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Beating quantum limits in interferometers with quantum locking of mirrors

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Abstract. The sensitivity in interferometric measurements such as gravitational-wave detectors is ultimately limited by quantum noise of light. We discuss the use of feedback mechanisms to reduce the quantum effects of radiation pressure. Recent experiments have shown that it is possible to reduce the thermal motion of a mirror by cold damping. The mirror motion is measured with an optomechanical sensor based on a high-finesse cavity, and reduced by a feedback loop. We show that this technique can be extended to lock the mirror at the quantum level. In gravitational-waves interferometers with Fabry-Perot cavities in each arms, it is even possible to use a single feedback mechanism to lock one cavity mirror on the other. This quantum locking greatly improves the sensitivity of the interferometric measurement. It is furthermore insensitive to imperfections such as losses in the interferometer.

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1. Introduction

Quantum fluctuations of light play an important role in the sensitivity limits of optical measurements such as interferometric measurements considered for gravitational-wave detection [1, 2]. A gravitational wave induces a differential variation of the optical pathes in the two arms of a Michelson interferometer. The detection of the phase difference between the two optical pathes is ultimately limited by the quantum noises of light: the phase fluctuations of the incident laser beam introduce noise in the measurement whereas radiation pressure of light induces unwanted mirrors displacements. Due to the Heisenberg's inequality, both noises are conjugate and lead to the so-called standard quantum limit for the sensitivity of the measurement when coherent states of light are used [3, 4, 5].

Potential applications of squeezed states to overcome this limit have motivated a large number of works in quantum optics. The injection of a squeezed state in the unused port of the interferometer can improve the sensitivity of the measurement [4, 5, 6]. Another possibility is to take advantage of the quantum effects of radiation pressure in the interferometer to perform a quantum nondemolition measurement [5]. Since radiation pressure effects are frequency dependent, a main issue is to find simple

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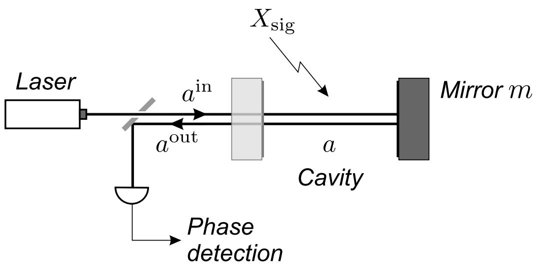

Figure 1. An interferometric measurement is equivalent to a length measurement by a single Fabry-Perot cavity. A cavity length variation $X_{\rm sig}$ is detected through the phase shift induced on the field $a^{\rm out}$ reflected by the cavity.

schemes which improve the sensitivity over a wide frequency band [7]. Another issue is to precisely examine the constraints imposed to the interferometer by the use of such quantum techniques. Losses in particular may have drastic effects on the sensitivity improvement.

An alternative approach consists in using feedback mechanisms working in the quantum regime. This technique has been proposed to generate squeezed states [8] or to perform quantum nondemolition measurements [9], and experimentally demonstrated on laser oscillators [10] and twin beams [11]. Active controls are also widely used in the classical regime as for example in cold-damped mechanical systems [12, 13]. Cold damping is able to reduce the mechanical thermal displacements of a mirror [14, 15], and it may in principle be used to reduce the displacement noises in a quantum regime, down to the zero-point quantum fluctuations of the mirror [16, 17].

We discuss in this paper the possibility to increase the sensitivity in interferometers by reducing radiation pressure effects with such a feedback mechanism. We propose to use a compact optomechanical sensor made of a high-finesse cavity to measure the mirror displacements induced by radiation pressure. The information is fed back to the mirror in order to lock its position at the quantum level. We show that the sensor sensitivity is transferred to the interferometric measurement, resulting in a reduction of the back-action noise associated with the radiation pressure in the interferometer [18].

We present in section 2 the main characteristics of the quantum noises in an interferometric measurement. Section 3 is devoted to the active control of a mirror and to the resulting reduction of back-action noise in the interferometer. As shown in section 4, it is possible to completely suppress back-action noise if the optomechanical sensor performs a quantum nondemolition measurement of the mirror motion [19]. We finally study in section 5 the control of the whole Fabry-Perot cavity in each interferometer arms with a single optomechanical sensor. We show that the information provided by the measurement allows one to lock the cavity length in such a way that it is no longer sensitive to radiation-pressure effects.

2. Quantum limits in interferometric measurements

A gravitational-wave interferometer is based on a Michelson interferometer with kilometric arms and Fabry-Perot cavities inserted in each arm [1, 2]. A gravitational wave induces a length variation of the cavities and is detected as a change in the interference fringes at the output of the interferometer. As long as we are concerned with quantum and mirror-induced noises, the interferometer is equivalent to a simpler scheme consisting in a single resonant optical cavity, as shown in figure 1, which actually corresponds to one interferometer arm. A variation of the cavity length changes the optical path followed by the intracavity field and induces a phase shift of the reflected field which can be detected by an homodyne detection.

To study the effects of quantum fluctuations of light, we describe the fields by a complex mean amplitude α and quantum annihilation operators $a\,[\Omega]$ at frequency Ω. We define any quadrature a_θ of the field as

$$a_\theta\,[\Omega] = e^{-i\theta} a\,[\Omega] + e^{i\theta} a^\dagger\,[\Omega]\,. \tag{1}$$

For a lossless single-ended cavity resonant with the laser field, the input-output relations for the fields can be written in a simple way. The reflected mean field $\alpha^{\rm out}$ is equal to the incident mean field $\alpha^{\rm in}$ and both can be taken as real. The amplitude and phase quadratures of the fields then correspond to the quadratures a_0 and $a_{\frac{\pi}{2}}$, respectively aligned and orthogonal to the mean field. Assuming the frequency Ω of interest smaller than the cavity bandwidth, the input-output relations for the field fluctuations are given by [17],

$$a_0^{\rm out} = a_0^{\rm in}, \tag{2}$$

$$a_{\frac{\pi}{2}}^{\rm out} = a_{\frac{\pi}{2}}^{\rm in} + 2\xi_{\rm a} X, \tag{3}$$

where X is the cavity length variation and $\xi_{\rm a}$ is an optomechanical parameter related to the intracavity mean field α, the cavity finesse $\mathcal{F}_{\rm a}$, and the optical wavelength λ,

$$\xi_{\rm a} = \frac{4\pi}{\lambda}\alpha\sqrt{2\mathcal{F}_{\rm a}/\pi}. \tag{4}$$

Equations (2) and (3) show that the reflected fluctuations reproduce the incident ones, but the phase quadrature is also sensitive to the cavity length variation X. Neglecting any mirror displacement, this variation corresponds to the length change $X_{\rm sig}$ due to the gravitational wave. The sensitivity of the measurement is only limited by the incident phase noise $a_{\frac{\pi}{2}}^{\rm in}$. For a coherent incident field, quantum fluctuations are characterized by a noise spectrum equal to 1 for any quadrature. One thus expects to be able to detect length variations small compared to the optical wavelength λ by using large values of the optomechanical parameter $\xi_{\rm a}$, that is for a high-finesse cavity and an intense incident beam.

Mirror displacements also limit the sensitivity of the measurement. In the following we focus on the displacements of a single mirror of the cavity, namely, the end mirror m in figure 1. The cavity length variation X in (3) is then the sum of the signal $X_{\rm sig}$ and the displacement $X_{\rm m}$ of mirror m, which corresponds to the back-action noise due to radiation pressure, and to classical noises such as seismic or thermal fluctuations. Its Fourier component at frequency Ω is related to the applied forces by [18],

$$-\,i\Omega Z_{\rm m} X_{\rm m} = \hbar\xi_{\rm a} a_0^{\rm in} + F_{\rm m}, \tag{5}$$

where $Z_{\rm m}$ is the mechanical impedance of the mirror. The first force is the radiation pressure of the intracavity field, expressed in terms of the incident intensity fluctuations $a_0^{\rm in}$. The second force $F_{\rm m}$ represents the classical coupling with the environment.

The information provided by the phase of the reflected field is described by an estimator $\hat{X}_{\rm sig}$ of the measurement which is obtained through a normalization of the output phase (3) as a displacement. It appears as the sum of the signal and extra noise terms,

$$\hat{X}_{\rm sig} = \frac{1}{2\xi_{\rm a}} a^{\rm out}_{\frac{\pi}{2}} = X_{\rm sig} + \frac{1}{2\xi_{\rm a}} a^{\rm in}_{\frac{\pi}{2}} + X_{\rm m}. \tag{6}$$

The sensitivity is limited by an equivalent input noise equal to the noises added in the estimator. Since all these noises are uncorrelated for an incident coherent light, the equivalent input noise spectrum $\Sigma_{\rm sig}$ is given by,

$$\Sigma_{\rm sig} = \frac{1}{4\xi_{\rm a}^2} + \sigma_{X_{\rm m}X_{\rm m}}, \tag{7}$$

$$= \frac{1}{4\xi_{\rm a}^2} + \frac{\hbar^2\xi_{\rm a}^2}{\Omega^2 \left|Z_{\rm m}\right|^2} + \frac{\sigma_{F_{\rm m}F_{\rm m}}}{\Omega^2 \left|Z_{\rm m}\right|^2}, \tag{8}$$

where $\sigma_{X_{\rm m}X_{\rm m}}$ and $\sigma_{F_{\rm m}F_{\rm m}}$ are the noise spectra of the displacement $X_{\rm m}$ and of the classical force $F_{\rm m}$. The first term in (8) is the measurement error due to the incident phase noise, the second term the back-action noise due to radiation pressure, and the last one the classical noise. Curve *a* in figure 3 shows the quantum-limited sensitivity obtained by neglecting this last term, and considering a suspended mirror for which the impedance reduces to

$$Z_{\rm m} \simeq -i\Omega M_{\rm m}, \tag{9}$$

where $M_{\rm m}$ is the mirror mass. Radiation pressure is dominant at low frequency with a $1/\Omega^4$ dependence of the noise power spectrum, whereas phase noise is dominant at high frequency with a flat frequency dependence, at least for frequencies smaller than the cavity bandwidth. Curve *b* is the so-called standard quantum limit which corresponds to the minimum noise reachable at a given frequency by varying the optomechanical coupling $\xi_{\rm a}$. For a given value of $\xi_{\rm a}$, the sensitivity is optimal at only one frequency defined as,

$$\Omega_{\rm a}^{\rm SQL} = \sqrt{2\hbar\xi_{\rm a}^2/M_{\rm m}}. \tag{10}$$

Squeezed states may change this behavior. Equations (5) and (6) show that the input noise is related to a specific combination of the incident intensity and phase quadratures. For a suspended mirror it is proportional to a particular incident quadrature $a^{\rm in}_{-\theta}$, with an angle θ defined by,

$$\hat{X}_{\rm sig} = X_{\rm sig} - \frac{1}{2\xi_{\rm a}\sin\theta} a^{\rm in}_{-\theta}, \tag{11}$$

$$\cot\theta = \left(\Omega_{\rm a}^{\rm SQL}/\Omega\right)^2. \tag{12}$$

The sensitivity of the measurement is then improved by using an incident squeezed state for which the noise of this quadrature is reduced [4]. Note that the optimal angle θ is frequency dependent so that the squeezing angle must vary with frequency. This can be done by sending a squeezed state with a constant squeezing angle in a detuned cavity [7].

Another possibility to improve the sensitivity is to take advantage of the self phase-modulation induced by radiation pressure to perform a back-action evading measurement. Instead of detecting the output phase quadrature, we measure an other quadrature $a^{\rm out}_{\theta}$. According to equations (2) and (3), the input-output relation for this quadrature is given by,

$$\frac{1}{2\xi_{\rm a}\sin\theta} a^{\rm out}_{\theta} = \frac{1}{2\xi_{\rm a}} a^{\rm in}_{\frac{\pi}{2}} + X_{\rm sig} + X_{\rm m} + \frac{\cot\theta}{2\xi_{\rm a}} a^{\rm in}_{0}. \tag{13}$$

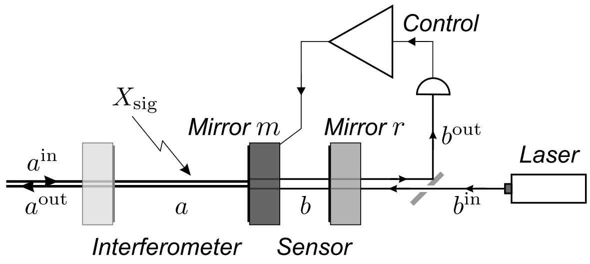

Figure 2. Active control of mirror m. Its motion is monitored by a second cavity based on a reference mirror r, and controlled by a feedback loop.

In the case of a suspended mirror, the two last terms cancel out for the particular quadrature angle given by (12). Radiation-pressure effects associated with the motion of mirror m then disappear in the measurement and the sensitivity is only limited by the phase noise [first term in (13)]. As in the case of the injection of squeezed state, beating the standard quantum limit over a wide bandwidth requires to adapt the detected quadrature to the frequency dependence of radiation-pressure effects [7]. This can be done by sending the reflected field in properly optimized detuned cavities.

In both cases, the sensitivity improvement strongly depends on the quantum properties of the optomechanical coupling between the light and the suspended mirror. It is in particular necessary to avoid losses in the interferometric measurement.

3. Quantum locking of a mirror

Figure 2 shows the scheme used to perform a quantum locking of mirror m. The mirror motion is monitored by another cavity made of the mirror m itself and a reference mirror r. The information is fed back to the mirror in order to control its displacements. The field b in this control cavity obeys equations similar to the ones of field a, except for the cavity length variation now equal to the relative displacement $X_{\rm r} - X_{\rm m}$ between the two mirrors. According to equations (2) and (3), the phase of the reflected field provides an estimator $\hat{X}_{\rm m}$ for the motion of mirror m, with a sensitivity limited by the incident phase noise and the motion of mirror r,

$$\hat{X}_{\rm m} = -\frac{1}{2\xi_{\rm b}} b^{\rm out}_{\frac{\pi}{2}} = X_{\rm m} - \frac{1}{2\xi_{\rm b}} b^{\rm in}_{\frac{\pi}{2}} - X_{\rm r}, \tag{14}$$

where $\xi_{\rm b}$ is the optomechanical parameter for the control cavity.

The mirror m is submitted to a feedback force proportional to this estimator. One has also to take into account the radiation pressures from both cavities, so that the motions of mirrors m and r are given by,

$$-i\Omega Z_{\rm m} X_{\rm m} = \hbar\xi_{\rm a} a_0^{\rm in} + F_{\rm m} - \hbar\xi_{\rm b} b_0^{\rm in} + i\Omega Z_{\rm fb} \hat{X}_{\rm m}, \tag{15}$$

$$-i\Omega Z_{\rm r} X_{\rm r} = \hbar\xi_{\rm b} b_0^{\rm in} + F_{\rm r}, \tag{16}$$

where $F_{\rm r}$ represents the classical noise of mirror r and $Z_{\rm fb}$ is the transfer function of the feedback loop. The resulting motion of mirror m is obtained from equations (14) and (15),

$$-i\Omega \left(Z_{\rm m} + Z_{\rm fb}\right) X_{\rm m} = \hbar\xi_{\rm a} a_0^{\rm in} + F_{\rm m} - \hbar\xi_{\rm b} b_0^{\rm in} - i\Omega Z_{\rm fb} \left(X_{\rm r} + \frac{1}{2\xi_{\rm b}} b^{\rm in}_{\frac{\pi}{2}}\right). \tag{17}$$

The control changes the response of mirror m by adding a feedback-induced impedance Z_{fb} to the mechanical impedance Z_{m}. For a large feedback gain, the effective impedance is increased and the mirror displacements are reduced. The control also contaminates the mirror motion by the noises in the control cavity [last terms in (17)].

We first examine the effect of the control on classical noise, neglecting all the quantum noises. According to equations (16) and (17), the resulting motion of mirror m is given by,

$$-i\Omega Z_{\mathrm{m}} X_{\mathrm{m}} \simeq \frac{Z_{\mathrm{m}} F_{\mathrm{m}} + Z_{\mathrm{fb}} F_{\mathrm{r}}}{Z_{\mathrm{m}} + Z_{\mathrm{fb}}}, \tag{18}$$

where we have assumed for simplicity the two mirrors identical so that $Z_{\mathrm{r}} = Z_{\mathrm{m}}$. As the feedback gain increases, the classical force F_{m} applied on mirror m is replaced by the force F_{r} acting on mirror r, which can be less noisy if the reference mirror is less coupled to its environment. From the expression (7) of the equivalent input noise Σ_{sig}, the classical noise in the interferometric measurement is reduced down to the displacement noise of mirror r. In other words, the control locks the motion of mirror m to the one of the reference mirror r, leading to a transfer of noise from the sensor measurement to the interferometric one.

A similar transfer of sensitivity occurs at the quantum level. For a very large feedback gain ($Z_{\mathrm{fb}} \to \infty$), only the last term in (17) is significant, and one gets

$$X_{\mathrm{m}} \simeq X_{\mathrm{r}} + \frac{1}{2\xi_{\mathrm{b}}} b^{\mathrm{in}}_{\frac{\pi}{2}}. \tag{19}$$

The motion of mirror m no longer depends on the radiation pressure in the interferometric measurement. Apart from the phase noise of beam b, the mirror m is locked at the quantum level on the reference mirror r. Its displacement noise reproduces the quantum noises in the sensor measurement, and the resulting sensitivity for the interferometric measurement, deduced from (7) and (16), is given by

$$\Sigma^{\infty}_{\mathrm{sig}} = \frac{1}{4\xi_{\mathrm{a}}^2} + \frac{1}{4\xi_{\mathrm{b}}^2} + \frac{\hbar^2 \xi_{\mathrm{b}}^2}{\Omega^2 \left|Z_{\mathrm{r}}\right|^2}. \tag{20}$$

The two last terms in this equation correspond to the equivalent input noise for the sensor measurement [compare to equation (8) with ξ_{a} replaced by ξ_{b}, and Z_{m} by Z_{r}]. This quantum transfer of noises is shown in curve c of figure 3, obtained with an optomechanical parameter ξ_{b} equal to $\xi_{\mathrm{a}}/5$. Since the sensor measurement is less sensitive than the interferometric one ($\xi_{\mathrm{b}} < \xi_{\mathrm{a}}$), radiation-pressure effects of beam b are smaller than the ones of beam a. At low frequency where these effects are dominant, the mirror m reproduces the motion of the reference mirror, leading to a clear reduction of noise.

At high frequency, the sensitivity is contaminated by the phase noise in the sensor measurement. This can easily be improved by using a frequency-dependent feedback gain in such a way that the control is efficient at low frequency whereas it plays no significant role at high frequency. Curve d of figure 3 shows the result obtained by an optimization of the feedback gain at every frequency. One gets a very clear noise reduction at low frequency while the sensitivity at high frequency is preserved.

4. Back-action cancellation

The quantum locking presented in the previous section cancels the radiation-pressure effects in the interferometer, replacing them by the less noisy effects in the sensor

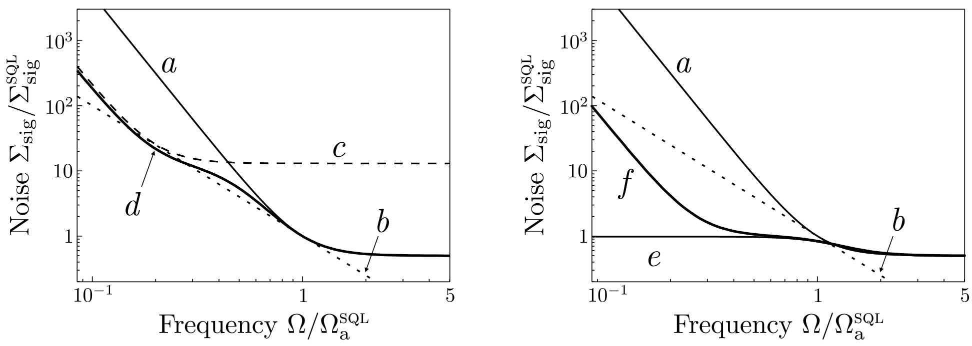

Figure 3. Equivalent input noise $\Sigma_{\rm sig}$ in the interferometric measurement as a function of frequency Ω: free interferometer (*a*), standard quantum limit (*b*), quantum locking with $\xi_{\rm b} = \xi_{\rm a}/5$ for infinite (*c*) and optimum (*d*) gains, back-action cancellation with $\xi_{\rm b} = \xi_{\rm a}$ (*e*), and with 1% loss in the control cavity (*f*). Noise is normalized to $1/2\xi_{\rm a}^2$ and frequency to $\Omega_{\rm a}^{\rm SQL}$.

measurement. It is possible to completely suppress the back-action noise in the sensor measurement by using the quantum optics techniques presented in section 2, such as the injection of squeezed states [4] or the optimization of the detected quadrature [7]. Using these techniques in the sensor measurement rather than in the interferometer itself presents the advantage that all the necessary adaptations has to be made on the sensor and not on the interferometer.

As in the case of the optimization of the detected quadrature for the free interferometer [equation (13)], measuring a quadrature $b_\theta^{\rm out}$ different from the phase quadrature changes the estimator $\hat{X}_{\rm m}$ by adding a term proportional to the incident intensity fluctuations $b_0^{\rm in}$,

$$\hat{X}_{\rm m} = -\frac{1}{2\xi_{\rm b}\sin\theta} b_\theta^{\rm out} = X_{\rm m} - \frac{1}{2\xi_{\rm b}} b_{\frac{\pi}{2}}^{\rm in} - X_{\rm r} - \frac{\cot\theta}{2\xi_{\rm b}} b_0^{\rm in}. \tag{21}$$

Different optimizations of the detected quadrature are possible. Since both mirror motions $X_{\rm m}$ and $X_{\rm r}$ depend on radiation pressure in the sensor cavity, one can eliminate the whole contribution or only the contribution due to the reference mirror. Considering two identical and suspended mirrors, the second solution is simpler since it corresponds to an angle θ given by an equation similar to (12) with $\Omega_{\rm a}^{\rm SQL}$ replaced by the frequency $\Omega_{\rm b}^{\rm SQL}$ defined as in (10). This angle is experimentally accessible by sending the field in a single detuned cavity [19]. It furthermore corresponds to a back-action evading measurement of the motion of mirror m by the sensor cavity. Neglecting the classical noise in the motion (16) of the reference mirror, the estimator $\hat{X}_{\rm m}$ of the sensor measurement indeed reduces for this value of θ to,

$$\hat{X}_{\rm m} = X_{\rm m} - \frac{1}{2\xi_{\rm b}} b_{\frac{\pi}{2}}^{\rm in}. \tag{22}$$

As compared to the standard detection scheme [equation (14)], the measurement is no longer sensitive to radiation-pressure effects in the sensor. Its sensitivity is only limited by the incident phase noise which can be made arbitrarily small by increasing the optomechanical coupling $\xi_{\rm b}$. For an infinite feedback gain, the control freezes the motion of mirror m down to a limit associated with the phase noise,

$$X_{\rm m} \simeq \frac{1}{2\xi_{\rm b}} b_{\frac{\pi}{2}}^{\rm in}, \tag{23}$$

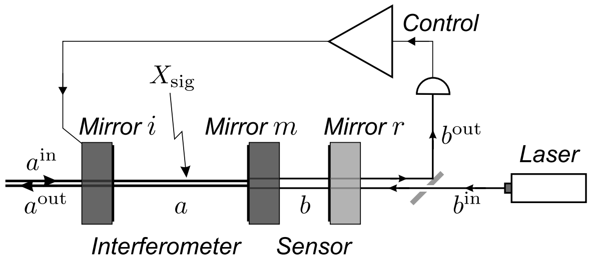

Figure 4. Active control of the whole interferometric cavity by a single feedback mechanism. The motions of mirrors m and r are measured by the sensor cavity. The information is fed back to the input mirror i in order to suppress the length variations of the interferometric cavity induced by radiation pressure.

and the resulting sensitivity for the interferometric measurement is deduced from (7),

$$\Sigma_{\rm sig}^{\infty} = \frac{1}{4\xi_{\rm a}^2} + \frac{1}{4\xi_{\rm b}^2}. \tag{24}$$

The sensitivity reduces to the sum of phase noises of both cavities. Curve e of figure 3 shows the sensitivity obtained with an optimized feedback gain. Radiation-pressure effects are completely suppressed, resulting in a sensitivity only limited by the phase noises and almost flat over the whole frequency band.

An essential feature of the active control is to be decoupled from other optimizations of the interferometer. Losses in the interferometer usually have a drastic effect on the noise reduction obtained by quantum optics techniques [7]. Quantum locking, however, is insensitive to such losses and they have no effect on the sensitivity improvement obtained with this technique. We have actually made no assumption on the exact motion of mirror m. The back-action cancellation in the sensor measurement and the quantum locking of the mirror do not depend on the optomechanical coupling between the mirror and the light in the interferometer. Imperfections in the interferometer thus do not affect the control.

As usual in quantum optics, losses must be avoided in the optomechanical sensor. Curve f in figure 3 shows the sensitivity obtained with 1% loss in the control cavity [19]. In contrast to the lossless case (curve e), back-action cancellation is no longer perfect. One however still has a very large reduction of radiation-pressure noise as compared to the free interferometer.

5. Quantum locking of a cavity

The input mirror of the interferometric cavity (mirror i in figure 4) moves as well in response to radiation pressure and a complete control of the interferometric measurement would require a local control of each mirror of the cavity. We show in this section that a single control mechanism can lock the cavity at the quantum level [20].

Taking into account the motion of mirror i, the estimator of the interferometric measurement now depends on the differential motion between mirrors m and i, and equation (6) is modified to

$$\hat{X}_{\rm sig} = X_{\rm sig} + \frac{1}{2\xi_{\rm a}} a_{\frac{\pi}{2}}^{\rm in} + X_{\rm m} - X_{\rm i}. \tag{25}$$

As shown in figure 4, the principle of the locking is to use the information provided by the sensor measurement to apply a feedforward force to the input mirror i, in such a way that its motion follows the one of mirror m. The key point is that the sensor gives access to the differential motion $X_{\rm m} - X_{\rm i}$ between mirrors m and i, for an appropriate choice of the detected quadrature. Neglecting the classical noise, the motion of mirror i without feedback is indeed related to the radiation pressure in the interferometric cavity,

$$- i\Omega Z_{\rm i} X_{\rm i} = -\hbar\xi_{\rm a} a_0^{\rm in}. \tag{26}$$

Equations (15), (16) and (26) show that the total radiation-pressure force exerted on the system composed of the three mirrors is equal to zero. The motions of the three mirrors are not independent and one gets,

$$Z_{\rm i} X_{\rm i} + Z_{\rm m} X_{\rm m} + Z_{\rm r} X_{\rm r} = 0. \tag{27}$$

In the case of identical and suspended mirrors, the sum of the three displacements cancels out. We then detect a quadrature $b_\theta^{\rm out}$ at the output of the sensor cavity with an angle θ defined by

$$\cot\theta = \frac{3}{2}\left(\Omega_{\rm b}^{\rm SQL}/\Omega\right)^2. \tag{28}$$

As in the previous section, this quadrature is obtained by sending the field $b^{\rm out}$ in a properly detuned cavity [19]. According to (21), the estimator of the sensor measurement is given by

$$\hat{X}_{\rm m} = X_{\rm m} + \frac{1}{2} X_{\rm r} - \frac{1}{2\xi_{\rm b}} b_{\frac{\pi}{2}}^{\rm in} = \frac{1}{2}\left(X_{\rm m} - X_{\rm i}\right) - \frac{1}{2\xi_{\rm b}} b_{\frac{\pi}{2}}^{\rm in}, \tag{29}$$

where we have used the relation $X_{\rm r} = -X_{\rm m} - X_{\rm i}$. Apart from the phase noise of beam b, the sensor measures the differential motion $X_{\rm m} - X_{\rm i}$.

The quantum locking is obtained by applying a feedforward force $-i\Omega Z_{\rm fb}\hat{X}_{\rm m}$ to mirror i, with a feedforward gain $Z_{\rm fb}$ equal to 2. This force induces a displacement of mirror i by a quantity $2\hat{X}_{\rm m}$, and its resulting motion is given by,

$$X_{\rm i} \rightarrow X_{\rm i} + 2\hat{X}_{\rm m} = X_{\rm m} - \frac{1}{\xi_{\rm b}} b_{\frac{\pi}{2}}^{\rm in}. \tag{30}$$

The mirror i is then locked on the mirror m. The differential motion $X_{\rm m} - X_{\rm i}$ no longer depends on radiation pressure and the sensitivity of the measurement, deduced from (25), reduces to the phase noises,

$$\Sigma_{\rm sig} = \frac{1}{4\xi_{\rm a}^2} + \frac{1}{\xi_{\rm b}^2}. \tag{31}$$

For an optomechanical parameter $\xi_{\rm b}$ larger than $\xi_{\rm a}$, the sensitivity is only limited by the phase noise $1/4\xi_{\rm a}^2$ in the interferometer. Radiation-pressure effects are completely suppressed.

Although this quantum locking is not a local control as in the previous sections, an important feature is that it is still insensitive to losses in the interferometer. As a matter of fact, we have made no assumption on the radiation pressure in the interferometer, except that both mirrors i and m are submitted to the same force [equations (15) and (26)]. Losses in a gravitational-wave interferometer are mainly due to imperfections on the mirrors, but the propagation between the two mirrors usually is lossless. Radiation pressures exerted on each mirror are the sum of the radiation pressures of the incoming and outgoing fields, and they are the same on both mirrors whatever the mirror losses are. Relation (27) and the principle of the quantum locking are then valid even in presence of losses in the interferometer.

6. Conclusion

The active control studied in this paper is based on a local optomechanical sensor which measures the position fluctuations of the mirror and locks its position with respect to a reference mirror. The main characteristic of the sensor is its sensitivity, defined by an optomechanical parameter ξ_{b} which depends on the cavity finesse and the light power. Quantum locking is efficient when the optomechanical parameter of the sensor is of the same order as the one of the interferometer. This seems easy to achieve with currently available technology [21, 14]. Taking for example the parameters of the VIRGO interferometer (15 kW light power in each Fabry-Perot arms with a global finesse of 600 [1]), this condition corresponds to a sensor of finesse 10^5 with an intracavity light power of 90 W, that is an incident light power of 1.5 mW only. Due to the high finesse of the cavity, the same sensitivity is reached for the sensor than for the interferometer itself, while the intracavity and incident light powers are much smaller.

This technique is useful to reduce classical noise such as thermal fluctuations, as long as the reference mirror of the sensor cavity is less noisy than the mirror of the interferometer. This has already been experimentally demonstrated for the thermal noise of internal acoustic modes of a mirror [14]. In this case, only the cooled mirror is resonant at frequency of interest and thermal noise reduction as large as 1000 have been obtained [15]. This technique may be of some help for cryogenic gravitational-wave interferometers. A major issue is the heat generation due to the absorption of the high-power light in the interferometer, which prevents from cooling to very low temperatures [22]. Since the light in the sensor is much less intense, a low temperature can be reached for the reference mirror by passive cryogenic cooling, and transferred to the interferometer mirror by active control. Concerning the internal thermal noise of mirrors, however, the sensor cavity must detect the noise as it is seen by the interferometric measurement. The optical waist in the sensor cavity must be adapted to the one in the interferometer, requiring to develop high-finesse cavities with large effective waists [23].

We have shown that a local control of mirrors allows one to efficiently reduce the quantum effects of radiation pressure in an interferometric measurement. The back-action noise is completely suppressed by using an optimized detection strategy. The sensitivity is thus greatly improved in the low-frequency domain where radiation-pressure effects are dominant, without alteration in the high-frequency domain where phase noise prevails.

In a practical implementation, the complete control of a gravitational-wave interferometer would require the use of optomechanical sensors for each sensitive mirrors, that is the four mirrors of the Fabry-Perot cavities in the interferometer arms. We have shown that a single control mechanism can lock the whole cavity at the quantum level. Adjusting the detected quadrature, an optomechanical sensor placed near the end mirror of the cavity can monitor the cavity length variations induced by radiation pressure. The information is then fed back to the front mirror in order to suppress radiation-pressure effects in the interferometric measurement.

Finally note that it is also possible to perform a correction of the signal delivered by the interferometer rather than to control the mirror motion. In that case, the optomechanical sensor detects the radiation-pressure effects in the interferometer, and the information is numerically subtracted from the result of the interferometric measurement. As long as the effect of the mirror motions on the output of the interferometer is known with a sufficient accuracy, the two techniques are in principle equivalent.

These results show that active control is a powerful technique to reduce quantum noise. An essential characteristic of this approach is to be decoupled from other optimizations of the interferometer. As usual in quantum optics, losses must be avoided in the optomechanical sensor. Quantum locking, however, is insensitive to imperfections in the interferometer. All the necessary adaptations has to be made on the control measurement, but the quantum locking does not induce any additional constraint on the interferometer.

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