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Bounded confidence and Social networks

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Abstract. In the so-called bounded confidence model proposed by Deffuant et al, agents can influence each other's opinion provided that opinions are already sufficiently close enough. We here discuss the influence of possible social networks topologies on the dynamics of this model.

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1 Introduction

Many models about opinion dynamics, [1], [2], [3], are based on binary opinions which social actors update as a result of social influence, often according to some version of a majority rule. Binary opinion dynamics have been well studied, such as the herd behaviour described by economists ([1], [3], [4]). When binary interactions can occur about any pair of agents randomly chosen, the attractors of the dynamics display uniformity of opinions, either 0 or 1. Clusters of opposite opinions appear when the dynamics occur on a social network with exchanges restricted to connected agents. These patterns remind of magnetic domains in Ising ferromagnets.

The spreading of epidemics on scale free networks [5] is also an instance of a binary state dynamics [6].

One issue of interest concerns the importance of the binary assumption: what would happen if opinion were a continuous variable such as the worthiness of a choice (a utility in economics), or some belief about the adjustment of a control parameter? These situations are encountered in economic and social science:

- In the case of technological changes economic agents have to compare the utilities of a new technology with respect to the old one, and e.g. surveys concerning the adoption of environment friendly practicies following the 1992 new agricultural policies [7] showed that agents have uncertainties about the evaluation of the profits when they adopt the new technique and thus partially rest on evaluations made by their "neighbours".
- Some social norms such as how to share the profit of the crop among landlords and tenants [8] do display the kind of clustering that we will further describe.

In the bounded confidence model of continuous opinion dynamics proposed by Deffuant etal [9], agents can influence each other's opinion provided that opinions are already sufficiently close enough. A tolerance threshold d is defined, such that agents with difference in opinion larger than the threshold can't interact. Several variants of the model have been proposed in [9] [10]. In these models, the only restriction for interaction is the threshold condition and interactions among any pair of agents can occur. The attractor of the dynamics are clusters which number increases by steps when the tolerance threshold is decreased.

The dynamics which we will describe here can be compared to the cultural diffusion model introduced by Axelrod: agents culture is represented by strings of integer in these models [11].

The purpose of this paper is to check the role of specific interaction structures on the result of the dynamics. We will investigate a bounded confidence interaction process on scale free networks and compare the obtained dynamics to what was already observed when all interactions are possible and when they occur on square lattices among nearest neighbours.

The paper is organised as follows:

- We first expose the simple case of complete mixing among agents.
- We then check the genericity of the results obtained for the simplest model to other topologies, mostly scale free networks.

We are mainly interested in:

- the clustering process,
- the possible existence of regime transitions according to the value of the threshold of influence d
- the relative importance of the clustering process with respect to the whole population. Do all or at least most agents participate into this process?

2 The basic case: Complete Mixing

Let us consider a population of N agents i with continuous opinion x_i. We start from an initial distribution of opinions, most often taken uniform on [0,1] in the computer simulations. At each time step any two randomly chosen agents meet: they re-adjust their opinion when their difference in opinion is smaller in magnitude than a threshold d. Suppose that the two agents have opinion x and x'.

Iff $|x - x'| < d$ opinions are adjusted according to:

$$x = x + \mu \cdot (x' - x) \tag{1}$$
$$x' = x' + \mu \cdot (x - x') \tag{2}$$

where μ is a convergence rate whose values may range from 0 to 0.5.

In the basic model [9], the threshold d is taken as constant in time and across the whole population. Note that we here apply a complete mixing hypothesis plus a random serial iteration mode[1].

For finite thresholds, computer simulations show that the distribution of opinions evolves at large times towards clusters of homogeneous opinions. The number of clusters varies as the integer part of $1/2d$: this is to be further referred to as the "1/2d rule" (see figure 1[2]).

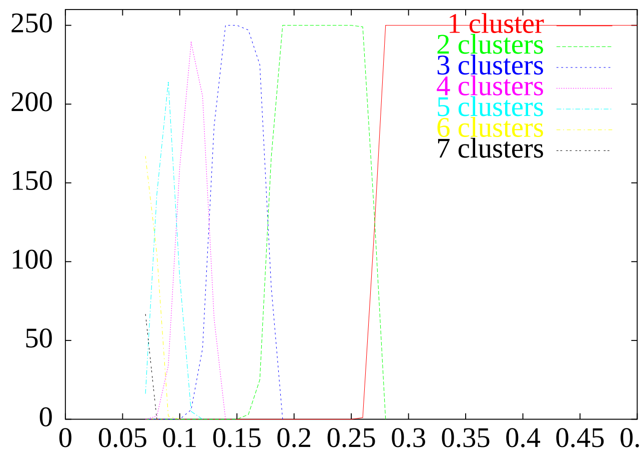

Fig. 1. Statistics of the number of opinion clusters as a function of d on the x axis for 250 samples ($\mu = 0.5$, $N = 1000$).

[1] The "consensus" literature [10] most often uses parallel iteration mode when they suppose that agents average at each time step the opinions of their neighbourhood. Their implicit rationale for parallel iteration is that they model successive meetings among experts.

[2] Notice the continuous transitions in the average number of clusters when d varies. Because of the randomness of the initial distribution and pair sampling, any prediction on the outcome of dynamics such as the 1/2d rule only becomes true with a probability close to one in the limit of large N.

3 The scale free network topology and opinion updating process

We use a standard method, see e.g. Stauffer and Meyer-Ortmanns [12]:

Starting from a fully connected network of 3 nodes, we add iteratively nodes (in general up to 900 nodes) and connect them to previously created nodes in proportion to their degree. We have chosen to draw two symmetrical connections per new added node in order to achieve the same average connection degree (4) as in the 30x30 square lattice taken as reference. But obviously the obtained networks are scale free as shown by Barabasi and Albert[5].

In fact scale free networks [5] display a lot of heterogeneity in nodes connectivity. In the context of opinion dynamics, well connected nodes might be supposed more influential, but not necessarily more easily influenced. At least this is the hypothesis that we choose here. We have then assumed asymmetric updating: a random node is first chosen, and then one of its neighbours. But only the first node in the pair might update his position according to equ.1, not both. As a result, well connected nodes are influenced as often as others, but they influence others in proportion to their connectivity. This particular choice of updating is intermediate between what Stauffer and Meyer-Ortmanns [12] call directed and undirected versions.

4 Clustering and transitions

A simple way to check clustering, and especially on average, for any topology is the dispersion index y proposed by Derrida and Flyberg [13]. y is the relative value of the ratio of the sum of the squared cluster sizes s_i^2 to the squared number of agents.

$$y = \frac{\sum_{i=1}^{n} s_i^2}{(\sum_{i=1}^{n} s_i)^2} \tag{3}$$

For m clusters of equal size, one would have $y = 1/m$. The smaller y, the more important is the dispersion in opinions.

When averaging over network topology and initial conditions the step structure (fig. 2) observed in the case of full mixing seems to be completely blurred. For scale free networks one observes a continuous increase of the Derrida Flyberg parameter as a function of the tolerance threshold with only a kink in the $d = 0.25, y = 0.7$ region; while two distinct steps at $y = 0.5$ and $y = 0.33$ are observed in the well mixed case, corresponding to the occurence of 2 and 3 large clusters respectively.

In fact the blurring of the transition in scale free networks is due to two effects:

– the S curve is the result of averaging over many network topologies and initial conditions.

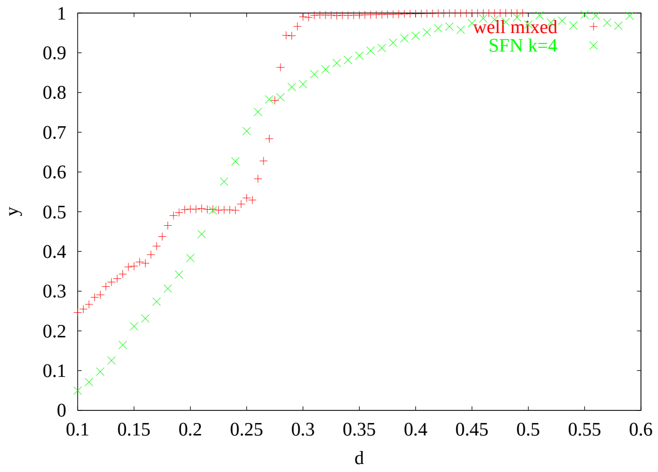

Fig. 2. Dispersion index y as a function of the tolerance threshold d for well mixed systems (red '+') scale free networks (green 'x') with 900 nodes. Each data point is the result of an average over 100 simulations.

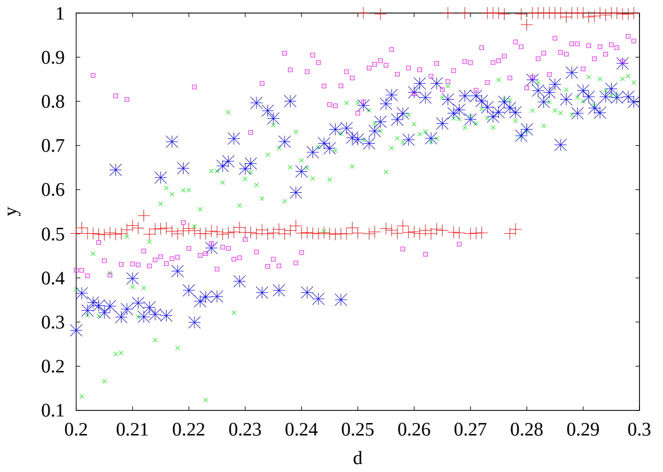

Fig. 3. Variation of the dispersion index y as a function of the tolerance threshold d. Big red '+' correspond to the well-mixed case, small green 'x' to square lattice , big blue '*' to scale free network with connectivity 4 and small violet squares to scale free network with connectivity 8

– Presence of outlying[3] nodes [14] in scale free networks , which remain out of the clustering process, decrease y, especially at low tolerance values.

When measurements are done on single instances of network topology and initial conditions, one observes y values corresponding to either one (larger y values) or two clusters (smaller y values) in the $0.2 < d < 0.3$ region. The proportion of these two y values varies with d, larger y values being more often obtained with larger d values. For the sake of comparison figure 3 displays the variations of the dispersion index with the tolerance threshold for three different topologies: the standard well-mixed case where any agent might interact with any other one, the square lattice and the scale free network with an average connectivity k equal to 4 and 8 ($k = 4$ is the same as the connectivity of the square lattice).

One observes that in the well mixed case the y values are either 0.5 or 1, with a rather narrow ambiguous region in d. For scale free networks, y values are smaller, an indication of the existence of many outlying agents which opinion does not cluster because they are too isolated (see further). Their distribution looks bimodal in a larger ambiguous region. The magnitude and dispersion of y values is similar for scale free network with connectivity 4 and square lattices. Increasing the average connectivity by a factor 2 brings the scale free network results closer to those of the well-mixed case. Connectivity at this stage seems more important than topology.

One of the most important questions in scale free networks is the role of the most connected nodes with respect to the less connected ones. In the context of opinion dynamics, we might want to figure out whether they are more influential, or eventually more influenced? One answer is provided by checking how far their opinion is changed by the clustering process. Figure 4 is a plot of final opinions of agents as a function of their initial opinion. Nodes connectivity are indicated by the size of the vertical bars. The importance of clustering is indicated by the density of points on horizontal lines while outlying agents are located on the first bisectrix.

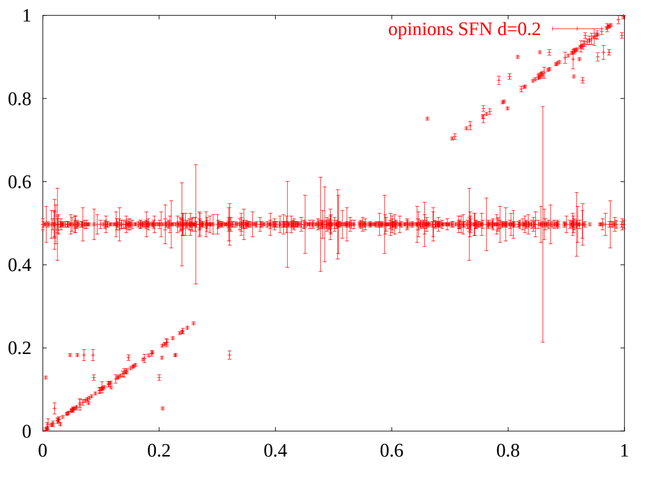

Fig. 4. Final opinions versus initial opinions on a scale free network with average connectivity 4 and tolerance 0.2. Vertical bars give the number of neighbours of each node (the largest correspond to 85).

Most of the well connected nodes belong to horizontal cluster at $x_\infty = 0.5$. They are far from the first bisectrix, which imply that they have been influenced during the clustering process.

[3] During the iterative process of opinion exchange, nodes with few connections have less chances to interact with a neighbour which opinion is close enough from their own opinion to actually interact. Many of them are not affected by the convergence process and remain outside the distribution of clustered opinions. We call them outlying nodes.

The first bisectrix is composed of less connected nodes, which initial and final opinion are more than $d = 0.2$ away from the cluster. These nodes have not changed their opinion. In scale free networks, static isolation (due to lower connectivity) often results in being kept out of the clustering process and remaining outlying. The effect is systematically observed for all tolerance thresholds less than 0.5. The outlying number explains why the highest values of y are lower than 1 in figure 3: only one central cluster is present, but it only contains a fraction of the nodes.

For the same k values, well mixed systems display horizontal clusters in this $[x_0, x_\infty]$ representation but very few outlying agents. Their occurence relates to dynamics: when the dynamics is fast some agents remain outlying when they are reached for a possible updating after the convergence process has been already well engaged, because of the randomness of the iteration process. Agents with initial extreme values have more chances to become outlying, but those who actually do, depend upon the particular instance of the random iteration.

Stauffer et al [12] have done extended statistics of the total number of different opinions after convergence in scale free networks . Since the number of outlying agents is much bigger than the number of big clusters, their figures give a very good characterisation of the number of outlying nodes.

For the sake of comparison we give the equivalent display for square lattices (fig. 5). The results are pretty similar to those obtained with scale free networks . The less populated horizontal lines correspond to small connected clusters on the lattice.

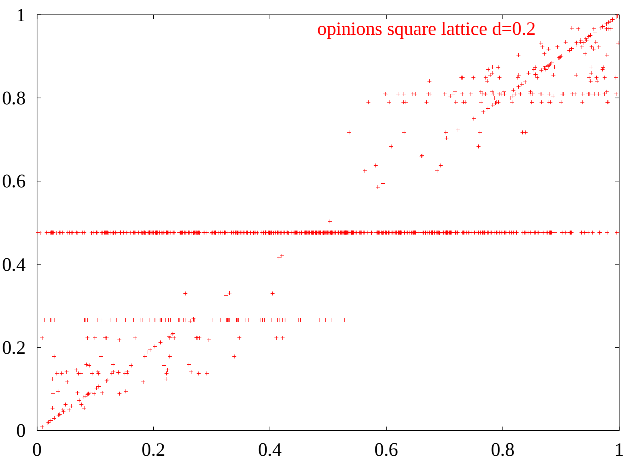

Fig. 5. Final opinions versus initial opinions on a 30x30 square lattice with tolerance 0.2.

5 Conclusions

In conclusion, restricting influence by a network topology does not drastically change the behaviour of these models of social influence as compared to the well mixed case. To summarize some of the resemblances and differences:

- One does observe clustering effects, and the number of observed main clusters does not largely differ for what is observed for equivalent tolerance thresholds in the well mixed case. Caution: we have only been discussing clusters in terms of opinions, not in terms of connections across the network. For small d values, clustering in opinion might structure the network in smaller connected regions with clustered opinions. One can expect the number of such non-interacting regions to be larger than the number of clusters (as observed on square lattices [9]).
- Stairs of y, the dispersion index, do appear: at least when measured without averaging on single instances of networks and initial conditions. But y values are decreased by a larger proportion of outlying agents and the transition regions in tolerance are larger.
- Well connected nodes are influenced by other nodes and are themselves influential. Most of them belong to the big cluster(s) after the clustering process.
- Larger connectivities bring scale free networks dynamic behaviour closer to well mixed systems.

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