Magneto-electro-optical properties of the quantum vacuum and Lorentz invariance

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Résumé

We consider the magneto-electric optical properties of the quantum vacuum and show that all the different phenomenona are related by Lorentz invariance. As a model calculation we show how crossed fields properties can be calculated starting from single field properties by using Lorentz transformations. Using this method we have studied for the first time the case of a crossed static magnetic field and electric field applied with one of these two fields parallel to the direction of light propagation. We also show that parallel field properties can be found using general symmetry properties.

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Introduction

Several new magneto-electric optical phenomena in centrosymmetric media have recently been observed for the first time [1] [2] [3]. When a static magnetic field $\mathbf{B}_0$ and electric field $\mathbf{E}_0$ are applied perpendicular to each other and to the propagation vector of the light $\mathbf{k}$, the existence of magneto-electric linear birefringence [2], and the existence of a polarization-independent anisotropy, proportional to $\mathbf{B}_0 \times \mathbf{E}_0$ [3] have been proven. When, on the other hand, $\mathbf{B}_0$ and $\mathbf{E}_0$ are parallel, the existence of magneto-electric Jones birefringence was demonstrated for the first time [1].

The most elementary centrosymmetric medium is the quantum vacuum. Non-linear optical phenomena in vacuum have been predicted since 1935 [4] in the framework of quantum electrodynamics [5] [6]. In particular, the existence in the vacuum of a linear birefringence induced by a transverse static magnetic field (the Cotton-Mouton effect), or by a transverse static electric field (the Kerr effect), has been predicted [7], but not yet observed. Recently, magneto-electric linear birefringence, magneto-electric Jones birefringence [8] and polarization-independent magneto-electric anisotropy [9] have also been predicted for the quantum vacuum.

Here we perform a complete study of the magneto-electro-optical properties of the quantum vacuum using Lorentz invariance. Using a suggestion by the authors of Ref. [3], we relate single field properties to crossed field properties like magneto-electric anisotropy. This method is based on the magnetic (electric) field behavior under Lorentz transformation that allows us to look for an appropriate reference frame in which the two crossed fields are transformed into only one field. In this reference frame the only existing effect is a magnetic (electric) linear birefringence whose characteristic values of index of refraction are known. These refractive index values can be transformed back to the laboratory frame values by using the well-known result of the electrodynamics of moving media, as e.g. used to describe Fizeau's experiment [5]. This method is ideally suited for the quantum vacuum since it is invariant under Lorentz transformation, and one can calculate the analytical value of the refractive index in any reference frame.

Magneto-electro-optics of the quantum vacuum

With this method, we reproduce in a straightforward way the results, obtained in previous publications by more elaborate calculations, concerning magneto-electric birefringences, and polarization-independent anisotropy, proportional to $\mathbf{B}_0 \times \mathbf{E}_0$. The case where a static magnetic field $\mathbf{B}_0$ and a static electric field $\mathbf{E}_0$ are simultaneously applied perpendicular to each other but with one of these two static fields parallel to the direction of propagation has never been treated before. Here we demonstrate for the first time that only the field perpendicular to the direction of propagation gives an effect and that no bilinear phenomenon exists for this geometry. In the same context we also examine the case of both fields parallel to the direction of propagation. By symmetry arguments we relate parallel fields properties to crossed fields properties and show that no effect exists for this geometry.

The starting point of any calculation of the propagation of light in the quantum vacuum [8], [9] is the Heisenberg-Euler Lagrangian [10]. The form of the effective Lagrangian L_{HE} of the electromagnetic interaction is essentially determined by the fact that the Lagrangian has to be relativistic and CPT invariant and therefore can only be a function of the Lorentz invariants [11] F, G that in Heaviside-Lorentz units can be written as

$$F = (E^2 - B^2) \tag{1}$$

$$G = (\mathbf{E} \cdot \mathbf{B}) \tag{2}$$

Up to second order in the fields, L_{HE} can be written as $L_{HE} = L_0 + L_{EK}$ where L_0 is the usual Maxwell‘s term $\frac{1}{2}F$ and L_{EK} is the first order non linear term first calculated by Euler and Kockel [4]. L_{EK} is valid in the approximation that the fields vary slowly over the Compton wavelength of the electron $\lambda = \hbar/m_e c$ and during a time $t_e = \lambda/c$. Moreover E and B have to be smaller than the critical field $E_{cr} = m_e^2 c^3/e\hbar$ i.e. $B \ll 4.4 \times 10^9$ T and $E \ll 1.3 \times 10^{18}$ V/m. L_{HE} can be written in Heaviside-Lorentz units as

$$L_{HE} = L_0 + \frac{1}{2}(aF^2 + bG^2) \tag{3}$$

A term proportional to FG is Lorentz invariant but not CPT invariant and therefore does not appear in the expression of L_{EK}. Higher order terms of L_{HE} can be written in the same way by looking for combinations of the two invariants F and G that also respect CPT. In the case of a plane wave in vacuum, both F and G are equal to zero. The propagation of a plane wave in vacuum is thereby not affected by non linear interactions since $L_{HE} = 0$.

The values of a and b are provided by QED. The calculation by Heisenberg and Euler gives $a = e^4\hbar/45\pi m^4 c^7 = 2.67\ 10^{-32}\ G^{-2}$ and $b = 7a$. Based on this Lagrangian, in the case of crossed static fields $\mathbf{E}_0$ and $\mathbf{B}_0$, we showed in Ref. [8] the existence of a Cotton-Mouton birefringence $\Delta n_{CM} \propto B_0^2$, a Kerr birefringence $\Delta n_K \propto E_0^2$ and a magneto-electric linear birefringence $\Delta n_{MELB} \propto E_0 B_0$. In Ref. [9] we also showed the existence of a magneto-electric anisotropy, independent of polarization, Δn_{MEA} which is also proportional to $E_0 B_0$. In the case of parallel static fields we also showed in Ref. [8] the existence of a magneto-electric Jones birefringence corresponding to a $\Delta n_J \propto E_0 B_0$. Faraday and Pockel effects are not permitted in vacuum since no terms containing three electromagnetic fields exist in the Heisenberg-Euler Lagrangian. As far as we know, the case of crossed electric and magnetic fields with one of the two parallel to the direction of propagation of light has never been treated, neither in vacuum, nor in any other medium. All the effects predicted for the quantum vacuum have been observed in centrosymmetric media. The quantum vacuum behaves exactly like any other centrosymmetric medium. The predicted values for all these effects in vacuum are unfortunately so small that observation has not yet been possible.

For our purpose, we just need to suppose that when a static magnetic field $\mathbf{B}_0$ is present in a vacuum, perpendicular to the direction of light propagation, the light velocity changes in such a way that

$$n_{\|} = 1 + \eta_{\|} B_0^2 \tag{4}$$

and

$$n_{\perp} = 1 + \eta_{\perp} B_0^2 \tag{5}$$

where $n_{\|}$ and $n_{\perp}$ are the index of refraction for light polarized parallel and orthogonal to the static magnetic field, respectively. Since the velocity of light has to be smaller than c, $\eta_{\|}$ and $\eta_{\perp}$ are positive.

E and B fields under Lorentz transformations

The case of two crossed fields $\mathbf{E}_0$ and $\mathbf{B}_0$ can be completely solved by an appropriate Lorentz transformation thanks to the Lorentz invariance of the quantum vacuum. Using Lorentz transformations one can express the fields $\mathbf{E}$ and $\mathbf{B}$ in an inertial frame K' in terms of the values in another inertial frame K. For a general Lorentz transformation from frame K to a frame K' moving with velocity $\boldsymbol{\beta} = \mathbf{v}/c$ relative to K, the transformation of the fields can be written [12]

$$\mathbf{E}' = \gamma(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B}) - \frac{\gamma^2}{\gamma + 1}\beta(\boldsymbol{\beta} \cdot \mathbf{E}) \tag{6}$$

$$\mathbf{B}' = \gamma(\mathbf{B} - \boldsymbol{\beta} \times \mathbf{E}) - \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}\,(\boldsymbol{\beta} \cdot \mathbf{B}) \tag{7}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields in the frame K, $\mathbf{E}'$ and $\mathbf{B}'$ are the electric and magnetic fields in the frame $K^{'}$ and $\gamma \equiv \left(1-\beta^2\right)^{-1/2}$ as usual. Let's suppose that $\mathbf{E}$ and $\mathbf{B}$ are perpendicular with $E > B$. It is possible to choose the velocity $\mathbf{v}$ so that in the corresponding frame $K^{'}$ only the electric field $\mathbf{E}'$ exists. To calculate $\mathbf{v}$ it is sufficient to put $B^{'} = 0$ in Eq. 7 and to remark that a solution can be found only if $\mathbf{v}$ is perpendicular to $\mathbf{E}$ and $\mathbf{B}$. The result is that

$$\mathbf{v} = c\frac{\mathbf{E} \times \mathbf{B}}{E^2} \tag{8}$$

and therefore $\beta = B/E$. It is evident that this solution is only valid if $E > B$. In the case when $B > E$, one can find a velocity $\mathbf{v}$ so that in the corresponding frame $K^{'}$ only the magnetic field $\mathbf{B}'$ exists. In this case

$$\mathbf{v} = c\frac{\mathbf{E} \times \mathbf{B}}{B^2} \tag{9}$$

and therefore $\beta = E/B$. The value of $\mathbf{E}'$ for $E > B$ (resp. of $\mathbf{B}'$ for $B > E$) can be calculated by inserting the corresponding value of $\mathbf{v}$ in Eq. 6 (resp. Eq. 7). One obtains

$$\mathbf{E}' = \sqrt{1-\beta^2}\mathbf{E} \tag{10}$$

and

$$\mathbf{B}' = \sqrt{1-\beta^2}\mathbf{B} \tag{11}$$

respectively.

Light propagation in a moving medium

The basic formulas to study the propagation of light in a moving frame are the ones that give the magnitude and the direction of a velocity $\mathbf{u}$ obtained by adding relativistically two velocities $\mathbf{u}'$ and $\mathbf{v}$ [12]. In particular, one finds that

$$u^2 = \frac{u^{'2} + v^2 + 2u^{'}v\cos(\theta^{'}) - (\frac{u^{'}v}{c})^2\sin(\theta^{'})^2}{(1+\frac{u^{'}v}{c^2}\cos(\theta^{'}))^2} \tag{12}$$

and

$$\tan(\theta) = \frac{u^{'}\sin(\theta^{'})\sqrt{1-\beta^2}}{u^{'}\cos(\theta^{'}) + v} \tag{13}$$

where $\theta^{'}$ is the angle between $\mathbf{u}^{'}$ and $\mathbf{v}$ and θ the angle between $\mathbf{u}$ and $\mathbf{v}$. To apply these formulas to our calculation, let's write $u^{'} = c/n'$ and $u = c/n$, v being the moving frame velocity. In particular, if $\theta^{'} = 0$, $\theta = 0$ and moreover if $\beta << 1$, one obtains the well-known Fizeau formula

$$u = \frac{c}{n^{'}} + v(1 - \frac{1}{n^{'2}}) \tag{14}$$

For the quantum vacuum n and $n^{'}$ can be written as $n = 1 + \delta n$ with $\delta n << 1$ and $n^{'} = 1 + \delta n^{'}$ with $\delta n^{'} << 1$. If $\mathbf{u}$ and $\mathbf{v}$ are parallel, $\theta = 0$ and $\theta^{'} = 0$ so that Eq. 12 can be written, up to first order with respect to δn and $\delta n^{'}$, as

$$\delta n = \delta n^{'}\frac{1-\beta}{1+\beta} \tag{15}$$

If $\mathbf{u}$ and $\mathbf{v}$ are antiparallel, one has simply to change the sign of β. If $\mathbf{u}$ is perpendicular to $\mathbf{v}$, $\theta = \frac{\pi}{2}$. From Eq. 13, we can infer that $\cos(\theta^{'}) = -v/u'$ and $\sin(\theta^{'}) = \sqrt{1-(v/u')^2}$. This obviously means that in $K^{'}$, the direction of propagation of light is no longer perpendicular to the frame velocity. Upon inserting the values of $\cos(\theta^{'})$ and $\sin(\theta^{'})$ into Eq. 12, one obtains

$$\delta n = \delta n^{'}\frac{1}{1-\beta^2} \tag{16}$$

Light polarization in a moving medium

We are interested in effects that depend on the polarization of light. We therefore have to study how the polarization of light is transformed going from frame K to frame $K^{'}$. We first consider the case of a plane wave in K when no external fields are present and recall that the quantities $E^2 - B^2$ and $\mathbf{E}\cdot\mathbf{B}$ are invariant under Lorentz transformation. For a plane wave in vacuum $E_\omega = B_\omega$ and $\mathbf{E}_\omega \cdot \mathbf{B}_\omega = 0$ where $\mathbf{E}_\omega, \mathbf{B}_\omega$ are the optical fields. Since $E_\omega^2 - B_\omega^2$ and $\mathbf{E}_\omega \cdot \mathbf{B}_\omega$ are equal to zero in K, they are also equal to zero in $K^{'}$. This means that in $K^{'}$ we still have a plane wave. If in addition, static $\mathbf{E}_0$ and $\mathbf{B}_0$ fields are also present, the total fields can be written as $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_\omega$ and $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_\omega$. Using the linearity of Eqs. 6 and 7 with respect to the fields, it is straightforward to show that

$$\mathbf{E}' = \mathbf{E}'_{\mathbf{0}} + \mathbf{E}'_\omega \tag{17}$$

$$\mathbf{B}' = \mathbf{B}'_{\mathbf{0}} + \mathbf{B}'_\omega \tag{18}$$

where $\mathbf{E}'_\omega$ and $\mathbf{B}'_\omega$ are the transformations of the optical fields alone. We can therefore conclude that the optical fields transform as if no external field were present and thus a plane wave in K remains a plane wave in $K^{'}$. Let's now investigate how the orientation of $\mathbf{E}_\omega$ and $\mathbf{B}_\omega$ with respect to $\mathbf{E}_0$ and $\mathbf{B}_0$ changes in the $K^{'}$ frame. We know that

$$E^2 - B^2 = E'^2 - B'^2 \tag{19}$$

$$\mathbf{E}' \cdot \mathbf{B}' = \mathbf{E} \cdot \mathbf{B} \tag{20}$$

We also have that $E_0^2 - B_0^2 = E_0'^2 - B_0'^2$ and $\mathbf{E}'_0 \cdot \mathbf{B}'_0 = \mathbf{E}_0 \cdot \mathbf{B}_0$ (actually in our specific case $B_0^{'} = 0$ (resp. $E_0^{'} = 0$)). We finally derive from Eq. 19 that

$$\mathbf{E}_\omega \cdot \mathbf{E_0} - \mathbf{B}_\omega \cdot \mathbf{B_0} = \mathbf{E}'_\omega \cdot \mathbf{E}'_{\mathbf{0}} \text{ (resp. } \mathbf{B}'_\omega \cdot \mathbf{B}'_{\mathbf{0}}) \tag{21}$$

and, from Eq. 20 that

$$\mathbf{E}_\omega \cdot \mathbf{B_0} + \mathbf{E_0} \cdot \mathbf{B}_\omega = \mathbf{E}'_{\mathbf{0}} \cdot \mathbf{B}'_\omega \text{ (resp. } \mathbf{E}'_\omega \cdot \mathbf{B}'_{\mathbf{0}}) \tag{22}$$

If for example $\mathbf{E}_\omega \parallel \mathbf{E}_0$ and $\mathbf{B}_\omega \parallel \mathbf{B}_0$ Eq. 22 gives that $\mathbf{E}'_0 \cdot \mathbf{B}'_\omega = 0$ i.e. $\mathbf{E}'_\omega \parallel \mathbf{E}'_0$. If vice versa $\mathbf{E}_\omega \parallel \mathbf{B}_0$ and $\mathbf{B}_\omega \parallel \mathbf{E}_0$ Eq. 21 gives that $\mathbf{E}'_0 \cdot \mathbf{E}'_\omega = 0$ i.e. $\mathbf{B}'_\omega \parallel \mathbf{E}'_0$. Summarizing, the orientation of the polarization of the wave with respect to the only existing static field in the $K^{'}$ frame is the same with respect to that static field in the K frame.

Results

Let's finally calculate the value of n using the expressions derived in the previous paragraphs. We consider the situation when $\mathbf{E}_0 \perp \mathbf{B}_0$ and assume for the moment that $E_0 < B_0$ and that $\mathbf{k}$ is parallel to $\mathbf{E}_0 \times \mathbf{B}_0$ and therefore parallel to $\mathbf{v}$. In the frame $K^{'}$, the only existing effect is a Cotton-Mouton effect proportional to the square of B'_0 i.e. $\delta n'$ depends on the polarization of light. Thanks to eqs. 4, 5, we can write that

$$\delta n'_{\|} = \eta_{\|} B_0'^2 = \eta_{\|}(1-\beta^2)B_0^2 \tag{23}$$

and, using Eq. 15,

$$\delta n_{\|} = \eta_{\|}(1-\beta)^2 B_0^2 = \eta_{\|}(B_0^2 - 2E_0B_0 + E_0^2) \tag{24}$$

since $\mathbf{E}_0$ and $\mathbf{B}_0$ are perpendicular. For the same reason, we can write

$$\delta n_{\perp} = \eta_{\perp}(1-\beta)^2 B_0^2 = \eta_{\perp}(B_0^2 - 2E_0B_0 + E_0^2) \tag{25}$$

So if $\eta_{\|} \neq \eta_{\perp}$ i.e. if the Cotton-Mouton effect exists, we have demonstrated that the Kerr effect and the magneto-electric birefringence proportional to E_0B_0 must also exist. Moreover, because of Lorentz invariance, Kerr birefringence has to be of opposite sign compared to the Cotton-Mouton one since light retardation is equal for light polarized parallel to the $\mathbf{B}_0$ field and orthogonal to the $\mathbf{E}_0$ field and vice versa. As for magneto-electric birefringence, the coefficient that multiplies the fields is twice the one of the Cotton-Mouton or Kerr birefringence.

Let's now come to the polarization-independent magneto-electric anisotropy. If we write $\eta_{\|} = \eta_{\perp} + \Delta$ we find that light is retarded by the existence of the static fields independently of the polarization by a quantity corresponding to

$$\delta n_0(k) = \eta_{\perp}(B_0^2 - 2E_0B_0 + E_0^2) \tag{26}$$

If we change $\mathbf{k}$ into $-\mathbf{k}$, we have to change the sign of β and therefore the previous equation becomes

$$\delta n_0(-k) = \eta_\perp (B_0^2 + 2E_0 B_0 + E_0^2) \tag{27}$$

and finally we find for the anisotropy

$$\delta n_0(k) - \delta n_0(-k) = -4\eta_\perp E_0 B_0 \tag{28}$$

For the case when $B_0 < E_0$, one obtains exactly the same results and they are identical to those that we have obtained in ref. [8] and that we have complemented in ref. [9] where we have used for $\eta_{||}$ and $\eta_\perp$ the accepted values. We stress that the existence of the magneto-electric anisotropy is only related to the fact that $\eta_\perp$ is different from zero. This effect could exist even if the Cotton-Mouton effect would not exist, as the Cotton-Mouton effect exists only if Δ is different from zero, independent of the value of $\eta_\perp$.

We finally study the case when $E_0 = B_0$. If $\mathbf{k}$ is parallel to $\mathbf{E}_0 \times \mathbf{B}_0$, $\delta n_{||} = \delta n_\perp = 0$. This result is somewhat obvious since this case corresponds to the propagation of a plane wave in the field of a co-propagating plane wave. If $\mathbf{k}$ is antiparallel to $\mathbf{E}_0 \times \mathbf{B}_0$, the effect is not zero. This means that two counterpropagating plane waves can affect each other.

We note that $\eta_{||}$ and $\eta_\perp$ are directly related to the coefficients of the invariants F^2 and G^2 in the Lagrangian L_{EK} [13]. Actually, $\eta_{||} = \frac{b}{2}$ and $\eta_\perp = 2a$ and, for the Cotton Mouton effect, we have

$$\Delta n = n_{||} - n_\perp = \frac{1}{2}(b - 4a)B_0^2 \tag{29}$$

i.e. $\Delta n = \frac{3}{2} a B_0^2$ since $b = 7a$. The existence of the Cotton-Mouton effect in vacuum therefore depends on the ratio b/a. If for example b/a would be equal to 4, Δn would be zero and no Cotton-Mouton effect would exist. Nor would the Kerr effect, the magneto-electric birefringence or the magneto-electric Jones birefringence exist. The only existing effect would be the polarization-independent magneto-electric anisotropy. This fact is not without importance at least from a historical point of view, since Born and Infeld have developed around 1934 a QED theory [14] in which the value predicted for the ratio b/a was exactly 4.

The case of $\mathbf{E}_0$ perpendicular to $\mathbf{B}_0$ with $\mathbf{E}_0$ or $\mathbf{B}_0$ parallel to $\mathbf{k}$ has not been treated before and we will show in the following how we can solve it using our method. Let's assume that $E_0 < B_0$ and consider $\mathbf{B}_0 || \mathbf{k}$. The velocity $\mathbf{v}$ is perpendicular to $\mathbf{k}$ in frame K. In the frame $K^{'}$, the $\mathbf{B}_0'$ field lies in the plane containing $\mathbf{k}'$ and $\mathbf{v}$ and it is perpendicular to $\mathbf{v}$. The $\mathbf{k}'$ vector is no more perpendicular to $\mathbf{v}$ and therefore $\mathbf{B}_0'$ is no more collinear with the direction of propagation of light. In this frame, only the Cotton-Mouton effect can exist, and to calculate $\delta n'$, one has therefore only to consider the component of $\mathbf{B}_0'$ perpendicular to $\mathbf{k}'$. Using the result for $\cos(\theta^{'})$, this component $B_{0,\perp}'$ is equal to

$$B_{0,\perp}' = -\beta B_0' \tag{30}$$

and

$$\delta n^{'} \propto (1 - \beta^2) B_0^2 \beta^2 \tag{31}$$

and finally, using Eq. 16,

$$\delta n \propto B_0^2 \beta^2 = E_0^2 \tag{32}$$

Therefore only the Kerr effect exists for this case. Let's now consider the case $E_0 > B_0$. The resulting $\mathbf{E}_0'$ is still perpendicular to $\mathbf{v}$ and $\mathbf{k}'$. This means that in $K^{'}$, the only existing effect is a Kerr effect given by the entire electric field.

$$\delta n^{'} \propto (1 - \beta^2) E_0^2 \tag{33}$$

and finally by Eq. 16,

$$\delta n \propto E_0^2 \tag{34}$$

Again, the $\mathbf{B}_0$ field gives no contribution. It is straightforward to show that in order to solve the case when the $\mathbf{E}_0$ field is parallel to $\mathbf{k}$, it is sufficient to permute $\mathbf{E}_0$ with $\mathbf{B}_0$ in the previous formulas. Therefore, in general, the static field parallel to $\mathbf{k}$ does not contribute to a bilinear optical effect.

Parallel fields geometry

To complete our analysis, we consider the geometry where the two fields $\mathbf{B}_0$ and $\mathbf{E}_0$ are parallel to each other. The case in which the two fields are perpendicular to the $\mathbf{k}$ vector of light has been studied in detail by Ross, Sherborne and Stedman in ref. [15]. These authors proved by symmetry considerations that in this configuration, magneto-electric Jones birefringence should exist and that it should have the same magnitude as the magneto-electric linear birefringence in crossed fields. The two effects are actually two different facets of the same phenomenon.

What has not been studied yet is the case where the two parallel fields are parallel to the $\mathbf{k}$ vector. We will show that no bilinear effect exists for this geometry. First assume that such an effect exists i.e.

$$\delta n = \eta E_0 B_0$$

Let's regard the two fields in a K' reference frame moving at a velocity $\boldsymbol{\beta}$ in the direction of $\mathbf{k}$. Using equations 6 and 7, one can show that

$$\mathbf{E_0'} = \mathbf{E_0} \tag{35}$$

and

$$\mathbf{B_0'} = \mathbf{B_0} \tag{36}$$

In K' therefore

$$\delta n' = \eta E_0 B_0$$

Using equation 15 we must then conclude that

$$\delta n = \eta E_0 B_0 \frac{1-\beta}{1+\beta} \tag{37}$$

Since β can take any value between 0 and 1,this is in disagreement with our starting assumption, unless η is equal to zero. Thus the two parallel fields in the direction of $\mathbf{k}$ give no bilinear effect.

Conclusion

We have shown that the magneto–electric optical properties of the quantum vacuum can be deduced from the principle of Lorentz invariance. We have reproduced in a straightforward manner theoretical results obtained by much more elaborate methods, and have also considered new geometries, never treated before. In particular we have found the new result that in crossed static electric and magnetic fields, a static component parallel to the propagation of light does not give rise to an optical effect. Although our method could also be applied to material media, it is less convenient there, as one also has to take into account the transformations of the induced material responses.

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Références

[1] T. Roth and G.L.J.A. Rikken, Phys. Rev. Lett. **85**, 4478 (2000).

[2] T. Roth and G.L.J.A. Rikken, Phys. Rev. Lett. **88**, 063001 (2002).

[3] G.L.J.A. Rikken, C. Strohm and P. Wyder, Phys. Rev. Lett. **89**, 133005 (2002).

[4] H. Euler and B. Kockel, Naturwiss. **23**, 246 (1935).

[5] J.D. Jackson, "Classical Electrodynamics", Wiley, New York (1975).

[6] P.W. Milloni, "The Quantum Vacuum", Academic Press, San Diego (1994).

[7] Z. Bialynicka-Birula and I. Bialynicka-Birula, Phys. Rev. **D2**, 2341 (1970).

[8] G.L.J.A. Rikken and C. Rizzo, Phys. Rev. **A 63**, 12107 (2001).

[9] G.L.J.A. Rikken and C. Rizzo, Phys. Rev. **A 67**, 15801 (2003).

[10] W. Heisenberg and H. Euler, Z. Phys. **98** 714 (1936).

[11] L. Landau and E. Lifchitz, "Physique Théorique, tome IV", Editions Mir, Moscou (1973).

[12] R. Becker, "Electromagnetic fields and interactions", Dover, New York (1982).

[13] E. Zavattini, Comment. At. Mol. Phys., **33** 83 (1996).

[14] M. Born and L. Infeld, Proc. R. Soc. London **A 143** 410 (1934).

[15] H. J. Ross, B. S. Sherborne and G. E. Stedman, J. Phys. **B**. : At. Mol. Opt. Phys. **22** 459 (1989).