

Rheology of a two-phase material with applications to partially molten rocks, plastic deformation and saturated soils

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ABSTRACT

A global model is presented to account for the specific rheology of a two-phase material. Examples of observations are taken from a crystallising magma. Applications are ported to a partially molten rock, plastic deformation and soil liquefaction. The general behaviour of the viscosity is drawn as a function of the strain rate and the amount of solid phase. It constitutes a 3D diagram developing a cubic surface. The cubic equation is justified by thermodynamic considerations. It basically results from the mixing of a Newtonian ($n=1$) and a power law ($n=3$) type of deformation. The diagram shows two types of rheological response. At high strain rate values, the viscosity contrast between the two phases is the lowest. A bulk en masse behaviour results, as observed during tectonic activity. It manifests by a homogeneous transport of magma during emplacement and fabric development. An equivalent medium, with averaged viscosity is a good proxy. Conversely, at low strain rate values, the viscosity contrast between the two phases is the highest. The two end members behave according to their respective rheology. In between, a transitional state develops, in which instability occurs depending on the strain rate and stress conditions. It manifests in the 3D diagram by a cusp shape. Rheology presents continuous jumps between the liquid-like and the solid-like rheology. They result in strain localisation or phase segregation. The later preferentially develops during magma crystallisation. Deformation under a constant amount of each phase is also possible, resulting in pressure dissolution-like processes. A major cause for instability results from a bifurcation in the solution plane of the equation of viscous motion. It is comparable with strain softening. A similar situation should develop when mixing Newtonian and power law rheology as during diffusion and dislocation creep, or water saturated sediment deformation. Owing to the continual jumps between the two types of rheology, hysteresis, or memory effect may develop. Rapid cyclic deformation may drive strain to extreme straining. The effect of simple shear seems much more effective than pure shear (compaction) to segregate the weak phase out of its strong matrix. The development of instabilities and continuous jumps from one rheology to the other lead to discontinuous motion of the weak phase. In a molten region, it corresponds to discontinuous burst of magma that are extracted.

Keywords: rheology, two-phase, magma, liquefaction, hysteresis

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Two-phase materials and rheology

Rheology describes the response of a material to an applied force. Whatever the type of response (non-linear, time dependent), it pertains to continuum mechanics (Ranalli, 1995). It assumes continuity, homogeneity and isotropy of the parameters. In a two-phase material, a distinct rheology characterises each phase, representing a first departure to continuum mechanics. However, when the two properties are close enough, continuity can still be assumed. Homogeneity and isotropy have also to be re-considered except when the phases present a regular distribution, or when one phase is much preponderant compared to the other.

Most of geological objects are composed of several distinct phases. Rocks commonly include a limited number of minerals, each of them presenting a specific chemical, mechanical or rheological property. A fluid phase, in variable quantity, is also observed that adds to the mineral phases. It may reach saturation in sedimentary rocks. Igneous rocks are also a good example of two-phase material when they form. Partially molten rocks (PMR) represent good candidates for investigating the structural behaviour of such two-phase materials, as are also water saturated sediments or plastic deforming rocks.

The rheology of a two-phase material is characterised by non-linear instabilities. The stress-strain curve for porous material is typically non-linear (Wang, 2000). In saturated porous material, as Berea sandstone, the curve presents a shelf reflecting grain crushing and porosity collapse (Zhang et al., 1990). Compaction shear bands initiate associated, joint with shear bands (Olsson, 1999). They initially collect the fluid forming the second phase (Du Bernard et al., 2002; Schmocker et al., 2003). But, as the differential stress increases, they loose porosity (Issen & Rudnicki, 2000), expelling the fluid phase. Fluid extraction is discontinuous, like the matrix motion along those structures. The highly discontinuous response to stress appears characteristics of two-phase materials. It manifests through fault valve mechanism (Sibson, 1989), episodic fault motion (Heki et al., 1993), and cyclic magma extraction (Barraud et al., 2001; Vigneresse & Burg, 2001). However, at a larger scale, deformation seems continuous and regular, as observed during large scale deformation of migmatite massifs (Brun & Martin, 1978), homogeneous flow fabric in a sheared magma while flowing (Bouchez, 1997) or during soil liquefaction (Ishihara, 1993).

Two-phase materials incorporate a varying proportion of each phase, each with a characteristic reaction to strain. They are designed as the end-members. In a PMR, it would be the melt and its matrix. In a first step, the transition between the two end-members has been assigned to a binary model in PMR (Arzi, 1978). A sudden change in the viscosity marks the limit from a solid-like to a liquid-like behaviour. A coherent bulk behaviour is also observed during experimental deformation of two-phase systems (Burg & Wilson, 1987; Ji et al., 1993; Handy, 1994; Rosenberg & Handy, 2001). The two mineral phases, or their analogue equivalent, present a contrasted rheology, but they have the same magnitude in terms of relaxation rate and length. They define a binary model of deformation related either to the load-bearing framework (LBF) structure or to the interconnected weak layer (IWL) structure. Nevertheless, experimental deformation on PMR is restricted to very low melt fraction, less than 12%, because of experimental difficulties (Kohlstedt et al., 2000). They also require fast strain rates, typically of the order of 10^{-6} to 10^{-4} s⁻¹, though experiments last several hours, or even days. Composite materials also present two phases, but the rheological contrast between them must be low to guarantee cohesion (Hull & Clyne, 2002).

A bulk percentage of each phase can be estimated in a PMR. For instance, 25 % melt may be estimated in a migmatite. However, the proportion of the molten phase may vary from 100% in the leucosome to 0% in an unaffected zircon. The proportion varies from place to place, but also with time, owing to the large mobility of the weak phase. In consequence, the definition of a bulk property, or transport property, is always subject to a scale problem in a two-phase material. Depending on place and time, the local properties cannot be estimated, thus making local phenomena often unexplained.

The rheology of the two phases is much contrasted in a PMR (Petford, 2003). Viscosity contrast between the phases commonly ranges 10 to 15 orders of magnitude (Burg & Vigneresse, 2002). All experimental and numerical approaches reported instabilities

(Brown et al., 1995; Bercovici et al., 2001; Rabinowicz et al., 2001; Vigneresse & Burg, 2001). They document that a single binary transition is not appropriate, and that an intermediate state develops, highly unstable. Two thresholds have been defined that bracket the transitional state in a crystallising PMR. One, Rigid Percolation Threshold (RPT), defines the onset of connection of the solid phase represented by the forming crystals. The second, or Particle Locking Threshold (PLT), corresponds to the close packing reached by the crystals (Vigneresse et al., 1996). Thus, the rheology of PMR essentially appears as controlled by the respective amount of each phase. Other factors, as the fluid content of the magma, melt polymerisation (Petford, 2003) seem to be restricted to second order effects.

Two-phase materials are not restricted to Earth Sciences. They are commonly studied in liquid crystals (nematics), food engineering (pulp bearing products), or various pastes and slurries (Okagawa et al., 1973; Forest et al., 1997; Fan et al., 1998). Recent developments in those fields have brought a series of new materials (polymers, composite materials, shape memory alloys) for which the reaction to stress bears importance in industrial processes. They resulted in many textbooks or journal articles (e.g. Adler et al., 1990; Liu et al., 1996). They describe the various departures to a non-linear response of strain to stress, or thixotropic response (Barnes, 1997; 1999). The second point that comes out from those studies is the effect of time. Time enters in the definition of strain rate, i.e. of viscosity, but, surprisingly it is absent in most models concerning the rheology of two-phase mixture in Earth Sciences (e.g. Arzi, 1978).

The present paper focuses on the rheology of two-phase material during the transitional state. It incorporates several concepts that have been issued in a series of four papers on specific aspects of the rheology of PMR. Thresholds bracketing a specific behaviour of PMR in static conditions have been defined in Vigneresse et al. (1996). Later on, the role of the weak phase in partitioning strain and increasing vorticity, i.e., the non coaxial component of strain, was emphasised in Vigneresse & Tikoff (1999). It leads to the determination of the non-linear aspects of PMR rheology, with feedback loops that accelerate or damp the response to stress (Burg & Vigneresse, 2002). Finally, catastrophic jumps from the rheology of one state to the other have been suggested (Vigneresse & Burg, 2002). Here, a conceptual approach to the rheology of a two-phase material is presented. It takes examples from the different responses of PMR. It focuses on the strain rate dependence of the viscosity in a two-phase mixture. Applications are ported to water saturated sediments in undrained conditions which undergo liquefaction during earthquakes (Ishihara, 1993; Di Prisco & Imposimato, 2002) and to plastic deformation.

Main principles of rheology

The rheology of a rock characterises its reaction, or strain (ε), due to deformation when a stress (σ) is applied (Ranalli, 1995). Reaction may be elastic, and a linear equation relates the two parameters through an elastic modulus (K).

$$\varepsilon = \sigma / K \quad (1)$$

In case of shear deformation, a notation relating shear strain (γ) to tangential stress (τ) is preferred, with the shear modulus (G)

$$\gamma = \tau / G \quad (2)$$

Here, σ for stress and ε for strain, are used indifferently for pure or simple shear. In the elastic domain, strain returns to initial conditions when stress is removed. Strain is independent of time. In contrast, reaction to stress is time dependent for plastic bodies. In which case the strain rate ($\dot{\varepsilon}$) is related to stress by the viscosity (η).

$$\dot{\varepsilon} = \sigma / \eta \quad (3)$$

Melts present a linear relationship between stress and strain. A constant viscosity characterises those Newtonian bodies. Carbonatites magmas present a very low viscosity ($5 \cdot 10^{-3}$ to 10^{-2} Pa.s), making them comparable to a liquid (Dobson et al., 1996). In contrast, felsic magmas are more viscous than basalts, with viscosity values in the range 10^4 to 10^6 Pa.s (Clemens & Petford, 1999).

A specific case of linear relationship concerns plastic deformation activated by diffusion creep in crystal aggregates (Nicolas & Poirier, 1976). It is function of temperature (T), through an energy of activation (Q) and the gas perfect constant (R)

$$\dot{\epsilon} = A d^{-m} \sigma \exp (-Q/RT) \quad (4)$$

The relation may be in d^{-2} or d^{-3} , with d the grain size, depending on the way diffusion develops. It leads to Coble creep when diffusion develops along grain boundaries (Coble, 1963). In contrast, Nabarro-Herring creep develops through lattice diffusion within grains (Nabarro, 1950; Herring, 1950). In both cases, the relation between strain rate and stress is linear. Coble creep develops preferentially at lower temperature compared to Nabarro-Herring creep (Fig. 1).

In many other cases, non-linear descriptions are required, with an instantaneous viscosity. It is commonly taken as the local tangent to the curve relating stress to strain rate (Ranalli, 1995). Here, it is simply computed as the ratio of stress to strain rate. Departures to the linear curve determine strain hardening when the response is less than the elastic response, which is commonly the case. The hardening coefficient ($h = d\sigma/d\dot{\epsilon}$) remains positive, but decreases with increasing stress. Strain softening marks a increase of strain while stress decreases.

Solid state physics (Green, 1998) considers the mechanisms that rule plastic deformation (Fig. 1). In usual conditions, a power law describes the long term behaviour of crustal rocks (Kirby & Kronenberg, 1987).

$$\dot{\epsilon} = A \sigma^n \exp (-Q/RT) \quad (5)$$

The different parameters, as the stress exponent (n), the coefficient (A), and the activation energy (Q) are experimentally determined for each particular rock type. At high temperature, the stress exponent is close to 3 for amphibolitic rocks (Kirby & Kronenberg, 1997). It reflects a deformation accommodated by dislocation creep (Fig. 1). The mechanism develops at high temperature, and higher stress values compared to diffusion creep.

The adopted rheology for a PMR is typically that of melting or crystallisation of a granitic magma, for which amphibolites offer a good proxy. The temperature is fixed to 800 °C, corresponding to biotite breakdown reaction (Patiño Douce & Beard, 1995). The viscosity variations with temperature are not considered, since being a second order effect. Hence, the melt is Arrhenian, i.e. its viscosity exponentially decreases with temperature, according to an activation energy E

$$\eta = \eta_0 \exp (-E/RT) \quad (6)$$

A typical value for E is about 300 kJ/mole (Maaløe, 1985) close to that of amphibolites (243 kJ/mole). In consequence both phases, melt and solid, keep constant their viscosity contrast during the temperature interval of the phase transition. The viscosity of granitic melt varies with high strain rate (Burg & Vigneresse, 2002). But it occurs at very high strain rate values ($10^{-4.5} \text{ s}^{-1}$), rarely observed in geological situations.

Rheology of a two-phase material

The rheology of a two-phase material behaves according to the relative proportion of each phase. The case of a PMR is adopted for the numerical values. The amount of the strong phase (Φ) is the intrinsic variable. The two end-members have their own specific responses. They are progressively altered by the influence of the other phase. A transitional rheology develops that must assume connection between the two end-members. It should be examined within a 3D diagram ($\eta - \dot{\epsilon} - \Phi$). The use of the viscosity (η) instead of stress (σ) is better constrained by experimental values.

The two end-members

Owing to the large variations in the viscosity values, the strain response to stress plots in a log-log diagram. For the melt, assumed to be Newtonian in a first order, the equation (3) translates into a line:

$$\log \dot{\epsilon} = \log \sigma - \log \eta \quad (7)$$

Here, the adopted viscosity value is 10^6 Pa.s (Clemens & Petford, 1999). Conversely, the power law rheology (Eq. 5) of the matrix transforms into a linear plot, with a slope (n) and a constant value ϵ_0 at the origin of co-ordinates in the log-log diagram

$$\log \dot{\epsilon} = n \log \sigma + \epsilon_0 \quad (8)$$

Experimentally derived values for amphibolites (Kirby & Kronenberg, 1997) have been adopted. Values for the parameters in Eq. 5 are $n = 3$, $\log A = -4.9$ and $Q = 243$ kJ/mole and a temperature of 800 °C. Adopting those values, the pre-exponential and exponential terms combine, resulting in a constant value ϵ_0 of -32 and a slope of 3 .

The rheology of PMR plots in a log-log diagram within the two lines that define the rheology of the two end-members (Fig. 2). Their intersection occurs for an unrealistic value of strain rate value (10^8 s⁻¹). The two lines are separated in the usual range of stress and strain rate values. Those range between 10^{-20} and 1 s⁻¹, thus bracketing a common deformation rate estimated at 10^{-16} s⁻¹ (Pfiffner & Ramsay, 1982). In comparison, magma emplacement is estimated to take place at 10^{-10} s⁻¹ (Koenders & Petford, 2000). The fast deformation rate may be associated to seismic events. Stress values are bracketed between 0.01 and 100 MPa, thus resulting in logarithmic values of 4 to 8 in the diagram (Fig. 2).

The transitional state

Determining the rheology of a PMR requires filling the area between the two end-members, leading to two possibilities. In the first case, extrapolation from the end-members provides a solution. It has been the case for previous experiments (f.i. Arzi, 1978). However, the solution is non unique and remains poorly constrained. In the second case, thermodynamic considerations are necessary to provide additional constraints to the adopted solution, also bracketed by experimental results.

Extrapolating the end-members

For a suspension of solid particles in a viscous fluid of viscosity (η_0), the bulk relative viscosity increases with the amount of solid phase. Experiments have been carried out by Einstein (1906) and Roscoe (1952), resulting in a general equation

$$\eta = \eta_0 (1 - \Phi / \Phi_{\max})^{-n} \quad (9)$$

The value of the maximum packing value (Φ_{\max}) and (n) are experimentally determined. Packing values range between 0.63 to 0.75 for monodisperse spherical particles (Rogers et al., 1994). They depend on the shape of particles and on strain rate in a lesser extent (De Kruif et al., 1986). A common value for the exponent is 2.5 (Lejeune & Richet,

1995). The Einstein-Roscoe equation is valid for dilute solutions ($\Phi \ll 0.30$). For concentrated suspensions, the law is slightly altered by particle interactions. It leads to a similar equation in which the power coefficient changes from n to $n \cdot \Phi_{\max}$, resulting in a value of 1.8 (Krieger & Dougherty, 1959). The viscosity increase due to the solid suspension is about 3 orders of magnitude when approaching the maximum packing of 0.75 (Vigneresse et al., 1996). It alters the viscosity value, modifying the slope of the viscosity (Eq. 7).

The other end of the diagram corresponds to the solid phase with low residual melt content. The completely solidified matrix follows a power law (Eqs. 5 and 8) that is strain rate dependent. Its response to stress is locked until some connection exists between the melt phase, as it occurs for initial melting (Vigneresse et al., 1996). Connection starts at Φ_{\max} , but it extends up to loose packing. In both cases, the limit is a function, non defined at present, of the strain rate. By evidence, low strain rate provides more chance to the weak phase to be connected.

The two surfaces overlap on a wide range of Φ (0.50 – 0.75). The connection between the two end-members takes the form of a cusp surface (Fig. 3). The viscosity adopts a cusp shape in a $(\dot{\epsilon} - \eta - \Phi)$ diagram, as it has been suggested for other metastable situations dealing with catastrophe theory (Thom, 1990; Petford, 1995). However, the construction remains purely geometric, even if the two end members are experimentally controlled.

Thermodynamic considerations

At present, there exists no exact theory that physically and mathematically describes the transitional state. Partial solutions consist in determining the interactions between solid particles (Krieger & Dougherty, 1959; Batchelor, 1976). They rapidly face close packing problems. Eulerian descriptions examine melt segregation under compaction and shear (Rabinowicz & Vigneresse, 2003; Spiegelman, 2003). They identify non-linear instabilities, the interactions of which lead to concentrate the weakest phase into elongated pockets. They describe the consequences of the interactions, but they are not adapted to infer its problematics.

The total energy of a two-phase material should be considered. Assuming that pressure and volume are constant, and that temperature varies within a restricted range, the energy is the sum of the product of strain by stress, weighted for the respective amount of weak and strong phase. The work needed to alter the system is the derivative of the energy with respect of time. It is the weighted sum of the respective products of strain rate by stress for both phases, and a coupling term

$$W = \Phi (\sigma \dot{\epsilon})_{\text{melt}} + (1-\Phi) (\sigma \dot{\epsilon})_{\text{matrix}} + \text{coupling term} \quad (10)$$

The coupling term assumes continuity of strain between the two phases. Equilibrium conditions implies no work variation ($dW/d\dot{\epsilon} = 0$). Incorporating Eq. 4, 5 and 9 into the preceding one (Eq. 10), deriving with respect to strain rate and rearranging for viscosity leads to the equation for stability

$$\eta^3 + p \eta + q = 0 \quad (11)$$

in which the terms p and q are function of $\dot{\epsilon}$ and Φ . The cubic equation adopts a cusp shape when the term p becomes negative. At present, I am unable to exactly quantify the both terms p and especially q . A proxy for the change of sign of p is when the viscosity of the strong phase is the cube of the weak one ($\eta_{\text{matrix}} \approx \eta_{\text{melt}}^3$).

Significance of the cusp shape

Describing the viscosity variation by a cubic equation during a phase transition is not fortuitous. Viscosity is the transport parameter that relates the two extrinsic variables, stress and strain rate. It also depends on the respective amount of phase, the variable that

describes the phase transition. It corresponds to an equation of state, with specific reference to rheology. The equation of state, describing pressure and volume in function of temperature, during the liquid-gas transition is by essence a cubic equation (Landau & Lifchitz, 1967; Perez, 2001).

The cubic equation emphasises the role of strain rate, i.e. time in rheology. Viscosity is by essence time dependent. It manifests by the strain rate, the derivative of strain with respect of time. Consequently, it is obvious that all mixture of viscous fluid should incorporate a time dependence, which was not the case in previous models of rheology (f.i. Arzi, 1978). The strain rate dependence and the cubic equation clearly delimit two types of behaviour. One, for high strain rate consists in a continuous evolution of the curved surface (Fig. 3). It corresponds to an homogeneous strain in response to stress. The second occurs beyond a critical point corresponding to the cusp. It is the domain in which local instabilities develop.

To better illustrate the rheology of PMR, a new figure still referring to $(\eta - \dot{\epsilon} - \Phi)$, is shown, but simplified from Fig. 3 on which the rheological paths are indicated (Fig. 4). In a $(\eta - \Phi)$ plane, equivalent to a section at constant $\dot{\epsilon}$ in the 3D diagram, the cusp region manifests by sudden changes from a weak phase to a strong phase, or stress hardening (Fig. 4). The region of overlapping branches is metastable. Three viscosity values can be assigned to a given point of fixed Φ . Two are metastable and correspond to the viscosity of the weak and strong phase. The third point, on the connecting branch, is unstable. The metastable region corresponds to a local maximum of the energy of the system, reflecting the fourth order equation of the energy. The total energy, proportional to the product $\sigma \cdot \dot{\epsilon}$ increases during strain hardening. As soon as strain softening develops, corresponding to a decrease of stress with strain, the energy decreases as well.

Rheological responses

Owing to the cusp shape of the viscosity surfaces (Fig. 3 and 4), a different response to stress is expected, depending on whether it develops at high or low strain rate, i.e. if instabilities occur or not. The transition from high to low strain rate values may be compared to a critical state that appears in liquid-gas equation of state (Perez, 2001). It corresponds to the change of sign in the coefficient p in Eq. 4. For a PMR, it occurs when the two values of the viscosity are about in a cubic proportion. The critical point also corresponds to the coincidence between loose and maximum packing (Fig. 3). Because maximum packing is partly strain dependent, a proxy could be 0.66. It means that the strain rate imposed to the melt avoids a loose packing to develop. At present, no data allow estimating the corresponding strain value. An approximate value, taken from Fig. 2, brackets the critical strain rate below 10^{-12} s^{-1} , depending on the ambient stress value. Considering the strain rate during magma emplacement (10^{-10} s^{-1}), it would place the critical transition below, which fits with observations. This estimate is valid only for granitic magmas.

High strain rate behaviour

High strain rates result from a high stress level applied to the PMR. It may result from tectonic forces or relatively fast motion of PMR. Magma emplacement is the case, since the rate of emplacement, followed by crystallisation, is about one order of magnitude faster than tectonic strain rate (Harris et al., 1998; Améglio & Vigneresse, 2000). Experiments demonstrate that magma emplacement develops at any strain (Koenders & Petford, 2000). Strain estimates recorded by a magma pulse being emplaced to build a granitic pluton provides the range of strain rate recorded by a flowing magma by comparison with the time of intrusion. A sequence of a crystallising granitic magma, about 5 km in thickness, laterally displaced by about 15 km results in a shear value of about 3. Both distances correspond to those observed in granitic plutons (Vigneresse, 1995). If developing during the intrusion time, that is about 10^2 - 10^4 years, resulting in a strain rate of 10^{-9} to 10^{-11} s^{-1} . Faster strain rates are

certainly recorded for smaller displacement of a much thinner unit of magma, that would be emplaced in a considerably smaller time before it crystallises.

Owing to the high strain rate recorded by a flowing magma, the viscosity contrast is the smallest between the melt and its matrix (Fig. 5). Because of this small contrast, each phase has similar time to relax, leading to a bulk response of the PMR, the apparent viscosity of which is an average viscosity, depending on the respective amount of each phases.

By evidence, the strain recorded by a crystallising magma while settling, appears homogeneous within a granitic pluton, at least at the scale of the emplacement and motion of one specific petrographic facies (Bouchez, 1997). Identically, when a migmatite body is submitted to external tectonic forces, the internal response, as folds, are concordant with those of the surrounding (Brun & Martin, 1978). In such situation, averaging properties (Handy, 1994; Rosenberg & Handy, 2001) may perfectly apply to estimate a bulk rheology for a two-phase material.

Low strain rate response

At low strain rate, the response of a PMR varies according to the boundary conditions of the system. It depends on whether deformation develops under common stress, common stress rate or common percentage of each phase (Fig. 4). Three types of response with specific boundary conditions develop. Depending on the transition from solid to liquid or from liquid to solid, deformation may develop under common stress or common strain (see discussion). If the system is closed, the amount of each phase is consequently fixed, avoiding large scale motion of one phase.

Response with a fixed amount of each phase

It is the simplest case of stress-strain rate relationship, and corresponds to a closed system. The volume of each phase is conserved, and strain is limited within the considered volume. A cyclic path develops in the ($\dot{\epsilon}$ - η - Φ) diagram along a fixed Φ value (Fig. 6). It first follows the liquid-like surface, jumping suddenly to the solid-like surface. They manifest by a decrease of the strain rate, i.e. a decrease of the viscosity of the solid-like phase. For instance, at .60 crystals, the decrease in strain rate goes to about 10^{-9} s^{-1} , at which point the strain rate is the equivalent to that of the matrix. A jump to the solid-like behaviour allows return to the initial condition. The proposed path (Fig. 6) points to two problems which are the closed system and the decrease/increase in strain rate.

A closed system is unlikely to develop at the onset of crystallisation in a magma because melt mobility. However, at the end of the crystallisation, when the proportion of the solid phase is high, the possibility for the melt to move decreases drastically. It can lead to small scale closed systems (Fig. 6). The condition of varying the strain rate for the solid phase may be compared to pressure dissolution, that would lead to enhanced stress gradient in between touching crystals by dissolving one crystal to the detriment of the other (Grinfeld, 1993). Such observations have been described on analogue crystal mushes (Means & Park, 1994), leading to contact melting (Park & Means, 1996). Natural examples of contact melting show the growth of one plagioclase crystal to the detriment of another in a gabbro (Nicolas & Ildefonse, 1996). In both cases of contact melting, evidence exist that crystals were compacted with only a few residual liquid phase. In the case of the Oman gabbros, strain rate estimates are in the range 10^{-12} s^{-1} for crystals about 1 mm long, whereas melt fraction is less than 20 % (Nicolas & Ildefonse, 1996).

Response under a common stress

Under a common applied stress, the equivalent stress-strain rate diagram presents strain softening, which corresponds to the change from the solid-like curve to the liquid-like curve in a (σ - $\dot{\epsilon}$) diagram. Both curves have different viscosity, i.e. different slope. The transition between them, or strain softening, is unstable (Bazant, 1988; Goddard, 2002) and localises the strain (Fig. 7). Owing to the negative slope of the stress-strain rate curve, the weaker region localises strain. Shear bands develop with specific orientations with regard to

the major stress component orientation (Vermeer & De Borst, 1984). A complete theoretical analysis of shear band formation has been issued using a tensor notation (Rudnicki & Rice, 1975), or using crystal dislocation mechanisms (Estrin & Kubin, 1995). The constitutive tensor (\mathbf{L}) is expressed as a function of the elastic (Young and Poisson coefficients) and plastic (friction and dilatancy angles) parameters. The change of the sign of the eigenvalues of the tensor \mathbf{nLn} corresponds to shear bands development with direction (\mathbf{n}). It is equivalent to compute the quantity (Rudnicki & Rice, 1975; Bazant, 1988).

In a PMR, shear bands develop when an ambient stress applies, leading to strain partitioning in the weak phase (Vigneresse & Tikoff, 1999). In migmatites, shear bands are commonly observed, that collect melt. In a crystallising magma, shear zones develop, that disrupt the original fabric (Guineberteau et al., 1987; Pons et al., 1995; Smith, 1998). They are local shear zones developing in a magmatic stage and have been described as proto-faults (Brun et al., 1990). Proto-faults have been described, but no specific data on their respective orientation to the major stress component has been issued. Estimates of strain rate linked to such shear bands provides low values (10^{-14} s^{-1}) for tectonic loading (Koenders & Petford, 2000).

Response under a common strain rate

Instability develops under a common strain rate more easily when the load is transferred from the weak to the solid phase (Fig. 8). It results from the lower slope of the weak phase compared to the strong one in a $(\sigma - \dot{\epsilon})$ diagram. To keep continuity between both phases, the strained material must segregate leading to particle banding. Phase segregation has been described in flowing liquid crystals and in rigid rod-like particles in a shear flow (Olmsted & Golbart, 1992). Weakly aligned particles with the general flow motion is commonly the rule. But, two types of pattern are observed: onion-like and roll-like, that develop parallel to the shear gradient or to the vorticity, in case of a Couette flow, a rotational shear. They reflect segregation under a common stress or a common strain rate respectively (Olmsted, 1999).

Phase separation in a PMR corresponds to small scale features such as schlieren, banding or even melt rich segregation (Barrière, 1981; Clarke & Clarke, 1998; McCaffrey et al., 1999; Weinberg et al., 2001; Clarke et al., 2002). Schlieren are commonly interpreted as the final disruption of a mafic magma into a felsic magma. Segregation banding is associated with crystal sorting by size. Biotite and fine grain magma segregate on each rim, whereas coarse grained plagioclase and alkali feldspars are more abundant at the centre of the flow. If gravity driven forces were acting, it would not explain the observed symmetry of sorting. Bandings found in a vertical position rule out the action of gravity forces. In migmatites, banding manifests by the segregation of the residual biotite on each side of the flowing leucosome.

Discontinuous motion of both phases

The preceding behaviours reflect the type of response of a two-phase material. The cusp shape has also implications on the way the material responds. Because of the alternating jumps from one rheology to the other, the two-phase material responds discontinuously in time to stress. It alternatively adopts the strong and the weak phase response, with strain partitioning alternatively among the two phases. The situation is similar to stick-slip motion in dry (Marone, 1998) or wet friction experiments (Géminard et al., 1999).

Discontinuous extraction of the weakest phase, i.e. melt, in PMR has already been observed (Barraud et al., 2001; Vigneresse & Burg, 2001). It also manifests in mud volcanoes (Dimitrov, 2002), in which the discontinuous release of fluids is controlled by the advancing front of the overlying pressure (Pérez-Belzuz et al., 1997; Murton & Briggs, 2003).

An application for PMR relates to the old egg and chicken-like problem of the interaction between faults and plutons (Paterson & Schmidt, 1999). Considering the two-phase rheology of a fault zone filled with magma, the continuous jumps from one rheology to the other predict alternate motions of the fault and of the magma. Penetrative deformation in the magma and motion along the fault are observed in many places. Alternate strain

partitioning in both phases rules out the question on whether which of the pluton or the fault controls the system, both being required.

Implications for other two-phase materials

PMR represents a specific case of two-phase material. It manifests by a Newtonian melt and a power law matrix. It has been taken as an example because of many available situations for which an explanation could be suggested by two-phase rheology. Many other materials mix two contrasted types of rheology. They could manifest similar types of response to stress.

Diffusion/dislocation creep

Plastic deformation commonly develops through motion of atoms or defects within crystal aggregates. Diffusion or dislocation creep (Nicolas & Poirier, 1976) are such mechanisms. However, dislocation creep also involves other mechanisms, depending on the nature of the dislocations (single dislocation or dislocation loops), or on the nature of the motion, glide or climb controlled. They are described in solid state physics (Christian & Vitek, 1970; Allnatt & Lidiard, 1987, Green, 1998). However, they are not specific to moderate stress commonly registered in rocks (Fig. 1).

Strain rate is linear with stress during diffusion creep, whereas it responds to the cube of stress during dislocation creep (Eqs. 4-5). The energy involved when combining the two types of rheology writes like for PMR. An equivalent viscosity can be computed. The condition for equilibrium (Eq. 11) thus also writes as a cubic equation, implying a cusp shape. Low strain rate effects should manifest by different responses (and perhaps microstructures) on whether they develop at constant strain rate, constant stress or constant volume.

Acoustic wave propagation

Acoustic waves propagation in poro-elastic media is a special case of alternate stress cycles. The general form of the equation of displacement (u) under shear writes, as a 1D formulation for simplicity, with ρ the density:

$$\rho \frac{d^2u}{dt^2} - h \frac{du^2}{dy^2} = 0 \quad (12)$$

The second term incorporates the variation of stress with strain ($h = d\sigma/d\varepsilon$), the local tangent of the stress-strain curve. For strain hardening this term is positive and the equation is hyperbolic. Solutions write as complex exponential functions, i.e. usual trigonometric functions. They describe the propagation of an acoustic wave with velocity

$$c = (h / \rho)^{1/2} \quad (13)$$

Loss of hyperbolicity occurs when one eigenvalue becomes null, or changes its sign. Such situation develops with strain softening ($h < 0$). The system becomes elliptic, developing instabilities (Schaeffer, 1992; Bazant, 1988). Instability develops owing to a bifurcation jump. The situation mimics and exemplifies the formation of shear bands above introduced (Estrin & Kubin, 1995; Bésuelle, 2001).

Soil liquefaction

The rheology of a saturated sand pertains to that of two-phase materials, but under elastic conditions. Sand rheology is controlled by the shear modulus that relates stress to strain (Eq. 2). In usual conditions, G for sediments ranges 0.1 to 0.4 GPa. However, because of the large porosity of sediments, and particularly in sand, the relation is non-linear and G is found to increase proportionally to shear stress, or $\tau^{1/3}$. The power law relates to grain contacts and obeys the Hertz-Midlin relationship (Midlin, 1949; Bachrach et al., 2001). In

consequence, granular soils cannot be considered as purely elastic when submitted to stress for a long period of time. This non-linear behaviour bears similarity with viscosity that relates strain rate to stress. It corresponds to thixotropy (Barnes, 1997) and is due to irreversible gain in soil strength with time at a constant amount of water (Bauer, 1999).

Liquefaction takes place in saturated soils submitted to earthquakes under undrained conditions (Seed & Lee, 1966; Ishihara, 1993; Vaid & Sivathayalan, 2000). The material dilates or contracts with grain rearrangement under cyclic loading induced by the seismic waves. Under loading, packing normally increases. However, when compaction is prevented by the presence of a fluid within interstitial pores, pore pressure also increases. The latter also decreases the effective mean pressure. Liquefaction occurs when the effective pore pressure becomes null. The collapse of pore volume and simultaneous increase in pore pressure can lead to vanishing effective stress (Youd, 1972). Nevertheless, irreversible compaction is not likely since saturated sands in regions of frequent earthquakes manifest liquefaction at successive periods of time.

Importance of shear deformation in a two-phase material

Deformation occurs in pure, simple shear, or combination of both. Pure shear essentially consists in a convergent (or divergent) component, whereas simple shear manifests by a rotational component (Jaeger, 1969). A mixture of two viscous phases can accommodate the both types of deformation without problems. The situation gets more complex when one phase has a very low viscosity, i.e. low compressibility. In that case, the mixture, for instance rocks and water, presents contrasted reaction to deformation.

Deformation of water impregnated porous rocks manifests by alternate structures: dilatant bands, in which the fluid phase segregates and compaction bands in which the porosity is reduced (Antonellini & Aydin, 1994). During sediment compaction, the fluid phase is expelled (Mc Kenzie, 1984), giving place to alternate layers of pure compressional deformation, characterised by a reduced porosity (Olsson, 1999; Issen & Rudnicki, 2000). They manifest by tabular structures, being perpendicular to the major stress component (Antonellini & Aydin, 1994; Cashman & Cashman, 2000). When a shear component adds to simple compaction, shear bands develop, that are oriented $\pm 25^\circ$ to the major stress component. The dominant shear displacement gradient is accompanied by a porosity reduction, but also by the formation of shear zone, in which the fluid phase first accumulates, and is later expelled. They form Riedel conjugated shear zones (Schmocker et al., 2003). Compacting and dilating shear bands have been theoretically predicted (Bésuelle, 2001) and experimentally reproduced (Du Bernard et al., 2002).

They extend the concept of localized deformation, perpendicular to the maximum compressive stress, as a bifurcation due to strain softening (Rudnicki & Rice, 1975; Estrin & Kubin, 1995). Hence, the new approach takes into account the effect of the fluid phase. Sealing is a direct consequence of the development of alternate dilatant and compacted bands (Main et al., 2000).

A second consequence of shear resides in the long term effect of fault development. During simple shear, associated Riedel fractures are initiated, oblique to the plane of shear. They consequently weaken the material and constitute preferential planes for future shear motion when shear is incremented. The situation totally differs during pure compaction. The lack of rotational component in the strained material produces alternate bands that remain in same position regarding the stress pattern. Anisotropy develops within the two-phase material, but cannot be reactivated later on.

Re-addressing PMR, with in mind such effects, indicates that shear deformation is more effective to segregate melt than pure compaction. It was obvious after the former studies of pure compaction applied to magmas (Mc Kenzie, 1984). They resulted in unappropriate values of compaction length (about 100 m) that are actually not observed on the field. In contrast, when simple shear is added to compaction, the instabilities initiated by both types of deformation cooperate. Introducing values for magmas derived from the continental crust results in compaction length (cm to mm) comparable to values that are commonly

observed in migmatites (Rabinowicz & Vigneresse, 2003). A similar reduction operates for mantle derived rocks (Holtzman et al., 2003). In both case, the percentage of the fluid phase, through permeability, enters the equation of the compaction length.

Discussion

Experiments

The model of metastable rheology presented in the diagram ($\eta - \dot{\epsilon} - \Phi$) bears consequence on experimental studies. Few experiments document the rheological behaviour of PMR (Kohlstedt et al., 2000; Wirth & Franz, 2000), or water impregnated quartz (Schmocker et al., 2003). In the case of olivine, the melt percentage always remains low (< 12 %) owing to experimental difficulties. In case of dry rocks, creep experiments report coefficients for a power law that are intermediate ($1 < n < 3$) between those of diffusion and dislocation creep.

Experiments are conducted at high strain rate (10^{-6} to 10^{-4} s⁻¹). They document the effects predicted by the diagram in its stable range (Fig. 3). Results report a change from a power law behaviour ($n=3$) for high strain rates to linear behaviour ($n=1$) for lower strain rates. It is interpreted as a change from dislocation creep to a granular flow (Paterson, 1995).

The present results suggest to realise experiments in the region where instabilities develop, i.e. at much lower strain rate than actually. Because minerals may not be the better suited material, owing to their high viscosity, analogue materials should be tested. Simultaneously, theoretical and numerical models should address the development of instability.

Non-linear effects

The non-linear effects in the rheology of two-phase materials have important consequences on the bulk description of PMR. Non-linear aspects of the melting and crystallisation curves with temperature, varying viscosity during the phase transition, and a non-homogeneous spatial distribution of the two phases have been considered in a preceding paper (Burg & Vigneresse, 2002). They develop feedback loops that contribute to accelerate or to damp the rheological response during melting and crystallisation respectively.

A second aspect of non-linearity is the development of cyclic curves, i.e. hysteretic, or memory effect, during phase transitions. Hysteresis in stress/strain rate co-ordinates has long been known to develop in plastic material (Prandtl, 1928; Brokate & Sprekels, 1996). Viscosity variation according to the frequency of loading leads to several rules that are commonly used in soil studies. They are known as the Masing, or Cox-Mertz rules (Doraiswamy et al., 1991; Servais & Manson, 1999). Renewed attention to hysteresis related to wave propagation in sediments impregnated with fluids provided a new insight into non-linear elasticity (Guyer et al., 1995; Kadish et al., 1996; Tutuncu et al., 1998).

Hysteresis must be considered as a memory effect that delays return to the initial condition (null strain) when the applied force (stress) is removed. However, hysteresis does not necessarily implies that a full cycle in melting/crystallisation must be overcome to observe memory effects. It can also develop during partial cycling in ($\sigma - \dot{\epsilon}$) co-ordinates. This is presently the case when seismic waves pass through a PMR or water-saturated rock. Seismic waves consists of a the succession of compressional/extensional small scale deformation with a characteristic frequency. Body waves are commonly in the range 1 – 50 Hz (Scholz, 1990), which means that cycles of deformation alternate in the same range per second. When the material cannot relax, large strains add together, resulting in a large amplification of the strain. It is similar to shifting the hysteresis cycle toward increasing strain by a spiral motion. Accumulating and adding strain may result in strain value that seriously overcome the possibility of the material to deform within a single cycle of loading (Fig. 10). Amplification of strain by spiral cycles is known in saturated sediments as cyclic mobility (Manzari & Dafalias, 1997; Li & Ming, 2000). Superplastic deformation could be re-inspected within this new insight.

Development of instabilities

The proposed cusp instability describing the rheology of two-phase material (Fig. 3) pertains to the six elementary catastrophes (Thom, 1990). Catastrophe theory has already been applied to Earth Sciences (Henley, 1976; Petford, 1995). The present approach attempts quantifying the cusp shape, determining a critical point that could separate the homogeneous response from a region where instabilities develop.

The cusp corresponds to a third order surface, documented by the cubic equation (eq. 10). It implies two internal variables (η and Φ) and one control parameter ($\dot{\epsilon}$). When the strain rate is low enough, so that the respective relaxation times for each phase are too different to find accommodation, the only possibility left to the system is an abrupt transition from one state to the other. It corresponds to a change in sign of one eigenvalue of the motion equation. Such a process is often designed as a bifurcation in studies concerning instability (Needleman, 1979; Hunt, 1986).

A phase space in which bifurcations develop should be constructed documenting the mode of evolution of the instability. It is a diagram constructed from the assemblage of variables controlling the nature and type of the motion equation. In the case of planar strain, a combination of G/G' and a function that combine plastic hardening ($d\sigma/d\epsilon$) and G has been suggested (Needleman, 1979). It corresponds to an elastic-like material in which the plastic response remains low. It also delineates a parabolic domain for large hardening, i.e. for very stiff material. When the plastic component is high, then separation between hyperbolic and elliptic solutions partly depends on the ratio of the elastic shear module to the instantaneous shear module (G/G').

Conclusions

A global model suggests describing the rheology of a two-phase material according to strain rate. It is the first approach, still conceptual at present, that envisions a non binary transition from a weak rheology to a strong one. It focuses on the interaction between two end-members with a distinct and contrasted rheology. The model emphasises the role of a critical strain rate value that separates a region of bulk homogeneous response from a region in which instabilities develop. Under low strain rate, local effects manifest, that develop under constant stress, constant strain rate, or constant amount of each phase. Deformation while the amount of each phase is conserved leads to crystal melting. Deformation at a constant strain rate, which is the rule during crystallisation, leads to banded segregation of magma and schlieren formation. Deformation at constant stress results in proto-faults and shear bands.

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Figure captions

- Figure 1. Schematic map of plastic deformation. T_m is the melting temperature, whereas σ/G is the ratio of the stress to the shear modulus (about 5-50 GPa). The field of PMR is indicated by a grey shadow.
- Figure 2. Stress (σ) – strain rate ($\dot{\epsilon}$) in a log-log diagram for a crystallising magma. The Newtonian melt is taken at 10^6 Pa.s. The solid phase follows a power law ($n = 3$). Stress values range from 0.01 MPa to 100 MPa, whereas strain rates range 10^{-20} to 10^0 s $^{-1}$. Viscosity (η) values are indicated in grey. The zone corresponding to magma transport is indicated by a grey box. The zone where instabilities develop (see text) is hatched.
- Figure 3. Three-dimensional diagram ($\dot{\epsilon} - \eta - \Phi$) showing the cusp shape of viscosity for low strain rate values.
- Figure 4. Same diagram as Figure 3, but showing the high strain rate path (homogeneous deformation), and two paths at low strain rate under a common amount of phase and under a common strain rate value. They both show a cyclic path, determining hysteresis, or memory effect.
- Figure 5. Deformation at high strain rate, or high stress, as it occurs during tectonic activity. The deformation path depends on the average amount of solid, corresponding to an homogeneous, en masse, transport of the PMR.
- Figure 6. Deformation at constant amount of phase. It develops during the final stage of crystallisation, and acts as a closed system. Plagioclase crystals grow at the detriment of already formed crystals. Contact melting develops and crystals impinge altogether (inset redrawn from a photo of Oman gabbros, provided by B. Ildefonse (Nicolas & Ildefonse, 1996).
- Figure 7. Deformation under common stress at low strain rate, as it develops preferentially during crystallisation. Particles segregates, leading to banded features in the magma shown in inset.
- Figure 8. Deformation under common low strain rate. Overlapping of the two curves for the viscosity of the weak and strong phase induces sudden jumps from one rheology to the other. Deformation develops through cycles, inducing hysteresis, i.e. memory effects. In inset, shear localisation develops, leading to proto-faults.
- Figure 9. Differences between melting and crystallisation for the rheology of PMR. The transitional state from the respective end-members favours deformation under a common stress for melting since the system goes from the solid-like to the liquid-like rheology. In contrast, the opposite path that develops during magma crystallisation favours deformation under a common strain rate.
- Figure 10. Hysteresis during cyclic loading and unloading of a PMR. Repeated loading may bring the hysteresis curve (thin line) to unexpected large strain by spiraling cycles.