On the New-Stein Group and the Mass Dynamical Origin

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Abstract. We propose a model of "Fundamental Symmetries" leading to a dynamical origin of mass, based on the concept of "Historical Time" and described by a relativistic Schrödinger-type equation. In this framework, the matter spectrum is obtained by excitation, at the level of internal structure, of a fundamental system composed of two constituents, the relativistic free masses of which are null, with harmonic interaction and a possible null-energy vacuum (ground) state.

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1. Introduction and Motivations

The combination of the electroweak interactions theory, developed by Glashow, Salam and Weinberg [1] and based on the $SU(2)_W \times U(1)_Y$ group, where $SU(2)_W$ (resp $U(1)_Y$) is the weak isospin (resp hypercharge) symmetry, with the strong interactions theory, generated by the $SU(3)$-color gauge symmetry, successfully describes the phenomenology of these two types of interactions (see, for example, [2]), to such an extent that this model has been baptized the "Standard Model" of elementary particle physics. Moreover, this model plays an important role in the discussion of the primordial universe [3]. However, despite this undeniable success, this model presents weaknesses. The most important of the latter concerns the mass observable, observable which is, from our point of view, the most fundamental, since it is directly connected to energy, evolution and dynamics. In fact, no theory related to the Standard Model possesses a sound ground of mass generation

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[4]. All of them rely on Higgs's mechanism [5] to make the theory renormalizable and to explain why particles possess a mass. But, this mechanism, which has been artificially forged for the subsistence of the Standard Model, also presents weaknesses. One of the latter is the non-detection, in spite of considerable efforts for more than thirty years, of the particle necessary for its survival, namely the so-called (non-gauge) Higgs boson of spin zero ; this fact has led, as a matter of fact, to give it an important (lower) mass which would justify its non-observation at the scale of the presently available energies. Thus, if we wish to give a sound basis to the origin of mass, we should naturally adopt other approaches than those relating to the Standard Model. Everything indicates that this problem cannot be elucidated without simultaneously establishing an understanding of the particles internal structure and a consistent theory of the quantum relativistic dynamics. It is in this framework that our work should be integrated.

Our approach is based on master ideas formulated by a certain number of plysicists, inventors of the elementary particles physics and of modern cosmology ; these ideas will be briefly exposed below. In a report published posthumously by Touschek, Pauli comes back once more to the problem that "the fields of all particles are to be constructed from the minimum number of fields" [6]. As for Heisenberg, he suggested that it was more correct to speak of "matter spectrum" rather than "elementary particles spectrum" [7]-[8] (reference [8] concerns the text of Heisenberg's last lecture), and this, among other things, because the number of particles is not preserved in the interactions, due to the fact that mass may be transformed into energy and/or that a particle-antiparticle pair may be eventually created. Besides, for Heisenberg, the matter spectrum can only be apprehended if the subadjacent interactions are clarified and the first step must be the attempt to fromulate mathematically a natural law that defines the dynamics of matter. In other words, it is the matter dynamics which constitutes the central problem : "The dynamics must be taken seriously, and we should not be content with vaguely defined hypotheses that leave essential points open... The particle spectrum can be understood only if the underlying dynamics of matter is known" [8]. In this context, he suggests to replace the concept of a "Fundamental Particle" by the concept of a "Fundamental Symmetry". The fundamental symmetries define the underlying law which determines the spectrum of elementary particles. He considers as fundamental symmetries the external symmetries and the isospin symmetry. As a matter of fact, since its introduction by Heisenberg [9], the isospin symmetry, charateristic of strong interactions (and transforming, for example, the proton into neutron), has been incorporated, on the basis of a Pauli's idea [10], by Heisenberg [11] into his Nonlinear Spinor Theory of Elementary Particles. It has been extended, afterwards, to the weak interactions, by analogy, into a weak isospin (see, for example [12]) transforming, for example, a charged lepton into its neutrino. Heisenberg explicitly excludes the $SU(3)$ symmetry and its generalizations as fundamental symmetries, because they may be produced by the dynamics as approximate symmetries. In fact, the violation of the baryonic number is a necessary hypothesis in all unification models of leptons and quarks connected to the Standard Model [13]. This violation would explain the asymmetry between matter and antimatter in the universe [14]. Besides, the baryonic number non-conservation is always accompanied by leptonic number non-conservation [15]. Finally, Heisenberg asserts that this decisive change in concepts came about by Dirac's discovery of antimatter before concluding : "... I do not think that we need any further breakthrough to understand the elementary-or rather non- elementary-particles. We must only learn to work with this new and unfortunately rather abstract concept of the fundamental symmetries..." [7]. In the discussion following the article [7], Dirac adhered to the idea that the concept of "elementary particle" has no real meaning. Dirac also rejects the legitimacy of the renormalization procedure as an essential ingredient in the present standard theories and thinks it is a simple calculation trick, corresponding to an incomplete knowledge of the nature of interactions, which "...does not conform to the high standard of mathematical beauty that one would expect for a fundamental physical theory, and leads one to suspect that a drastic alteration of basic ideas is still needed" [16]. As for the first rigorous approach of the origin of mass, it was born with the inflationary models of the universe [17]. In these models, the scenario is radically different from that of the Big Bang Standard Model until about 10^{-34} second of the age of the universe. But, beyond 10^{-30} second, both scenarios coincide and all the Big Bang successes are preserved. Let us briefly clarify the *raison d'être* of these inflationary models. As successful as the Big Bang cosmology is, it suffers from diverse dilemmas among which we can quote the initial data and the extreme overproduction of superheavy magnetic monopoles which occurred early ($t \leq 10^{-34}$ second) in the history of the universe. It is when trying to solve this type of dilemma that Guth introduced his idea of inflation in his seminal paper [18]. Let us note that there is presently no standard model of inflation, just as, actually, there is no standard model for physics at these energies (typically $10^{15}GeV$). But all the inflationary models lead to the fact that all the matter of the universe burst out from almost nothing : this would be an *ex nihilo* creation as a consequence of a phase transition. A further advantage, and not the least, of these inflationary models, is that the evolution of the universe is nearly independent of the initial conditions which may be practically arbitrary, because the universe was, prior to the inflationary era, almost void of matter. This would allow us to avoid the difficulties linked to the initial singularity of the Big Bang Standard Model. The hypothesis of the creation of the universe from the absolute nothingness originated from Jordan's idea that the total energy of our universe is null [19], an idea which was afterwards resumed by Tryon [20] before being integrated within the inflationary theory, which made it plausible. Tryon suggested that the universe could originally have been only a spontaneous quantum fluctuation which would have developed from nothingness. Nothingness in the hypothesis of this *ex nihilo* creation could designate the universe at null total energy as Jordan puts it, or "quantum vacuum". One of our hypotheses is this creation of the *ex nihilo* matter to which we give a pure Lie group framework, and this from a null-energy ground state associated to a new relativistic covariant harmonic oscillator formalism (developed through a relativistic covariant method unifying the external and the internal spaces which avoids superfluous generators) where the treatment of time as a dynamical variable and the internal dynamics are explicit ; mass (energy) being created by the internal excitations.

The choice of our formalism has been motivated by the fact that the (non relativistic) harmonic oscillator has frequently served as the best laboratory for theoretical physicits and has served as the first concrete solution to many new physical theories [21], [22]. Using harmonic oscillators one may construct models at all levels of complexity : from a single classical oscillator in one dimension to relativistic quantum fields, passing by the statistical mechanics, the theories of specific heat, superconductivity, coherent light etc. Moreover, several works were carried out concerning the construction of a non-trivial relativistic covariant harmonic oscillator wave function for studying, essentially, hadronic stuctures and interactions (see, among others, [22] and [23]). However, several criticisms can be leveled towards all these attempts ; the most important of which bear on the violation of one of the basic canons of the relativity theory which stipulates a perfect symmetry between space and time. In order to satisfy this fundamental symmetry principle, we shall be compelled to distinguish, in our treatment, kinematical time as observable of relativistic space-time from dynamical time, an independant parameter of this space-time. This dynamical time is not an observable : It is the Hamiltonian itself which is the true unkwown to determine and which governs it as a parameter describing evolution. In this context, the time-evolution constitutes a one parameter group relativistically compatible with the considered fundamental symmetries. We shall determine the structure of the various relativistic covariant Hamiltonians from a very general principle. One of these Hamiltonians will lead to the new relativistic covariant harmonic oscillator formalism which we referred to above. Furthermore, the fact that this model permits defining a denumerable infinity of fermions (or of bosons) of integer or half-integer isospin (in an irreducible representation) and that its Hamiltonian presents a symmetry between the "internal moments" of its two constituents, which are "canonically conjugated" , relates our model to Born's hypotheses concerning his "Theory of Reciprocity" [24], a theory which was approved by Pauli as soon as it appeared and which Born considers as being "the unique means to unify the undulatory mechanics and relativity". It is worth mentioning here that our Hamiltonian looks as being a generalization of Born's fundamental invariant. Furthermore, the possibility that there exist, in this framework, an infinite number of different hadrons (and leptons) does not appear presently unrealistic. But in the framework of the Standard Model, such a situation is intenable, since it demands an infinite number of different quarks.

Some years ago [25], we proposed a model of elementary particles based on a new concept of internal structure and a relativistically covariant method of unifying the external and internal structures. This model led to an exact mass formula for hadrons, compatible with the experimental results. Prior to that [26], we had introduced a new symmetry group of relativistic quantum kinematics (defining the covariance of the moment-energy and position observables) which was baptized [27] "Einstein Group", and which has led, since then, to diverse applications (for a bibliography concerning this subject, see Chapter I of [28]). In [28], the non-relativistic equivalent of Einstein Group was called "Newton Group".

In this paper, we are going to adopt the concept of internal structure and that of unification, as well as the construction procedure of the unifying group, referred to above, by considering the internal structure, reduced to the isospin as suggested by Heisenbeg, described, as in [25], by the extended space Newton group, and the external structure described by Einstein group. Consequently, the obtained unifying group will be called "New-Stein Group".

Thus, in our model, the internal structure is supposed to be, essentially, non-relativistic. As a matter of fact, there is no reason, either theoretical or experimental, why the conditions verified by the internal and external structures should be of the same nature. Furthermore, the question of the validity of the usual space-time concept at the scale of elementary particles has been put forward very often [29], [30].

By adopting an evolution principle (unifying dynamics with fundamental symmetry defined by a Lie group S), generalizing the one introduced in [31], and the concept of "Historical Time" defined in [32] (which is distinct from the geometrical time of the relativistic space-time ; this geometrical time having, a priori, no relation with evolution), we introduce, in the context of the New-Stein group symmetry, a dynamical principle of internal evolution which leads us to a relativistic Schrödinger-type equation. It is a model of mass generation which gives a dynamical explanation of its origin. This is done from a composite fundamental system, having two constituents with a free mass which may be null and a harmonic interaction, the internal excitations of which lead to the creation of the matter spectrum. Besides, the fact that this model leads to a unitary theory of mass (energy) creation, both for the massive and the massless particles (and this as composite objects), can lead to a new approach of the unification of the ultimate constituents of matter, in relation with the models of leptons, quarks and gauge hosons, as composite objects [33], as well as with the supersymmetrical models [34].

The idea of considering the massless particles as being composite particles was introduced, from the outset of quantum mechanics, by Louis de Broglie [35], in the framework of his "Photon Undulatory Mechanics" where he considers the photon as being a complex particle consisting of two constituents of 1/2 spin. This may be regarded as an early forerunner of the supersymmetrical theories. This idea had no immediate follow-up , but it has regained intense activity since the beginning of the eighties [36] ; it has led to various models such as that of the "Singleton Theory" [37]. As for the central place reserved by our model to the massless particles (or constituents), it is motivated, on the one hand, by the fact that each fundamental interaction of Nature has its privileged family of massless particles (the photon for the electromagnetic, the neutrinos for the weak, the gluons for the strong and the graviton for the gravitational) and, on the other hand, by the fact that, in the inflationary models, the mass generation is due to a phase transition and that, historically, prior to the first phase transition, matter was in its most symmetrical state and almost all elementary particles were massless [3], [17].

2. Definition and Structural Properties of New-Stein Group

Let $E = SL(2, \mathbb{C}) \bullet (\mathbb{R}^4 \times \mathbb{R}'^4)$ the Einstein group [26], inhomogenization of $SL(2, \mathbb{C})$ relatively to its representation $D(1/2, 1/2) \oplus D(1/2, 1/2)$, where the symbols $\bullet$ and $\times$ designate, respectively, the semi-direct product and the direct product of Lie groups. The Heisenberg group (resp the Newton group [28]) will be denoted by H_n (resp F_n) and the (universal covering group of the) extended space Newton group by F. We have $F = SU(2) \bullet H_3$, where the semi-direct product is defined by the representation $\Delta \oplus \Delta \oplus \varepsilon$ of $SU(2), \Delta$ designating the fundamental representation and ε the trivial one. In what follows, Greek indexes run from 1 to 4, Roman indexes from 1 to 3, summation convention for a repeated index (in two distinct positions) is implied and $g_{\alpha\beta}$ is the usual metric tensor ($g_{ij} = \delta_{ij}, g_{4i} = 0$ and $g_{44} = -1$). Except when clearly stated, the Lie groups will be designated by $A, B, \cdots$ and the corresponding Lie algebras by $\mathcal{A}, \mathcal{B} \cdots \mathcal{U}(\mathcal{A})$ designates the enveloping algebra of $\mathcal{A}$. Let $(M_{\mu\nu}, P_\mu, P'_\mu)$ [resp $(p_i, q_j, I), (I_{ij})$] the canonical basis of $\mathcal{E}$ (resp $\mathcal{H}_3, \mathcal{SU}(2)$) and $\mathcal{G}$ the subalgebra of the tensor product $\mathcal{U}(\mathcal{E}) \otimes \mathcal{U}(\mathcal{F})$ generated by :

$$L_{\mu\nu} = M_{\mu\nu} \otimes I; \quad T_\mu = P_\mu \otimes I; \quad T'_\mu = P'_\mu \otimes I; \quad Q_{j\mu} = P_\mu \otimes q_j;$$

$$A_{j\mu} = P_\mu \otimes p_j; \quad C_{\mu\nu} = P_\mu P_\nu \otimes I; \quad J_{ij} = 1 \otimes I_{ij}.$$

Let $\mathcal{N}_3$ the subalgebra of $\mathcal{G}$ generated by $(A_{j\mu}, Q_{i\nu}, C_{\mu\nu})$. If we denote by $\oplus$ the direct sum of Lie algebras and by $D(j, j')$ (resp $D(j)$) the irreducible representation of $\mathcal{SL}(2, \mathbb{C})$ whose dimension is $(2j + 1)(2j' + 1)$ (resp of weight j of $\mathcal{SU}(2)$), then, we have the following proposition.

Proposition 2.1. *$\mathcal{G}$ is semi-direct sum of $\mathcal{SL}(2, \mathbb{C}) \oplus \mathcal{SU}(2)$ by the nilpotent ideal $\mathbb{R}^4 \oplus \mathbb{R}'^4 \oplus \mathcal{N}_3$ relatively to the representation* : $\{D(1/2, 1/2) \otimes D(0)\} \oplus \{D(1/2, 1/2) \otimes D(0)\} \oplus \{D(1/2, 1/2) \otimes D(1)\} \oplus \{D(1/2, 1/2) \otimes D(1)\} \oplus \{[D(1, 1) \oplus D(0, 0)] \otimes D(0)\}$.

Proof : This is a consequence of Proposition 2.1 of [25] and of the commutation relations of $\mathcal{G}$, the non-null of which are yielded by :

$$[L_{\mu\nu}, L_{\rho\sigma}] = -g_{\mu\rho}L_{\nu\sigma} - g_{\nu\sigma}L_{\mu\rho} + g_{\mu\sigma}L_{\nu\rho} + g_{\nu\rho}L_{\mu\sigma}; [L_{\mu\nu}, X_\rho] = g_{\nu\rho}X_\mu - g_{\mu\rho}X_\nu;$$

$$[A_{i\mu}, Q_{j\nu}] = \delta_{ij}C_{\mu\nu}; [L_{\mu\nu}, C_{\rho\sigma}] = -g_{\mu\rho}C_{\nu\sigma} - g_{\mu\sigma}C_{\nu\rho} + g_{\nu\sigma}C_{\mu\rho} + g_{\nu\rho}C_{\mu\sigma};$$

$$[J_{ij}, J_{kl}] = -\delta_{ik}J_{jl} - \delta_{jl}J_{ik} + \delta_{il}J_{jk} + \delta_{jk}J_{il}; [J_{ij}, Y_k] = \delta_{jk}Y_i - \delta_{ik}Y_j;$$

where X_ρ (resp Y_k) designates $T_\rho, T'_\rho, A_{i\rho}$ or $Q_{i\rho}$ for i fixed (resp $A_{k\rho}$, $Q_{k\rho}$ for ρ fixed).

Let $\mathbb{R}_c^{10}$ (resp $\mathbb{R}_a^{12}$, $\mathbb{R}_q^{12}$) the group generated by $(C_{\mu\nu})$ (resp $(A_{j\mu}), (Q_{j\mu})$). Now, we shall denote by t(resp $t', c, a, q, \wedge, R$) the generic element of $\mathbb{R}^4$(resp $\mathbb{R}'^4$, $\mathbb{R}_c^{10}$, $\mathbb{R}_a^{12}$, $\mathbb{R}_q^{12}, SL(2, \mathbb{C}), SU(2)$), by $\theta^{\mu\nu}$ the function equal to 1/2 if $\mu = \nu$ and 1 if not and by $g = (t, t', c, a, q, \wedge, R)$ the generic element of the connected and simply connected group G whose Lie algebra is $\mathcal{G}$.In what follows, we note $\wedge$ (resp$R, S(\wedge)$) instead of $D(1/2, 1/2) \wedge$ (resp $D(1)R, [D(1, 1) \oplus D(0, 0)](\wedge) \otimes D(0)R$). .

Proposition 2.2. *The group law of G is given by $g_1g_2 = (t_1 + \wedge_1 t_2, t'_1 + \wedge_1 t'_2, c_1 + S(\wedge_1)c_2 + \beta(a_1, \wedge_1 \otimes R_1 q_2), a_1 + \wedge_1 \otimes R_1 a_2, q_1 + \wedge_1 \otimes R_1 q_2, \wedge_1\wedge_2, R_1R_2)$, where β is defined by $\beta^{\mu\nu}(a_1, q_2) = \theta^{\mu\nu}\delta_{ij}(a_1^{i\mu}q_2^{j\nu} + a_1^{i\nu}q_2^{j\mu})$.*

Proof : Results form Proposition 2.1. and from the group law of N_3 [25] whose Lie algebra is $\mathcal{N}_3$.

Definition 2.1. *Group G is called the New-Stein Group.*

3. On a Class of Irreducible Unitary Representations of the New-Stein Group

Proposition 2.2. shows that G admits the decomposition $H \bullet K$ where $H = \mathbb{R}^4 \times \mathbb{R}'^4 \times \mathbb{R}_c^{10} \times \mathbb{R}_a^{12}$ and $K = \mathbb{R}_q^{12} \bullet (SL(2, \mathbb{C}) \times SU(2))$. Consequently, the most natural method of determining its strongly continuous irreducible unitary representaions (IUR) is the method of induced representations [38] (in stages), provided that it turns out to have the required properties. Let $\hat{H}$ be the dual of H, then $\hat{H} = \mathbb{R}_p^4 \times \mathbb{R}_\xi^4 \times \mathbb{R}_D^{10} \times \mathbb{R}_b^{12}$. As in [25], in order to determine the action of K on $\hat{H}$, we write D and b as a matrix. Thus, the element (p, ξ, D, b) of $\hat{H}$ will be represented by (p, ξ, DG, B), where $D = (D^{\mu\nu})$, with $D^{\mu\nu} = D^{\nu\mu}$, $G = (g_{\alpha\beta})$ and $B = (b^{i\mu})_{1\le i\le 3; 1\le\mu\le 4} \in M(4, 3, \mathbb{R})$. Thus, the action of $k = (0,0,0,0,q,\wedge,R) \in K$ on $\hat{h} = (p,\xi,DG,B) \in \hat{H}$ is defined by :

$$k(\hat{h}) = (\wedge p, \wedge\xi, \wedge DG^{-1}\wedge, \wedge BR^{-1}, \wedge DG^{-1} \wedge Q) = (p', \xi', D'G, B'); \qquad (1)$$

where Q denotes the matrix $(q^{i\mu})_{1\leq i\leq 3, 1\leq\mu\leq 4} \in M(4,3,\mathbb{R})$.

We note that the action of K on $\mathbb{R}^4 \times \mathbb{R}'^4$ is reduced to the action of $SL(2,\ \mathbb{C})$.

Remark 3.1. In view of the physical interpretations we wish to draw from our model, we shall confine ourselves in this article to the determination of IUR of G for which : $(g_{\nu\mu}p^\mu p^\nu = 0, p^4 > 0)$ and $(g_{\nu\mu}\xi^\mu\xi^\nu = -m_0^2, m_0 > 0)$. Thus, the IUR searched for are those which are associated to the orbits contained in $\Omega_+^0 \times \Omega_+^{m_0^2}$.

3.1. Determination of the stabilizer

It results from what preceded that the orbits looked for are written :

$$(S_\lambda \times \{\lambda\}, \Omega_+^{m_0^2}), \text{ with } m_0 > 0 \text{ and } \lambda > 0; \qquad (2)$$

where S_λ is the sphere having a radius λ in the hyperplan $p^4 = \lambda$.

They are generated by $(\vec{\eta}, \lambda, (0,0,0,m_0))$. For all $\vec{\eta} \in S_\lambda$, the stabilizer of point $(\overset{o}{p}, \overset{o}{\xi}) = ((0,0,\lambda,\lambda),(0,0,0,m_0))$ is the subgroup $\mathbb{R}_q^{12} \bullet (U(1) \times SU(2))$ the action of which on $\mathbb{R}_D^{10}$ is given, according to (1), by $(q,\wedge,R)(DG) = \wedge DG\wedge^{-1}$. Let the $\overset{o}{D}G$ matrix, all the elements of which are null except $(\overset{o}{D}G)^{44} = -\alpha < 0$. Then, we have a point-orbit, since we have $\wedge\overset{o}{D}G\wedge^{-1} = \overset{o}{D}G, \forall \wedge \in SU(2)$. The orbit deduced from the action of K on $\mathbb{R}_p^4 \times \mathbb{R}_\xi^4 \times \mathbb{R}_D^{10}$ is thus :

$$(S_\lambda \times \{\lambda\}, \Omega_+^{m_0^2}, \{\overset{o}{D}G\}), \quad \lambda > 0; \qquad (3)$$

it is generated by the point $(\overset{o}{p}, \overset{o}{\xi}, \overset{o}{D}G)$ stabilized by the subgroup $\mathbb{R}_q^{12} \bullet (U(1) \times SU(2))$. Let us study now the action of this subgroup on $\mathbb{R}_b^{12}$. According to (1) we have : $\forall\ (q,\wedge,R) \in \mathbb{R}_q^{12} \bullet ((U(1) \times SU(2)), (q,\wedge,R)(\overset{o}{p}, \overset{o}{\xi}, \overset{o}{D}G, B) = (\overset{o}{p}, \overset{o}{\xi}, \overset{o}{D}G, B')$ with $B' = \wedge BR^{-1} + \wedge\overset{o}{D}G^{-1} \wedge Q$. Let us write $B = \begin{pmatrix} \mathbf{b} \\ {}^t\vec{\beta} \end{pmatrix}$, with $\mathbf{b} = (b^{i\mu})_{i\leq 3, \mu\leq 3}$, where $\vec{\beta}$ is the column vector $(b^{i4})_{i\leq 3}$ and ${}^t\vec{\beta}$ the transposed of $\vec{\beta}$; hence, knowing that $\wedge \in U(1) \subset SU(2)$, then of the form

$\begin{pmatrix} (\wedge(\varphi)) & 0 \\ 0 & 1 \end{pmatrix}$, where $\wedge(\varphi)$ is the canonical matrix associated to the rotation of angle φ around the third space axis, we have : $B' = \begin{pmatrix} \mathbf{b}' \\ {}^t\vec{\beta}' \end{pmatrix} = \begin{pmatrix} \wedge(\varphi)\mathbf{b} \\ {}^t\vec{\beta} \end{pmatrix} R^{-1} + \overset{o}{D}G \begin{pmatrix} \mathbf{q} \\ {}^t\vec{q} \end{pmatrix}$, where we have written, like $B, Q = \begin{pmatrix} \mathbf{q} \\ {}^t\vec{q} \end{pmatrix}$; hence $B' = \begin{pmatrix} \wedge(\varphi)\mathbf{b}R^{-1} \\ {}^t(R\vec{\beta} - \alpha\vec{q}) \end{pmatrix}$. The orbit searched for is generated by $\overset{o}{B} = \begin{pmatrix} 0 \\ {}^t\vec{\beta} \end{pmatrix}$. It is then defined by $\{B' = \begin{pmatrix} 0 \\ {}^t(R\vec{\beta} - \alpha\vec{q}) \end{pmatrix} / R \in SU(2), \vec{q} \in \mathbb{R}^3\}$, that is $\left\{ \begin{pmatrix} 0 \\ {}^t\vec{\beta} \end{pmatrix} / \vec{\beta} \in \mathbb{R}^3 \right\}$. The orbit Ω to consider is generated by $\hat{h}_0 = (\overset{o}{p}, \overset{o}{\xi}, \overset{o}{D}G, 0)$ and finally written :

$$\Omega = (S_\lambda \times \{\lambda\}, \Omega_+^{m_0^2}, \{\overset{o}{D}G\}, \left\{ \begin{pmatrix} 0 \\ {}^t\vec{\beta} \end{pmatrix} / \vec{\beta} \in \mathbb{R}^3 \right\}). \tag{4}$$

An element $(q, \wedge, R) \in \mathbb{R}_q^{12} \bullet (U(1) \times SU(2))$, where q is identified with matrix $Q = \begin{pmatrix} \mathbf{q} \\ {}^t\vec{q} \end{pmatrix}$, stabilizes $\overset{o}{B} = 0$ if and only if we have $\overset{o}{D}GQ = 0$. But $\overset{o}{D}GQ = \begin{pmatrix} 0 \\ -\alpha{}^t\vec{q} \end{pmatrix}$ and $\alpha \neq 0$; hence the stabilizer of $\hat{h}_0$ is :

$$K_0 = \mathbb{R}_q^9 \bullet (U(1) \times SU(2)); \tag{5}$$

where $\mathbb{R}_q^9 = \{(q^{i\mu}) \in \mathbb{R}_q^{12}$ so as $q^{i4} = 0\}$.

3.2. Construction of section $\Gamma_{\hat{h}}$ and calculation of $\Gamma_{\hat{h}}^{-1} k_0 \Gamma_{k_0^{-1}(\hat{h})}$ for $\hat{h} \in \Omega$ and $k_0 \in K$

Each point of orbit Ω is written : $\hat{h} = (q, \wedge, R)\hat{h}_0 = (\wedge\overset{o}{p}, \wedge\overset{o}{\xi}, \wedge\overset{o}{D}G^{-1}\wedge, \wedge\overset{o}{D}G \wedge^{-1} Q)$. If we pose $Y = \wedge^{-1}Q = \begin{pmatrix} \tilde{y} \\ {}^t\vec{y} \end{pmatrix}$, then $\overset{o}{D}GY = \begin{pmatrix} 0 \\ -\alpha{}^t\vec{y} \end{pmatrix}$ and ${}^t\vec{y}$ is invariant by the subgroup K_0 (that is to say ${}^t\vec{y}$ is the same if we replace $(q, \wedge, R)$ by $(q, \wedge, R)k_0$, with $k_0 \in K_0$). If we denote for $\xi \in \Omega_+^{m_0^2}$ and $\vec{\eta} \in S_\lambda$, A_ξ and $R_{\vec{\eta}}$ the habitual sections defined in the canonical formalism, then each point of orbit Ω is written : $\hat{h} = (A_\xi R_{\vec{\eta}}\overset{o}{p}, A_\xi\overset{o}{\xi}, A_\xi\overset{o}{D}GA_\xi^{-1}, A_\xi \begin{pmatrix} 0 \\ -\alpha{}^t\vec{y} \end{pmatrix})$. As Ω is a variety of dimension 8 (isomorphic with K/K_0), a section associated to our orbit is given by $\Gamma_{\hat{h}} = (A_\xi \begin{pmatrix} 0 \\ {}^t\vec{y} \end{pmatrix}, A_\xi R_{\vec{\eta}}, 1)$, where 1 is the neutral element of $SU(2)$. It is to be noticed that orbit Ω is parametered by $(\xi, \vec{\eta}, \vec{y}) \in \Omega_+^{m_0^2} \times S_\lambda \times \mathbb{R}^3$. Hence : $\Gamma_{\hat{h}}^{-1} = (-\begin{pmatrix} 0 \\ {}^t\vec{y} \end{pmatrix}, R_{\vec{\eta}}^{-1} A_\xi^{-1}, 1)$.

Let $k_0 = (Q_0, \wedge_0, R_0) \in K$, with matrix writing of the component of k_0 belonging to $\mathbb{R}_q^{12}$, then $k_0^{-1} = (-\wedge_0^{-1} \; Q_0 R_0, \wedge_0^{-1}, R_0^{-1})$ and we obtain : $k_0^{-1}(\hat{h}) = (\wedge_0^{-1} A_\xi R_{\vec{\eta}} \overset{o}{p}, \wedge_0^{-1} A_\xi \overset{o}{\xi}, \wedge_0^{-1} A_\xi \overset{o}{D} G A_\xi^{-1} \wedge_0, \wedge_0^{-1} A_\xi \begin{pmatrix} 0 \\ -\alpha^t \vec{y} \end{pmatrix} R_0$ $- (\wedge_0^{-1} A_\xi \overset{o}{D} G A_\xi^{-1} \wedge_0)(\wedge_0^{-1} Q_0 R_0))$. But, $A_\xi^{-1} \wedge_0 A_{\wedge_0^{-1}\xi} \in SU(2)$ (since it stabilizes $\overset{o}{\xi}$), then it stabilizes $\overset{o}{D} G$; thus $k_0^{-1}(\hat{h}) = (A_{\wedge_0^{-1}\xi} R_{\vec{\eta}'} \overset{o}{p}$ ($\vec{\eta}'$ to determine), $A_{\wedge_0^{-1}\xi} \overset{o}{\xi}$, $A_{\wedge_0^{-1}\xi} \overset{o}{D} G A_{\wedge_0^{-1}\xi}^{-1}$, $A_{\wedge_0^{-1}\xi} (\begin{pmatrix} 0 \\ -\alpha^t \vec{y} \end{pmatrix} - \overset{o}{D} G A_\xi^{-1} Q_0) R_0 = d)$. Let $A_\xi^{-1} Q_0 = A_\xi^{-1}(q_0^1, q_0^2, q_0^3) = \begin{pmatrix} T \\ {}^t\vec{\tau} \end{pmatrix}$, where $\vec{\tau} \in \mathbb{R}^3 . {}^t\vec{\tau} = \{(A_\xi^{-1} q_0^1)^4, (A_\xi^{-1} q_0^2)^4$, $(A_\xi^{-1} q_0^3)^4\}$; which yields $d = A_{\wedge_0^{-1}\xi} \begin{pmatrix} 0 \\ -\alpha^t(\vec{\tau} - \vec{y}) \end{pmatrix}$.

Then, we have obtained : $k_0^{-1}(\hat{h}) = (A_{\wedge_0^{-1}\xi} R_{\vec{\eta}'} \overset{o}{p}, A_{\wedge_0^{-1}\xi} \overset{o}{\xi}, A_{\wedge_0 \xi} \overset{o}{D} G A_{\wedge_0^{-1}\xi}^{-1}$, $A_{\wedge_0^{-1}\xi} \begin{pmatrix} 0 \\ -\alpha^t(\vec{y} - \vec{\tau}) \end{pmatrix} R_0)$; the last component may be written $A_{\wedge_0^{-1}\xi} \begin{pmatrix} 0 \\ -\alpha^t(R_0^{-1}(\vec{y} - \vec{\tau})) \end{pmatrix}$ with $\vec{\eta}' = D(1)(A_{\wedge_0^{-1}\xi}^{-1} \wedge_0^{-1} A_\xi)\vec{\eta}$. Consequently, $\Gamma_{k_0^{-1}(\hat{h})} = (A_{\wedge_0^{-1}\xi} \begin{pmatrix} 0 \\ {}^t R_0^{-1}(\vec{y} - \vec{\tau}) \end{pmatrix}, A_{\wedge_0^{-1}\xi} R_{\vec{\eta}}, 1)$. In conclusion, taking into account the fact that $A_\xi^{-1} \wedge_0 A_{\wedge_0^{-1}\xi} \vec{\eta} \in SU(2))$, we obtain :

$$\Gamma_{\hat{h}}^{-1} k_0 \Gamma_{k_0^{-1}(\hat{h})} = (R_{\vec{\eta}}^{-1} \begin{pmatrix} T \\ 0 \end{pmatrix}, R_{\vec{\eta}}^{-1} A_\xi^{-1} \wedge_0 A_{\wedge_0^{-1}\xi} R_{\vec{\eta}}, R_0). \tag{6}$$

Remaks.3.2. (a) Orbit Ω being homeomorphic with $\Omega_+^{m_0^2} \times S_\lambda \times \mathbb{R}^3$, the action of $k = (Q, \wedge, R) \in K$ on $\hat{h} = (\xi, \vec{\eta}, \vec{y})$ is defined by $k^{-1}(\hat{h}) = (\wedge^{-1}\xi, D(1)(A_{\wedge^{-1}\xi}^{-1} \wedge^{-1} A_\xi)\vec{\eta}, R^{-1}(\vec{y} - \vec{\tau}))$, so as $A_\xi^{-1} Q = \begin{pmatrix} T \\ {}^t\vec{\tau} \end{pmatrix} \in M(4, 3, \mathbb{R})$.

(b) The decomposition $G = H \bullet K$ is also written $G = (\mathbb{R}^4 \times G_1) \bullet K$, where $G_1 \bullet K$ is the group considered in [25] Section 6, which is a regular semi-direct product, and the action of K on $\mathbb{R}^4$ is reduced to the standard action of $SL(2, \mathbb{C})$; it yields the habitual orbits of the Poincaré group. Then, the product $H \bullet K$ is regular.

(c) If a group G is a semi-direct product of two closed unimodular subgroups H and K, H being normal in G, it follows from [39], Chapter II Paragraph 7, that G is unimodular if, for every function $f \in \mathcal{L}^1_\mu(H)$, we have $\int_H f(khk^{-1})d\mu(h) = \int_H f(h)d\mu(h)$, where $d\mu$ is an invariant Haar measure on H. It follows, on applying this property in stages, that all the semi-direct products encountered are unimodular and that their invariant measures are obtained by simply considering the product of the measures of their factors.

3.3. IUR of the New-Stein Group associated to Ω

Let L an IUR of the stabilizer $K_0 = \mathbb{R}^9_q \bullet (U(1) \times SU(2))$ defined on the Hilbert space $\mathcal{H}_L$ and trivial on $\mathbb{R}^9_q$, then L is of the form : $L = D^{(s)} \otimes D(j)$, where $D^{(s)}$ denotes a unidimensional representation of $U(1)$ on $\mathbb{C}$ with $s \in \{0, \pm\frac{1}{2}, \pm 1, \cdots\}$. It follows that L is defined on $\mathbb{C} \otimes \mathbb{C}^{2j+1} = \mathbb{C}^{2j+1}$ by : for all $(Q, \wedge, R) \in K_0$ and $v \in \mathbb{C}^{2j+1}$, $L(Q, \wedge, R)v = D^{(s)}(\wedge) \otimes D(j)(R)v$. Then, we have the following Theorem.

Theorem 3.1. *The IUR of G induced by the IUR of $K_0 = \mathbb{R}^9_q \bullet (U(1) \times SU(2))$, associated to the trivial orbit of $U(1) \times SU(2)$ in $\mathbb{R}^9_q$, are given by :*
$\{U(t, t', c, a, q, \wedge, R)F\}(\xi, \vec{\eta}, \vec{y}) = \exp i\{\langle p, t\rangle + \langle \xi, t'\rangle + \langle b, a\rangle_1 + \langle D, c\rangle_2\} \times$
$D^{(s)}(R^{-1}_{\vec{\eta}} A^{-1}_\xi \wedge A_{\wedge^{-1}\xi} R_{\vec{\eta}'}) \otimes D(j)(R)(F(\wedge^{-1}\xi, \vec{\eta}', R^{-1}(\vec{y} - \vec{\tau}))),$
where : $p = A_\xi R_{\vec{\eta}} \overset{o}{p}$, $b = B = A_\xi \begin{pmatrix} 0 \\ -\alpha^t \vec{y} \end{pmatrix}$, $DG = A_\xi \overset{o}{D} G A^{-1}_\xi$,

$F \in \mathcal{H} = \mathcal{L}^2_\mu(\Omega^{m_0^2}_+ \times S_\lambda \times \mathbb{R}^3, \mathbb{C}^{2j+1})$, $d\mu = dw(\xi)d\sigma(\vec{\eta})dz$, *with* dw(resp $d\sigma$) *being the invariant measure by $SL(2, \mathbb{C})$ (resp $SU(2)$) concentrated on the hyperboloid $\Omega^{m_0^2}_+$ (resp the sphere S_λ), $\vec{\eta}' = D(1)(A^{-1}_{\wedge^{-1}\xi} \wedge^{-1} A_\xi)\vec{\eta}$, $\langle x_1, y_1\rangle = g_{\mu\nu} x_1^\mu y_1^\nu$, $\langle b, a\rangle_1 = \delta_{ij} g_{\mu\nu} b^{i\mu} a^{j\nu}$ and $\langle D, c\rangle_2 = \sum_{\mu \leq \nu} D_{\mu\nu} c^{\mu\nu}$.*

Remarks 3.3. (a) If $\vec{q}^\nu$ is the column vector $(q^{i\nu})_{i \leq 3}$, $\vec{\tau} = -\frac{1}{m_0} g_{\mu\nu} \xi^\mu \vec{q}^\nu$ and consequently $R^{-1}(\vec{y} - \vec{\tau}) = R^{-1}(\vec{y} + \frac{1}{m_0} g_{\mu\nu} \xi^\mu \vec{q}^\nu)$.

(b) If $R^{-1}_{\vec{\eta}} A^{-1}_\xi \wedge A_{\wedge^{-1}\xi} R_{\vec{\eta}}$ is the inverse image of $\wedge(\varphi)$, by the universal covering homomorphism, whose eigenvalues are $e^{\pm i\varphi/2}$, then for all $g \in G, F \in \mathcal{H}$ and $2s \in \mathbb{Z}$, $\{U(t, t', c, a, q, \wedge, R)F\}(\xi, \vec{\eta}, \vec{y}) = \exp i\{\langle p, t\rangle + \langle \xi, t'\rangle + \langle b, a\rangle_1 + \langle D, c\rangle_2 + s\varphi\} \times D(j)RF(\wedge^{-1}\xi, \vec{\eta}, R^{-1}(\vec{y} + \frac{1}{m_0} g_{\mu\nu} \xi^\mu \vec{q}^\nu)$; or, by writing $\vec{z} = -m_0 \vec{y}$, $(U(g)F)(\xi, \vec{\eta}, \vec{z}) = \exp i\{\langle p, t\rangle + \langle \xi, t'\rangle + \langle b, a\rangle_1 + \langle D, c\rangle_2 + s\varphi\} D(j)RF(\wedge^{-1}\xi, \vec{\eta}, R^{-1}(\vec{z} - g_{\mu\nu}\xi^\mu \vec{q}^\nu))$.

Differentiating this representation, we obtain for infinitesimal generators (defined on the space of its $\mathcal{C}^\infty$-vectors) :

$$
\begin{aligned}
T'_\mu &= \xi_\mu; \quad T_\mu = p_\mu \text{ such that } p = A_\xi R_{\vec{\eta}} p_0; \quad C_{\mu\nu} = \frac{\alpha}{m_0^2}\xi_\mu\xi_\nu; \\
A_{j\mu} &= \frac{\alpha}{m_0^2} z^j \xi_\mu; \quad Q_{j\mu} = i\frac{\partial}{\partial z^j}\xi_\mu; \\
J_{23} &= S_j^1 + i(\vec{z} \bigwedge \frac{\partial}{\partial \vec{z}})^1; \;\; J_{31} = S_j^2 + i(\vec{z} \bigwedge \frac{\partial}{\partial \vec{z}})^2; \;\; J_{12} = S_j^3 + i(\vec{z} \bigwedge \frac{\partial}{\partial \vec{z}})^3; \\
L_{23} &= \frac{s\eta^1}{k+\eta^3} + i\{(\vec{\xi} \bigwedge \frac{\partial}{\partial \vec{\xi}})^1 + (\vec{\eta} \bigwedge \frac{\partial}{\partial \vec{\eta}})^1\}; \\
L_{31} &= \frac{s\eta^2}{k+\eta^3} + i\{(\vec{\xi} \bigwedge \frac{\partial}{\partial \vec{\xi}})^2 + (\vec{\eta} \bigwedge \frac{\partial}{\partial \vec{\eta}})^2\}; \\
L_{12} &= s + i\{(\vec{\xi} \bigwedge \frac{\partial}{\partial \vec{\xi}})^3 + (\vec{\eta} \bigwedge \frac{\partial}{\partial \vec{\eta}})^3\}; \\
L_{14} &= \frac{s[\eta^2\xi^3 - \xi^2(k+\eta^3)]}{(k+\eta^3)(m_0+\xi^4)} + i\{\frac{\xi^3\eta^3 + \xi^2\eta^2}{\xi^4+m_0}\frac{\partial}{\partial\eta^1} - \frac{\eta^1\xi^2}{\xi^4+m_0}\frac{\partial}{\partial\eta^2} \\
&- \frac{\eta^1\xi^3}{\xi^4+m_0}\frac{\partial}{\partial\eta^3} + (\xi^4\frac{\partial}{\partial\xi^1} + \xi^1\frac{\partial}{\partial\xi^4})\}; \\
L_{24} &= \frac{s[-\eta^1\xi^3 + \xi^1(k+\eta^3)]}{(k+\eta^3)(m_0+\xi^4)} + \{-\frac{\xi^1\eta^2 + \xi^1\eta^2}{\xi^4+m_0}\frac{\partial}{\partial\eta^1} + \frac{\eta^1 + \xi^1 + \xi^3\eta^3}{\xi^4+m_0}\frac{\partial}{\partial\eta^2} \\
&- \frac{\eta^2\xi^3}{\xi^4+m_0}\frac{\partial}{\partial\eta^3} + (\xi^4\frac{\partial}{\partial\xi^2} + \xi^2\frac{\partial}{\partial\xi^4})\}; \\
L_{34} &= \frac{s[\eta^1\xi^2 - \xi^1\eta^2]}{(k+\eta^3)(m_0+\xi^4)} + i\{-\frac{\eta^3}{\xi^4+m_0}(\xi^1\frac{\partial}{\partial\eta^1} + \xi^2\frac{\partial}{\partial\eta^2}) + \frac{\eta^1\xi^1 + \eta^2\xi^2}{\xi^4+m_0}\frac{\partial}{\partial\eta^3} \\
&+ (\xi^4\frac{\partial}{\partial\xi^3} + \xi^3\frac{\partial}{\partial\xi^4})\};
\end{aligned}
$$

where $(S_j^l)_{l\leq 3}$ denote the infinitesimal generators of the representation $D(j)$ of $SU(2)$, $\vec{\xi}$ the vector $(\xi^i)_{1\leq i\leq 3}$ and $\bigwedge$ the vectorial product.

4. Cohomology and Deformations of the New-Stein Lie Algebra

The deformations theory [40] appears in a very natural way in symmetry contexts, mainly to discover all the possible symmetries that are connected, in some sense, to the given one. It also permits a better understanding of the transitions between the fundamental levels of physics (non-relativistic, relativistic, classical, quantum...). For instance, the passage from classical Newton mechanics to classical relativistic mechanics can be interpreted as a deformation of the Galilei group into the Poincaré group (which, in turn, may be deformed into the de Sitter group). The deformations of a Lie algebra can be searched for systematically by computing its cohomology groups. This cohomological calculation will also allow us, in Paragraph 5, to determine what we shall call the *relativistic invariant extensions* of $\mathbb{R}$ by the New-Stein Lie algebra. These extensions are at the basis, in Paragraph 7, of the unification of the non-relativistic and relativistic dynamics by making the latter a Newtonian-type dynamics.

The results of this paragraph, together with their demonstration, are similar to those of [41], *Mutatis Mutandis.* The non-explicited notations are those of [42].

Let $Z(\mathcal{SL}(2,\ \mathbb{C})\oplus\mathcal{SU}(2))$ (resp $Z(\mathcal{G})$) the centralizer of $\mathcal{SL}(2,\ \mathbb{C})\oplus\mathcal{SU}(2)$ in $\mathcal{G}$ (resp the center of $\mathcal{G}$).

Lemma 4.1. *The three vector spaces* $H^0(\mathcal{G},\mathcal{G}), Z(\mathcal{SL}(2,\ \mathbb{C})\oplus\mathcal{SU}(2))$ *and* $Z(\mathcal{G})$ *are unidimensional and generated by* $C = g^{\mu\nu}C_{\mu\nu}$.

Lemma 4.2. *For* $i \in \{1,2\}$, *the* $\mathcal{G}$*-modules* $H^i(\mathcal{G},\mathcal{G})$ *and* $H^i(\mathbb{R}^4\oplus\mathbb{R}'^4\oplus\mathcal{N}_3,\mathcal{G})^{\mathcal{G}}$ *are isomorphic.*

Lemma 4.3. (a) *Any element* $f \in Z^1(\mathbb{R}^4\oplus\mathbb{R}'^4\oplus\mathcal{N}_3,\mathcal{G})^{\mathcal{G}}$ *may be parametred by :* $f(T_\mu) = \alpha T_\mu, f(T'_\mu) = \alpha' T'_\mu, f(A_{i\mu}) = \beta A_{i\mu} + \gamma Q_{i\mu}, f(C_{\mu\nu}) = (\beta + \gamma')C_{\mu\nu}, f(Q_{i\mu}) = \beta' A_{i\mu} + \gamma' Q_{i\mu}$, *where* $\alpha, \alpha', \beta, \beta', \gamma$ and γ' *are real numbers.*

(b) $H^1(\mathbb{R}^4\oplus\mathbb{R}'^4\oplus\mathcal{N}_3,\mathcal{G})^{\mathcal{G}}$ *is trivial.*

Theorem 4.1. $H^1(\mathcal{G},\mathcal{G})$ *is a* $\mathcal{G}$*-module of dimension 6.*

Lemma 4.4. (a) *Any element* $g \in B^2(\mathbb{R}^4\oplus\mathbb{R}'^4\oplus\mathcal{N}_3,\mathcal{G})^{\mathcal{G}}$ *is defined by :* $g(X,Y) = 0$ *for all* $X,Y \in \{L_{\mu\nu}, T_\mu, T'_\mu, C_{\mu\nu}, J_{ij}\}$ *and* $g(A_{i\mu}, Q_{j\nu}) = \delta_{ij}(\alpha C_{\mu\nu} + \beta g_{\mu\nu} g^{\rho\sigma} C_{\rho\sigma})$, *where* α, β *are real numbers.*

(b) $Z^2(\mathbb{R}^4\oplus\mathbb{R}'^4\oplus\mathcal{N}_3,\mathcal{G})^{\mathcal{G}} = B^2(\mathbb{R}^4\oplus\mathbb{R}'^4\oplus\mathcal{N}_3,\mathcal{G})^{\mathcal{G}}$.

Theorem 4.2. $H^2(\mathcal{G},\mathcal{G}) = \{0\}$.

Corollary 4.1. $\mathcal{G}$ *is a rigid Lie algebra.*

5. New-Stein Group Relativistic Invariant Extensions

As first basic hypothesis of evolution in the framework of fundamental symmetry defined by a Lie group S, we suppose that evolution is described by a one-parameter Lie group $\mathbb{R}$, a subgroup of a Lie group $\tilde{S}$ (unifying fundamental symmetry and dynamics) so that S is a normal subgroup of $\tilde{S}$ and the quotient group $\tilde{S}/S$ is $\mathbb{R}$. Which defines $\tilde{S}$ as being an extension of $\mathbb{R}$ by S . In order to search for the possible $\tilde{S}$ groups, we are going to reason at the level of Lie algebras and to determine, afterwards, the corresponding Lie groups.

An extension of $\mathbb{R}$ by $\mathcal{S}$ is defined by an exact sequence : $0 \to \mathcal{S} \xrightarrow{\lambda} \tilde{\mathcal{S}} \xrightarrow{\mu} \mathbb{R} \to 0$ so as $\lambda(\mathcal{S}) = Ker\mu$. Such an extension is inessential [43], since $\mathbb{R}$ is one-dimensional, and it is defined by a linear mapping, associating to the generator K of $\mathbb{R}$ the derivation Φ of $\mathcal{S}$, given by : $\forall\, x \in \mathcal{S}, \Phi(x) = [K, x]$.

As second basic hypothesis, we suppose that K is invariant by any purely external or purely internal symmetry. We call such an extension a *relativistic invariant extension.*

If we apply these results to fundamental symmetry defined by the New-Stein group G, we have $\Phi \in Z^1(\mathbb{R}^4 \oplus \mathbb{R}'^4 \oplus \mathcal{N}_3, \mathcal{G})^{\mathcal{G}}$ and, in Lemma 4.3., $\alpha = \alpha' = 0$. Then, we obtain the following Proposition.

Proposition 5.1. *Any relativistic invariant extension $\tilde{\mathcal{G}}$ is defined by the commutation relations* : $[K, A_{i\rho}] = \beta A_{i\rho} + \gamma Q_{i\rho}; [K, Q_{i\rho}] = \beta' A_{i\rho} + \gamma' Q_{i\rho}$; $[K, C_{\mu\nu}] = (\beta+\gamma')C_{\mu\nu}$; *the other non-null commutation relations being those of* $\mathcal{G}$.

Remark 5.1. In order to classify such extensions, we consider the matrix $L = \begin{pmatrix} \beta & \beta' \\ \gamma & \gamma' \end{pmatrix}$ associated to the restriction of $ad\, K$ at the subspace generated by $A_{i\rho}, Q_{i\rho}$ (i, ρ fixed). We notice that if L is equivalent to L', then the associated extensions are equivalent ; consequently, we treat L under reduced form of Jordan. Let P the characteristic polynomial of $L, P = X^2 - X\, trL + detL$; if $det\ L \neq 0$, the transformation $K' = K : \sqrt{|detL|}$ does not change the brackets of $\mathcal{G}$; besides, if L' is associated to K', we have det $L' = \pm 1$; hence, we have six cases defined by ($detL \in \{0, 1, -1)$ *and* ($trL = 0$ *or* $trL \neq 0$)), which yield 9 Jordan reduced forms for L (following the values of $tr\ L$), leading to 9 non-equivalent extensions of $\mathbb{R}$ by $\mathcal{G}$, analogous to those defined in [31]. This leads to the following Proposition.

Proposition 5.2. *Any relativistic invariant extension $\tilde{\mathcal{G}}$ is equivalent to one* *of the Lie algebras defined by the following Lie brackets* :

(1) $[K, A_{i\rho}] = 0$; $[K, Q_{i\rho}] = 0$; $[K, C_{\mu\nu}] = 0$;

(2) $[K, A_{i\rho}] = A_{i\rho}$; $[K, Q_{i\rho}] = -Q_{i\rho}$; $[K, C_{\mu\nu}] = 0$;

(3) $[K, A_{i\rho}] = \zeta^2 A_{i\rho}$; $[K, Q_{i\rho}] = -\zeta^{-2} Q_{i\rho}$; $[K, C_{\mu\nu}] = (\zeta^2 - \zeta^{-2}) C_{\mu\nu}$; *where* $\zeta^2 \notin \{0, 1\}$;

(4) $[K, A_{i\rho}] = \zeta^2 A_{i\rho}$; $[K, Q_{i\rho}] = \zeta^{-2} Q_{i\rho}$; $[K, C_{\mu\nu}] = (\zeta^2 + \zeta^{-2}) C_{\mu\nu}$; *where* $\zeta \neq 0$;

(5) $[K, A_{i\rho}] = \zeta^2 A_{i\rho}$; $[K, Q_{i\rho}] = 0$; $[K, C_{\mu\nu}] = \zeta^2 C_{\mu\nu}$; *where* $\zeta \neq 0$;

(6) $[K, A_{i\rho}] = -Q_{i\rho}$; $[K, Q_{i\rho}] = 0$; $[K, C_{\mu\nu}] = 0$;

(7) $[K, A_{i\rho}] = -Q_{i\rho}$; $[K, Q_{i\rho}] = A_{i\rho}$; $[K, C_{\mu\nu}] = 0$;

(8) $[K, A_{i\rho}] = \cos\varphi A_{i\rho} - \sin\varphi Q_{i\rho}$; $[K, Q_{i\rho}] = Q_{i\rho}$; $[K, C_{\mu\nu}] = 2C_{\mu\nu}$;

(9) $[K, A_{i\rho}] = \cos\varphi A_{i\rho} - \sin\varphi Q_{i\rho}$; $[K, Q_{i\rho}] = \sin\varphi A_{i\rho} + \cos\varphi Q_{i\rho}$; $[K, C_{\mu\nu}] = 2\cos\varphi C_{\mu\nu}$; *where* $\varphi \neq (k + 1/2)\pi$.

The other Lie brackets being those of $\mathcal{G}$.

Remark 5.2. In order to determine the Lie groups corresponding to these Lie algebras, it is sufficient to apply the Campbell-Hausdorff formula [44]. In Paragraph 7, we shall study, in detail, the fundamental symmetry associated to the relativistic invariant extension defined by case (7) the group of which is given by the following Proposition.

Proposition 5.3. *The group law of* $\tilde{G}$ *(whose Lie algebra is defined by case (7) of Proposition* 5.2.*)is given by : for all* $g_i = (k_i, t_i, t'_i, c_i, a_i, q_i, \wedge_i, R_i) \in \tilde{G} (i = 1, 2)$, *then* $g_1 g_2 = (k_1 + k_2, t_1 + \wedge_1 t_2, t'_1 + \wedge_1 t'_2, c_1 + S(\wedge_1) c_2 + (\sin^2 k_2) \times \beta(a_1, q_1) - \frac{1}{4}(\sin 2k_2)[\beta(a_1, a_1) - \beta(q_1, q_1)] - \beta(q_1 \cos k_2 + a_1 \sin k_2, \wedge_1 \otimes R_1 a_2)$, $a_1 \cos k_2 - q_1 \sin k_2 + \wedge_1 \otimes R_1 a_2, q_1 \cos k_2 + a_1 \sin k_2 + \wedge_1 \otimes R_1 q_2, \wedge_1 \wedge_2, R_1 R_2)$.

6. Remarkable IUR of an Extension of $\mathbb{R}$ by the New-Stein Group

Let $\tilde{G}$ the extension of $\mathbb{R}$ by G defined in Proposition 5.3. ; then G is a normal (non-abelian) subgroup of $\tilde{G}$ and to determine the IUR of $\tilde{G}$ we apply the induction method [38]. Let us fix its notations : Let $\chi \in \hat{G}$, then $k \in \mathbb{R}$ acts on χ by : $\forall g \in G, (k\chi)(g) = \chi(kgk^{-1})$. Let $O(\mathcal{X})$ the orbit of χ and K_χ the stabilizer of χ. If $(\rho, \mathcal{H}_\rho)$ is a IUR of K_χ and $\mathcal{H}_\chi$ the support of χ, we obtain a representation π of $K_\chi \bullet G$, defined on $\mathcal{H}_\rho \otimes \mathcal{H}_\chi$, by writing $\pi(k, g) = \rho(k) \otimes W(k)\chi(g)$, where $W(k)$ is the isomorphism of $\mathcal{H}_\chi$ realizing the equivalence of the representations χ and $k\chi$. From this IUR of $K_\chi \bullet G$, we obtain, by induction, an IUR of $\tilde{G}$. Besides, all the IUR of $\tilde{G}$ are of this form.

Proposition 6.1. *The IUR of* $\tilde{G}$, *induced by an IUR* $(U, \mathcal{H})$ *of* G *of the type obtained in Paragraph 3 and by the character* $k \to e^{ik\ell/2}$ *of* $\mathbb{R}$, *is given by*

$$\tilde{U}(k,g)F = \exp i\frac{k}{2}(\ell - \frac{m_0^2}{\alpha}\Delta + \frac{\alpha}{m_0^2}z^{(2)})U(g)F,$$

where $(k,g) \in \tilde{G}(= \mathbb{R}\bullet G)$, $F \in \mathcal{H}$, $\Delta = \sum_j \frac{\partial^2}{(\partial z^j)^2}$ *and* $z^{(2)} = \sum_j (z^j)^2$.

Proof : Knowing that $k \in \mathbb{R}$ commutes with $t, t', c, \wedge$ and R, it is sufficient to consider $(kU)(a)$ and $(kU)(q)$, where $a \in \mathbb{R}_a^{12}$ and $q \in \mathbb{R}_q^{12}$. Theorem 3.1. gives : $U(kak^{-1}) = \exp i\{\cos ka^{j\mu}\frac{\alpha}{m_0^2}z_j\xi_\mu + \frac{\sin 2k}{4}\frac{\alpha}{m_0^2}\sum_{\mu\leq\nu}\beta(a,a)^{\mu\nu}\xi_\mu\xi_\mu\} \times \exp i\{-\sin ka^{j\mu}[i\frac{\partial}{\partial z^j}\xi_\mu]\}$ and $U(kqk^{-1}) = \exp i\{\sin kq^{j\mu}\frac{\alpha}{m_0^2}z_j\xi_\mu - \frac{\sin 2k}{4}\frac{\alpha}{m_0^2}\sum_{\mu\leq\nu}\beta(q,q)^{\mu\nu}\xi_\mu\xi_\nu\} \times \exp i\{\cos kq^{j\mu}[i\frac{\partial}{\partial z^j}\xi_\mu]\}$; by writing $U(a) = \exp ia^{j\mu}\frac{\alpha}{m_0^2}[z_j\xi_\mu]$ and $U(q) = \exp iq^{j\mu}[i\frac{\partial}{\partial z^j}\xi_\mu]$, we obtain $U(kak^{-1}) = W(k)U(a)W(k^{-1})$ and $U(kqk^{-1}) = W(k)U(q)W(k^{-1})$, where $W(k)$ is the unitary operator $W(k) = \exp\frac{ikm_0^2}{2\alpha}(-\Delta + \frac{\alpha^2}{m_0^4}z^{(2)})$. Finally $U(kgk^{-1}) = W(k)U(g)W(k^{-1})$, $\forall g \in G$, and the orbit of U is $\{U\}$, whose stabilizer is $K_U = \mathbb{R}$. Consequently, to the character $k \to e^{ik\ell/2}$ of $\mathbb{R}$ is associated the IUR $\tilde{U}$ of $\tilde{G}$ defined on $\mathbb{C} \otimes \mathcal{H}$ by : $\tilde{U}(k,g) = \exp\frac{ilk}{2}W(k)U(g)$.

7. Physical Interpretation

In the same way as the non-relativistic classical mechanics, the standard quantum mechanics privileges time with regard to space, which is contrary to the very spirit of the Einstein relativity theory. In fact, in quantum mechanics, time is a parameter, whereas the spatial position observables are dynamical variables. Similary, in non-relativistic classical mechanics, it is possible to define the spatial position of a particle, whereas it is not possible to say that a particle has a well defined time. Moreover, the Minkowski space-time seems to favor the statical conception of time with regard to its dynamical conception. Thus, if we want to re-establish the space-time symmetry in relativistic quantum mechanics, we must distinguish the geometrical time of the space-time, which is an observable, associated to a clock, defining the state of the system (time of the event in the laboratory referential), from dynamical time, an independant parameter of space-time, which is not an observable, but the function of which is to describe the evolution of the system. This distinction will permit the unification of the relativistic and non-relativistic dynamics by turning both of them into a Newtonian dynamics : in both cases, the evolution time is managed by a dynamical concept which makes it "Universal Time" passing "uniformly and inexorably" as Newton imagined. Such an additional parameter was introduced by various authors, but, at the beginning [45], simply, as a mathematical convenience, without physical interpretation. It is only afterwards that this evolution parameter was taken into consideration, out of necessity, in order to conciliate the ideas of Einstein and those of Newton and to build up a Hamiltonian-type relativistic dynamics, and this by admitting the existence of a space-time in conformity with the geometry defined by the Poincaré group, but occupying, vis-à-vis evolution, a position analogous to that of the three-dimensional space in Newton theory. This parameter has been baptized " Historical Time" [32], and this is not the proper time of any particular system.

It is in this framework that we shall interpret symmetry in accordance with the New-Stein group, an interpretation which will permit to give "Historical Time" and the dynamics ensuing from it a pure theoretical group basis.

First of all, the generators of $\tilde{\mathcal{G}}$, other than those of the Poincaré algebra, are relativistic covariant since they are invariant under translations and transform, under the action of the Lorentz group, like a Lorentz tensor. In order to define the dynamics associated to the fundamental symmetry sub-jacent to the New-Stein group, we adopt the following Postulate.

Postulate : *To any relativistic invariant extension $\tilde{G}$, of the New-Stein group G, corresponds a dynamics, generated by the infinitesimal generator K of $\tilde{G}$, the evolution parameter of which is identified with "Historical Time" τ .*

At this stage, two important implications of this Postulate deserve mentioning : the first is that the specification of the dynamics may not be attempted, even in principle, before the symmetry group is decided upon ; as for the second implication, it stipulates that symmetry entirely determines the dynamics structure, that is to say the concept which governs the system modifications during time. According to Weinberg [46], such a connection between symmetry and dynamics was encountered, for the first time, in the "Theory of Strings". In general, we introduce dynamics in a model by adding an interaction term.

Generator K, consequently, generates evolution in historical time and, being an invariant with regard to anything which is purely external or internal symmetry, it is preserved during evolution. Therefore, we shall interpret it as being the total mass (energy) of the system. By its definition, it also appears as being a relativistic Hamiltonian. It is convenient to note, in this context, that, in general, the Hamiltonian invariance is slightly stronger than the motion equation invariance ; for example, for an isolated system, the Galilei group does leave the motion equation invariant, but alters the Hamiltonian.

In what follows, we are going to study, in detail, the fundamental symmetry associated to the relativistic invariant extension $\tilde{G}$ defined by Proposition 5.3. and in the context of its IUR defined by Proposition 6.1. . The privilege of this fundamental symmetry essentially lies in the fact that it is going to allow us to generalize the standard formalism of the harmonic oscillator [21] as well as Bohr's fundamental invariant, associated to his "Theory of Reciprocity" [24], in the relativistic framework with presence of an internal dynamical structure.

Taking into account our interpretation, the mass observable of our system is represented by $d\tilde{U}(K)$, where $d\tilde{U}$ is the differential representation of the $\tilde{U}$ representation defined on the space of its C^{∞}-vectors. We, consequently, obtain :

$$d\tilde{U}(K) = \frac{1}{2}(-\frac{m_0^2}{\alpha}\Delta + \frac{\alpha}{m_0^2}z^{(2)} + \ell).$$

The operator $d\tilde{U}(K)$ coincides with the representative of element B of the enveloping algebra $\mathcal{U}(\tilde{\mathcal{G}})$ defined by

$$B = (\frac{1}{2\alpha}M_N^2 + \frac{\ell}{4}) + (\frac{1}{2\alpha}M_A^2 + \frac{\ell}{4}),$$

where : $M_N^2 = -g^{\mu\nu}\delta^{ij}Q_{i\mu}Q_{j\nu}, M_A^2 = -g^{\mu\nu}\delta^{ij}A_{i\mu}A_{j\nu}.$

As for the representatives of these two last elements of $\mathcal{U}(\tilde{\mathcal{G}})$ in the representation $d\tilde{U}$, they are given by :

$$d\tilde{U}(M_N^2) = -m_0^2\Delta, d\tilde{U}(M_A^2) = \frac{\alpha^2}{m_0^2}z^{(2)}.$$

The mass observable $d\tilde{U}(K)$ admits a discrete spectrum leading to the exact mass formula :

$$m_n^2 = n + \frac{3}{2} + \frac{\ell}{2},$$

where n is a non-negative integer and ℓ a real number ; whereas the operators $d\tilde{U}(M_N^2)$ and $d\tilde{U}(M_A^2)$ admit continuous spectra.

This leads us to the following interpretation based, partially, on the ideas developed in Paragraph 5 of [25]. We interpret the $Q_{i\mu}$ and the $A_{i\mu}$ (and the $C_{\mu\nu}$ which depend on them algebraically) as describing the internal dynamics of a composite system with two constituents in interaction, conjugated from each other, which we shall call "Nazzamion" N and "Antinazzamion" A [47], in the following way : the Nazzamion (resp Antinazzamion) energy-momentum is given by an "external" part T_μ and an "internal" part $Q_{i\mu}$ (resp $A_{i\mu}$). Thus, the relativistic free mass of its constituents is null, since $d\tilde{U}(T_\mu T^u) = 0$, and, consequently, $d\tilde{U}(K)$ effectively represents the mass (energy) of this composite system. This does not contradict the relativistic quantum dynamics where the mass of a composite system (for instance, the mass of the resonance state) can be larger than the sum of the masses of the constituents ; the excess mass can be visualized as the positive kinetic energy of the constituents-particles in the potential well. In this framework, the internal degrees of freedom are thus assumed to contribute to the creation of mass via a term which is (for each space-time direction) the Hamiltonian of the (three-dimensional) harmonic oscillator. This term can be viewed as the sum of the self-interactions M_N^2 and M_A^2 of the Nazzamion and the Antinazzamion ; each of these self-interaction terms has a continuous spectrum in the representation $d\tilde{U}$, but the coupling of both leads to our composite system which we shall call "Nazzamium". The Nazzamium dynamics is defined by the Hamiltonian $d\tilde{U}(K)$ which has a discrete spectrum of the harmonic oscillator-type, capable, by its nature, of describing the matter spectrum of fundamental symmetry managed by the New-Stein group. In view of this, we should have a proper relativistic covariant formalism for the harmonic oscillator mass spectrum where the basic constituents can be of vanishing mass. As for the isospin content of this model, it is absolutely analogous with that of the harmonic oscillator model developed in paragraph 6.A of [25]. In particular, the eigenfunctions of $d\tilde{U}(K)$ will be classified by a principal quantum number connected with the internal excitation level and a secondary quantum number associated with the weight of a IUR of the factor $SU(2)$ of the Newton group (the isospin symmetry). Thus, we arrive to a dynamical explanation of mass (energy) origin : it is created by excitation of the Nazzamium internal structure. An important consequence of this analysis is that the generation of mass is associated with the generation of the internal symmetry and that its origin appears as a bound state effect or an interaction effect linked to that symmetry. In this context, we must point out that the presence of oscillations in our model relates it to the Theory of Strings (see, for instance, [49]). In fact, in many versions of this theory, the strings are closed into loops, and it is not these loops that represent particles but the various ways in which the loops can oscillate. The energies of these oscillations, expressed as mass by the Einstein equivalence, are the mass of the particle we know.

In a way, this interpretation seems to be a generalization of the standard description of a system of n quantum particles possessing only properties that have a classical analog. As a matter of fact, there exist, for such a system, n pairs of canonically conjugate variables $(P_i, Q_i)_{1\leq i\leq n}$, representing the generators of Heisenberg algebra $\mathcal{H}_n$, so that the Hamiltonian (and also every other physical observable) is a function of it. In our model, $\mathcal{N}_3$, which characterizes the internal dynamics, would generalize the Heisenberg algebra, and element B, associated with $d\tilde{U}(K)$ in $\mathcal{U}(\tilde{\mathcal{G}})$, would generalize the fundamental invariant of Born [24]. Finally, this interpretation has also the advantage that none of the generators of $\tilde{\mathcal{G}}$ is overabundant, contrarily to the various models related to the Standard Model.

The New-Stein group symmetry presents four important advantages by comparison with that developed in [25] :

(a) It permits, not only, to determine the matter spectrum, but, it also fixes its diverse dynamics which are classified by the extensions of the dynamic group $\mathbb{R}$ by the fundamental symmetry group G.

(b) Thanks to the concept of "Historical Time" τ, evolution can be described, rigorously, by a relativistic Schrödinger-type equation

$$i\partial_\tau \Psi_\tau = d\tilde{U}(K)\Psi_\tau,$$

analogous to the standard way to describe the dynamics of a non-relativistic particle where the change of states is given by unitary transformations. Such unitary transformations are supposed to form a one-parameter group representation. According to Stone's theorem such an evolution is completely defined by a self-adjoint operator leading to the Schrödinger equation.

(c) The energy of the quantum vacuum (fundamental state of the Nazzamium) may be chosen equal to zero : the only condition to satisfy is to have $\ell = -3$ in the mass formula. Consequently, the ground state is completely devoid of dynamics, but not of kinematics.

(d) The mass term can be achieved from the very dynamics even when the constituents are massless.

Consequently, the New-Stein group symmetry leads to a unified description of massless and massive particles : the passage of the former to the latter is carried out by excitations at the level of the Nazzamium internal structure. This might be the basis of a new approach to the unification of all the fundamental interactions of Nature. In this context, we may suppose that, like in the model of the *ex nihilo* creation of matter associated with the inflationary theory of the primordial universe, the generation of mass in our model is also a consequence of a phase transition. Thus, there exists a phase structure between massive and massless particles : quantum vacuum goes into a series of phase transitions ; the first of which, historically speaking (that which is, as a matter of fact, located in the most speculative era of contemporaneous cosmology [3] and during which the three types of strong, weak and electromagnetic interactions are unified), corresponds to the creation of mass. The estimate of the critical temperature T_c of this phase transition has been confirmed by the work of Weinberg [50] ; it is approximately $10^{27}K$ and corresponds to a typical energy of 10^{15} GeV located at the instant $10^{-34}s$ after the Big Bang [51]. At temperature $T > T_c$, mass will disappear and matter is in a highly symmetrical state. This evidently suggests that in the primitive universe, at very high temperature, there were only massless particles, an epoch which we may call the *massless era*. Later, the *mass era* began at $T = T_c$ due to a phase transition during which the massive particles appear as a bound state of Nazzamium. As for the physical reality of the Nazzamium constituents, they may possibly be only mathematical fictions, having nothing to do with the notion of particle as we commonly conceive it, but leading, effectively, to the fundamental symmetries, as expressed by Heisenberg.

8. Discussion and Outlook

We end this work by briefly making a few remarks and suggestions concerning its possible continuations :

(1) Since the New-Stein group is not a group of transformations admitting a passive interpretation in space-time, in order to study the consequences of the symmetry it induces, it has not been necessary to determine its unitary

or anti-unitary projective representations, which can be obtained from the study of its central extensions by $\mathbb{R}$. However, these extensions may lead us to possible new symmetry groups which are not "very far" from the New-Stein group. The space of equivalence classes of the central extensions of the New-Stein Lie algebra by $\mathbb{R}$ is of dimension eleven. The most promising of the correspondent groups is group G_β associated with the local exponent β of the New-Stein group defined by $\beta(g_1, g_2) = \langle t_1, \wedge_1 t'_2 \rangle$, for any two elements $g_i(t_i, t'_i, c_i, a_i, q_i, \wedge_i, R_i), i \leq 2$, of the New-Stein group. Group G_β contains the extended Einstein group and leads, like it [26], to generalized Heisenberg relations and to a new definition of the relativistic spin.

(2) The deformations of the extended Newton groups are analogous to those of the extended Einstein group [41], *Mutatis Mutandis.* Thus, taking into account the rigidity of the New-Stein group (consequence of Corollary 4.1.) and of the non-rigidity of the extended Einstein and Newton groups, the replacement, in this framework, of one of these two groups by one of its deformed groups would lead to a fundamental symmetry which is (separately) "neighboring" (from the external or internal structure view point) the one studied in our work. Besides, if we replace the extended space Newton group by that of a higher dimension (by substituting $Spin(n)$ for $SU(2)$ and H_n for H_3, with $n > 3$), the corresponding New-Stein group is still rigid.

(3) In what was exposed above, the Nazzamium Hamiltonian $d\tilde{U}(K)$ coincides with the representative of element B of the enveloping algebra $\mathcal{U}(\tilde{\mathcal{G}})$. Now, B possesses the property of being a symmetrical homogeneous polynomial of the second degree in the conjugate canonical variables $(A_{i\nu})$ and $(Q_{i\nu})$ which describe the internal dynamics. This, naturally, suggests the study of other evolution kinds generated by such polynomials. Thus, for instance, if we suppose that, at least in a first approximation, the "internal" energy-momentum observables of the Nazzamion and the Antinazzamion add linearly, we are led to consider the composite system the evolution of which is generated by $-g^{\mu\nu}\delta^{ij}(Q_{i\mu} + A_{i\mu})(Q_{j\nu} + A_{j\nu})$.

There remains, of course, the study of the other dynamics associated with the other New-Stein group relativistic invariant extensions of Proposition 5.2.

(4) It can be intuitively judged that an oscillator is not sufficient to describe a system as extended object and, similarly to molecular physics which combines the oscillator and the rotator to obtain a vibration-rotation energy, it would be necessary to add a term which would correspond to a quantum relativistic rotator. For that purpose, the central extension G_β of the

New-Stein group can be an ad hoc framework, since it is possible to consider $S_{\mu\nu}S^{\mu\nu}$ as the term describing the rotation energy, where $S^{\mu\nu}$ is the relativistic spin tensor defined in [26] and which has been constructed by analogy with the Galilean mechanics.

(5) In another perspective, the fact that the formalism of the creation and annihilation operators explicity includes the mass creation concept from energy, it would be interesting to define (working from internal canonical variables $(A_{i\mu})$ and $(Q_{i\mu})$) operators $X^{\pm}_{i\mu}$ of the creation and annihilation type and adapt our model to a field theory (with an infinite number of particles).

(6) The notion of the dimensionality of space-time is a fact which lies at the very foundations of geometry and physics and one can find, in the literature, considerations giving reasons for the four dimensionality of space-time which are related to effects calculated for some physical law (for a review, see [52]). Besides, various heuristic reasons may be given for space-time to have this dimension. However, in spite of all this, the mechanism (if it exists) responsible for fixing this effective number of dimensions to four is still a mystery. As a matter of fact, in some physical theories, other dimensions have been considered. In certain cases, they are used as a purely mathematical trick (see, for example, [53]). In other cases, the fact that the dimension of space-time differs from four has both a practical and a theoretical interest and is taken to descibe physical reality as it is in models based on the original suggestion of Kaluza and Klein with the aim of unifying all known interactions (see, for example, [54]), or in the framework of the physics of low dimensions quantum structures [55]. This new branch of physics, which has proved as important for fundamental research as for the applied one, has numerous theoretical links with other domains of physics and maintains close relations between experimental works and advanced technology such as the domain of numerous new materials produced and studied nowadays among which we can mention magnetic materials and electronic components. As for the space sub-jacent to the internal structure of matter, there is nothing to prevent it from having a given dimension, especially that it has been introduced, from the beginning (i.e from the introduction of the isospin [9]), essentially by analogy with the space-time. Moreover, to our knowledge, no experiment mentions any observable difference between the relativistic and non-relativistic intrinsic internal structures. In this context, it would be interesting to think of other dimensions for this structure, but always in conformity with the New-Stein symmetry. In this perspective, we are encour-

aged by the fact that several two and three-dimensional space-time systems have been intensely studied over the last two decades, especially when the structure of relevant phenomena is effectively confined in two or one spatial dimensions. Some of these models have already led to immediate and spectacular applications. Among these phenomena and models, we may quote [55], [56] : non-linear optics, theory of anyons and its connection with the fractional quantum Hall effect, Chern-Simmons gauge theories, high temperature superconductivity... Besides, according to Salam [16], the Theory of Strings must rather be considered as equivalent to a theory of fields in a two-dimensional space-time. All this has, in fact, recently led to the study of various two or three-dimensional space-time symmetry groups [57] such as : Galilei, Galilei-Similitude, Schrödinger, Poincaré and Conformal groups.

In this perspective, let us consider the New-Stein group G_2 associated with a two-dimensional internal space. This is achieved by replacing, in the definition of $\mathcal{G}, \mathcal{H}_3$ (resp $\mathcal{SU}(2)$) by $\mathcal{H}_2$ (resp $\mathcal{SO}(2)$). This dimension change will induce a radical change in interpretation and in structure. In fact, the isospin in two dimensions differs fundamentally from isospin in higher dimensions, because the angular momentum algebra in two dimensions is the trivial commutative algebra $\mathbb{R}$; there is no analog of the quantization of angular momentum in higher dimensions, associated with the $\mathcal{SO}(n)$ rotation algebra, $n \geq 3$. Thus, in the G_2 context, a particle can have arbitrary real isospin, like the anyon which may be considered as being a particle with arbitrary spin and statistics [56]. It would be interesting to explore , in the framework of low dimensions internal structure, the new ideas linked with the anyons physics and, especially, those associated with phase transitions and "critical phenomena" of small-dimensional systems [55], [58]. As for the structural change, it lies in the fact that G_2 has significantly richer extensions than G, since the space of equivalence classes of the central extensions of $\mathcal{G}_2$ (resp $\mathcal{G}$) by $\mathbb{R}$ is of dimension thirteen (resp. eleven). In a near perspective, it appears that the various symmetries associated with the bidimensional internal space differ fundamentally from those associated with the other dimensions, since the equivalence classes space of the infinitesimal exponents of $F_n (n = 1$ or $n \geq 3)$ is, as that of the Einstein group [26], unidimensional, whereas that of F_2 is of dimension four. Consequently, it would be interesting to include the other internal symmetries, defined by the other central extensions of F_2, in a fundamental global symmetry, in similitude with that studied in this paper, which we can call a New-Stein-type symmetry.

Dedication

We dedicate this paper to the spirit of Moshé Flato, teacher and friend, in gratitude for many stimulating conversations regarding mass problem and for his careful reading of the French version of the manuscript some months before his sudden disappearance.

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