Chaos gives quantum tunnelling a hand

Macroscopic objects follow the laws of classical mechanics. For microscopic objects, such as atoms or nuclei, the wave character of the dynamics has to be taken into account, and that is done within the framework of quantum mechanics. Their wave-like behaviour shows up in various ways. One of the most striking is the tunnelling effect: some events occur in spite of being classically forbidden either for energetic reasons (the quantum particles have to "tunnel" under the potential barrier), or because of some other constraint of the dynamics; in this second case the phenomenon is referred to as dynamical tunnelling. Tunnelling plays a major role in a variety of physical phenomena, from α-particle radioactivity of nuclei to the current-voltage characteristic of transistors.

Being genuinely quantal in origin, one might think that tunnelling of an atom is unrelated to the motion of a classical particle subject to the same forces. In particular, at first sight, chaos, a classical concept, seems a priori irrelevant for understanding tunnelling since it deals with the extreme sensitivity of *classical* trajectories to initial conditions. However, theoretical analyses and numerical studies of few degrees of freedom systems have shown that this is actually not the case, and that the nature of the underlying classical dynamics can alter drastically the way tunnelling takes place. Bohigas, Tomsovic, and the second author, at Orsay, have demonstrated for instance that the presence of chaotic trajectories in the classical system can increase the quantum tunnelling rates by orders of magnitude, as well as make them highly fluctuating functions of external parameters. The first experimental realization for which such effects were studied were not quantum mechanical objects, but microwave or optical resonators which can be described by wave equations similar to the Schrödinger equation.

Both the Phillips's team from the National Institute of Standards and Technology, Maryland [Nature vol.412 (2001) p.52] and Raizen's team from the University of Texas [Science vol.293 (2001) p.274] have recently taken advantage of the sophisticated tools developed in the context of cold atoms and Bose-Einstein condensation to study experimentally the physics of tunnelling in the presence of chaos for atoms.

At the energy scale of visible light, say a fraction of micrometer, an isolated motionless atom can be considered as an apparatus which obeys the laws of quantum physics with a rather simple internal structure. Of course, this is an oversimplified description: under ordinary physical conditions atoms are neither isolated nor motionless. At room temperature (say, about 300 K) the speed of atoms is so high ($\sim 100 m/s$ in a gas) that usually they quickly interact many times with their environment. To recover the richness of quantum dynamics, one must "domesticate" the atoms by cooling them down. At microKelvins, atoms can be trapped and manipulated with laser light, electrostatic or magnetostatic fields. The atomic waves offer an alternative to light waves and become an extremely accurate and sensitive tool. Examples of this occur in metrology, atomic interferometry, lithography with individual atoms for micro or nano electronics and atomic clocks. Even more astonishingly, recent work on Bose-Einstein condensates seems to demonstrate the feasibility of a laser-like atomic beam.

Moreover, model systems can also be built and tuned using atomic waves to study with better control many phenomena of interest in other field of physics. As long as the dissipative processes can be neglected, a neutral atom exposed to monochromatic light feels a dipolar force

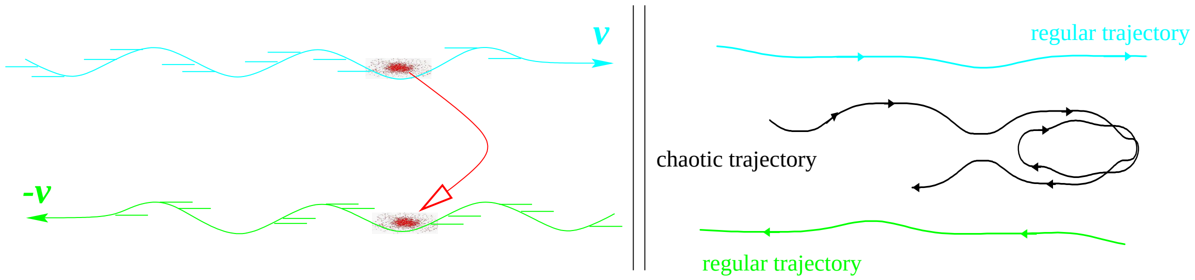

Figure 1: Schematic view of the quantum tunneling process (left), compared to the classical dynamics (right).

proportional to the variations of the intensity of the light. This for instance made it possible to mimic electronic waves in a crystalline potential, provinding a deeper understanding of conductivity by electric charges at the quantum level. In this spirit, the teams of Raizen and Phillips placed atoms within two monochromatic plane standing waves of slightly different frequencies. They produce in this way a time-dependent one-dimensional potential, which is a minimal condition for chaos to appear. Such a configuration for cold atoms was proposed and theoretically studied in great detail by Delande, the first author, and collaborators, in Paris. Each atom can be considered as independent from the others. Dissipation due to spontaneous emission can be neglected if the motion of the atoms is studied within a so-called coherence time which typically can be made of order a few milliseconds. This guarantees the classical dynamics to be described by a Hamiltonian (non-dissipative chaos) and preserves the wave character of the atoms at the quantum level.

Let us for a moment imagine a surfer in the sea where winds would produce a superposition of counter-propagating waves of speed v and $-v$. In such circumstances, the surfer will usually feel randomly the effect of the left and right moving waves, being more shaken than pushed in any definite direction. If, on the other hand, she is able to get an initial velocity close enough to v $(-v)$ just before being reached by a right (left) moving wave, she will be able to ride the wave. In the setup described above, the forces on the atoms produce a very similar situation. As illustrated in Fig 1, their motion would be essentially chaotic for most initial conditions, but if their initial speed and position are properly chosen they would be pushed either to the right or to the left by the electromagnetic wave created by the lasers, with classical laws of motion definitely forbidding any U-turn.

In principle, if they are cold enough, atoms can be prepared with sufficiently small uncertainty on their initial velocity, but this is a rather non-trivial task to achieve. Phillips and coworkers obtain this result by first producing a Bose-Einstein condensate and releasing it while switching on adiabatically a standing wave potential, in such a way that an atomic wavepacket with zero momentum is obtained; they then shift the position of the standing wave to give the proper initial momentum to the atoms. The approach used by Raizen and coworkers is quite similar, except that they do not actually use a Bose Einstein condensate. In both cases, the technology required to produce ultra-cold atoms is a prerequisite. Furthermore it is necessary to manipulate the obtained cold atoms in a very precise way.

It then becomes possible to follow the quantum mechanical evolution of the wavepacket.

At first the atoms follow a classical-like evolution, appearing for instance trapped by a right moving wave. But after some time, some atoms surf a wave moving in the *opposite* direction, although one never observes them stopping or changing direction. At longer times a coherent oscillation is found between the left and right motion. This classically forbidden back and forth "Houdinisation"[1] between two regular motions with different velocities is measured in both experiments and corresponds to the first observation of dynamical tunnelling for atoms with a non-integrable dynamics. Furthermore, Raizen and coworkers are able to identify a significant increase of the tunnelling rate due to the presence of a third, presumably chaotic delocalised state, which is typical of chaos-assisted tunnelling. Therefore, such experiments clearly open the possibility to study in depth the effect of chaos on quantum mechanical tunnelling.

Some of the most exotic and fascinating concepts developed in the context of quantum chaos are therefore now within range of experimental study, thanks to the remarkable progress made in the production and manipulation of cold atoms. This should lead to better understanding and perhaps also to new applications, for instance in the context of quantum information.

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[1] After H. Houdini (1874-1926), famous magician who invented the concept of "escape artist".