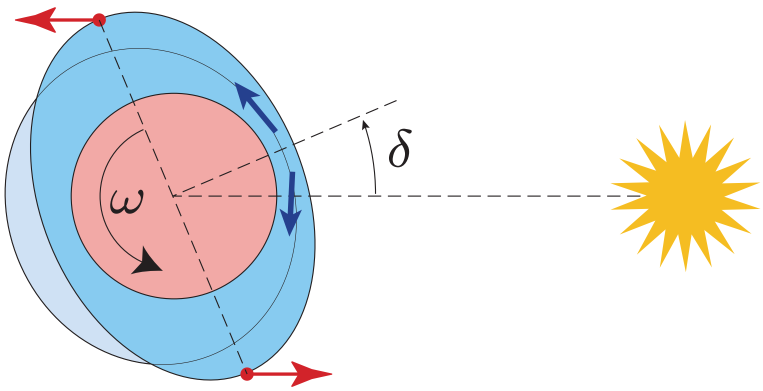

Figure 5. Atmospheric tides

2.2. Are synchronous rotations possible ?

One would think that the planets that are close to the central star, with nearly zero eccentricity, are in synchronous rotation with their orbital period, as predicted by formula (9). But here other mechanisms can take place. Indeed, these planets are supposed to be gaseous Jupiter-like planets, and atmospheric tides may have a large effect on their rotation motion as for Venus (Gold and Soter, 1969, Correia and Laskar, 2001, 2003 Correia *et al.*, 2003, and references therein).

The atmospheric tides result from the heating of the atmosphere of the planet at the subsolar point. In order to equilibrate the pressure, the particles will move away from the subsolar point, and a tidal bulge will result, with an important quadrupolar component, perpendicular to the direction of the Sun (Fig. 5) at synchronization. If $\omega > n$, the tidal bulge will be displaced by a small angle δ, and an accelerating torque will appear (Fig. 5), while the torque will be decelerating when $\omega < n$. The tidal potential at $\mathbf{r}$ at time t can be written in an analougs way as for gravitation tides (Eq. 4):

$$\tilde{V}_a(\mathbf{r}) = -\frac{3}{5}\frac{\tilde{p}_2}{\bar{\rho}}\left(\frac{R}{r}\right)^3 P_2(\cos(\mathcal{R}_\omega(\tilde{\Delta t})\mathbf{r}^*, \mathbf{r}))\ , \tag{10}$$

where $\tilde{p}_2$ is the second harmonic of the pressure amplitude variations at the base of the atmosphere (Correia et al., 2003).

Although it is difficult to have a proper estimate of this effect, one can nevertheless assume that it will dominate in the vicinity of synchronization where the Solar heating is maximal. Indeed, according to Chapman and Lindzen (1970) we have $\tilde{p}_2 \propto 1/\sigma$ and therefore atmospheric tides destabilize the synchronous limit fixed point. In the same way as for Venus, additional possible stable limits values will appear, two fixed points with obliquity $\varepsilon = 0$ and two for $\varepsilon = \pi$ (Fig.6). If the thermal tides are large enough, this should lead to the possibility of retrograde rotations, as for Venus (Correia and Laskar, 2001, 2003, Correia et al., 2003).

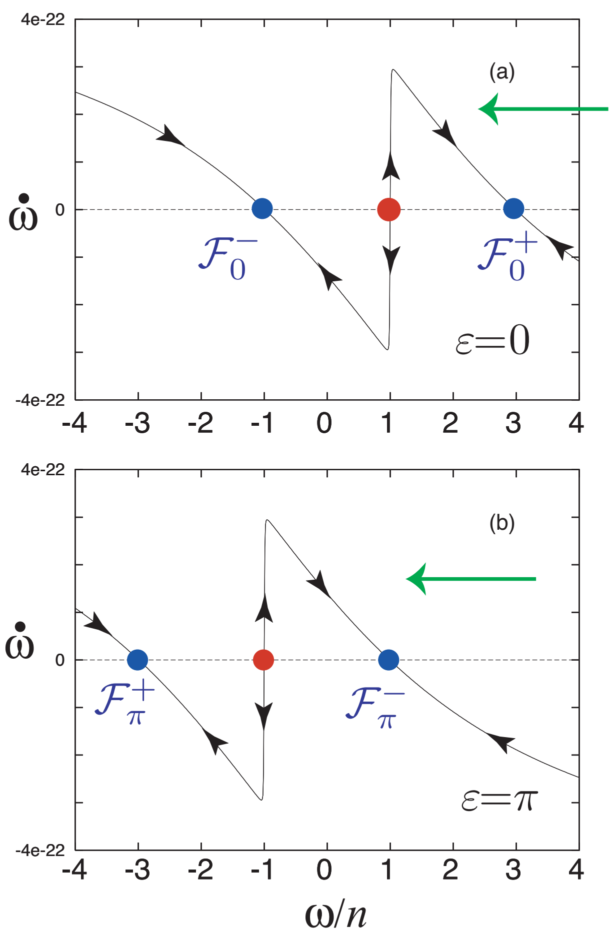

Figure 6. Final states for a planet with strong atmosperic thermal tides. The original equilibrium point obtained at synchronization ($\omega/n = 1$) when considering uniquely the body tides, becomes unstable, and bifurcates at $\varepsilon = 0$ into two new stable fixed points $\mathcal{F}_0^-$ and $\mathcal{F}_0^+$, and at $\varepsilon = \pi$ into $\mathcal{F}_\pi^-$ and $\mathcal{F}_\pi^+$ (Correia and Laskar, 2001, 2003).

2.3. Conclusions

Due to their large eccentricity, the rotation of many of the observed extra-solar planets should be tidally evolved even if they are not very close from their central stars (Figs 3,4). For all these planets, we can conjecture that their rotation period are the limit values $P_l(e)$ given in Table 2. It is thus possible to make precise predictions on their rotations periods, and it becomes a new challenge for the observers to be able to confirm these predictions.

Nevertheless, although it was largely assumed that the hot Jupiters in nearly circular orbits within 0.1 AU of the central star are in 1/1 spin-orbit resonance, the atmospheric tides may very well have destabilized these 1/1 fixed point, and created additional possible stable limit values, with the possibility of retrograde rotations, as for Venus.

In a paradoxical way, the final rotation rate of these planets are the most difficult to predict. On the opposite, once the rotation rate of one of them will be determined, we can probably assume that the amplitude of the atmospheric tides is the same for all similar planets, and thus deduce for all of them, the location of the limit stable fixed points (Fig.6), and thus the possible rotation periods for these planets.

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