PRECISION ANALYSIS OF THE LOCAL GROUP ACCELERATION

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We reexamine likelihood analyses of the Local Group (LG) acceleration, paying particular attention to nonlinear effects. Under the approximation that the joint distribution of the LG acceleration and velocity is Gaussian, two quantities describing nonlinear effects enter these analyses. The first one is the coherence function, i.e. the cross-correlation coefficient of the Fourier modes of gravity and velocity fields. The second one is the ratio of velocity power spectrum to gravity power spectrum. To date, in all analyses of the LG acceleration the second quantity was not accounted for. Extending our previous work, we study both the coherence function and the ratio of the power spectra. With the aid of numerical simulations we obtain expressions for the two as functions of wavevector and σ_8. Adopting WMAP's best determination of σ_8, we estimate the most likely value of the parameter β and its errors. As the observed values of the LG velocity and gravity, we adopt respectively a CMB-based estimate of the LG velocity, and Schmoldt et al.'s (1999) estimate of the LG acceleration from the PSCz catalog. We obtain $\beta = 0.66^{+0.21}_{-0.07}$; thus our errorbars are significantly smaller than those of Schmoldt et al. This is not surprising, because the coherence function they used greatly overestimates actual decoherence between nonlinear gravity and velocity.

1 Introduction

Comparisons between the CMB dipole and the Local Group (LG) gravitational acceleration can serve not only as a test for the kinematic origin of the former but also as a constraint on cosmological parameters. A commonly applied method of constraining the parameters by the LG velocity–gravity comparison is a maximum-likelihood analysis, elaborated by several authors (especially by Strauss et al. [1], hereafter S92). In a maximum-likelihood analysis, proper objects describing nonlinear effects are the *coherence function* (CF), i.e. the cross-correlation coefficient of the Fourier modes of the gravity and velocity fields, and the ratio of the power spectrum of velocity to the power spectrum of gravity. Here, with aid of numerical simulations we model the two quantities as functions of the wavevector and of cosmological parameters. We then combine these results with observational estimates of $\mathbf{v}_{\rm LG}$ and $\mathbf{g}_{\rm LG}$, and obtain the 'best' value of β and its errors.

2 Modelling nonlinear effects

We follow the evolution of the dark matter distribution using the pressureless hydrodynamic code CPPA (Cosmological Pressureless Parabolic Advection, see Kudlicki, Plewa & Różyczka 1996, Kudlicki et al. 2000 for details). It employs an Eulerian scheme with third order accuracy in space and second order in time, which assures low numerical diffusion and an accurate treatment of high density contrasts. Standard applications of hydrodynamic codes involve a collisional fluid; however, we implemented a simple flux interchange procedure to mimic collisionless fluid behaviour.

We studied the CF in an earlier paper.[2] Here we use a different fitting function, which is more accurate for low values of k:

$$C(k) = \left[1 + (a_0 k - a_2 k^{1.5} + a_1 k^2)^{2.5}\right]^{-0.2} . \tag{1}$$

Parameters a_l were obtained for 35 different values of σ_8 in the range [0.1,1], and we found the following, power-law, scaling relations:

$$\begin{aligned} a_0 &= 4.908\ \sigma_8^{0.750} \\ a_1 &= 2.663\ \sigma_8^{0.734} \\ a_2 &= 5.889\ \sigma_8^{0.714} . \end{aligned} \tag{2}$$

The fit was calculated for $k \in [0, 1]$ h/Mpc, with the imposed constraint $C(k = 0) = 1$.

We have found that the ratio of the power spectra, $\mathcal{R}$, obtained from simulation can be fitted with the following formula:

$$\mathcal{R}(k) = [1 + (7.071k)^4]^{-\alpha} , \tag{3}$$

with

$$\alpha = -0.06574 + 0.29195\sigma_8 \qquad \text{for } 0.3 < \sigma_8 < 1 . \tag{4}$$

3 Parameter estimation

We apply our formalism to the PSCz survey. As the value of σ_8 we adopt its WMAP's estimate, $\sigma_8 = 0.84$ (± 0.04; Spergel et al. 2003). This specifies the coherence function and the ratio of the power spectrum of velocity to the power spectrum of gravity. Also, this provides a normalization for the power spectrum of density.

3.1 Parameter dependence of the model

The likelihood of specific values of β and of the linear bias, b, is determined by the following distribution (Juszkiewicz et al. 1990, Lahav, Kaiser & Hoffman 1990):

$$f(\mathbf{g}, \mathbf{v}) = \frac{(1 - r^2)^{-3/2}}{(2\pi)^3 \sigma_{\mathbf{g}}^3 \sigma_{\mathbf{v}}^3} \exp\left[-\frac{x^2 + y^2 - 2r\cos\psi\, xy}{2(1 - r^2)}\right] , \tag{5}$$

where $\sigma_{\mathbf{g}}$ and $\sigma_{\mathbf{v}}$ are respectively the r.m.s. values of a single Cartesian component of gravity and velocity, $(\mathbf{x}, \mathbf{y}) = (\mathbf{g}/\sigma_{\mathbf{g}}, \mathbf{v}/\sigma_{\mathbf{v}})$, and ψ is the misalignment angle between $\mathbf{g}$ and $\mathbf{v}$. Finally, r is the cross-correlation coefficient of g_m with v_m, where g_m (v_m) denotes an arbitrary Cartesian component of $\mathbf{g}$ ($\mathbf{v}$).

In this distribution, the observables are $\mathbf{g}$ and $\mathbf{v}$, or g, v, and the misalignment angle, ψ. Following Schmoldt et al. [3] (hereafter S99), we adopt for them the following values: $g = 933$ km $\cdot$ s^{-1} (from the distribution of the PSCz galaxies up to 150 h^{-1} Mpc), $v = 627$ km $\cdot$ s^{-1} (inferred from the 4-year COBE data by Lineweaver et al. 1996), and $\psi = 15°$.

The theoretical quantities are $\sigma_{\mathbf{g}}$, $\sigma_{\mathbf{v}}$, and r. The variance of a single spatial component of measured gravity, $\sigma_{\mathbf{g}}^2$, is a sum of the cosmological component, $\sigma_{\mathbf{g},c}^2$, and errors, ϵ^2. Since gravity here is inferred from a galaxian, rather than mass, density field, we have $\sigma_{\mathbf{g},c}^2 = b^2 s_{\mathbf{g}}^2$, where $s_{\mathbf{g}}^2$ is the variance of a single component of the true (i.e., mass-induced) gravity,

$$s_{\mathbf{g}}^2 = \frac{1}{6\pi^2} \int_0^\infty \widehat{W}_g^2(k) P(k) dk . \tag{6}$$

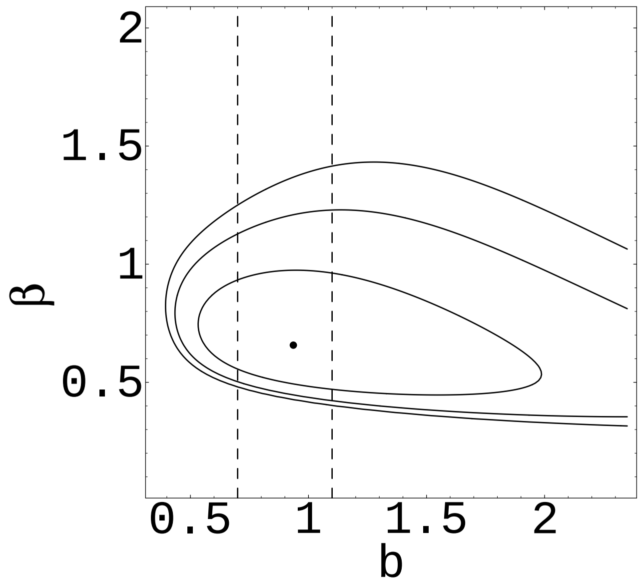

Figure 1: Likelihood contours for the parameters β and b, corresponding to the confidence levels of 68, 90 and 95%. The maximum of the likelihood function is denoted by a dot.

The gravity errors are twofold: due to finite sampling of the galaxy density field, and due to the reconstruction of the galaxy density field in real space. Therefore, $\epsilon^2 = (\sigma_{\rm SN}^2 + \sigma_{\rm rec}^2)/3$, where $\sigma_{\rm SN}^2$ and $\sigma_{\rm rec}^2$ are respectively the shot noise (or, sampling) variance and the reconstruction variance. In brief,

$$\sigma_{\mathbf{g}}^2 = b^2 s_{\mathbf{g}}^2 + \frac{\sigma_{\rm SN}^2 + \sigma_{\rm rec}^2}{3}\,, \tag{7}$$

where $\sigma_{\rm SN} = 160\ \mathrm{km}\cdot\mathrm{s}^{-1}$, and $\sigma_{\rm rec} = 58\ \mathrm{km}\cdot\mathrm{s}^{-1}$ (S99). Next, we have

$$\sigma_{\mathbf{v}} = \frac{\Omega_m^{0.6}}{6\pi^2}\int_0^\infty \widehat{W}_v^2(k)\mathcal{R}(k)P(k)dk\,. \tag{8}$$

Finally, errors in the estimate of the LG gravity do not affect the cross-correlation between the LG gravity and velocity, but increase the gravity variance. This has the effect of lowering the value of the cross-correlation coefficient. Specifically, we have

$$r = \rho\left(1 + \frac{\sigma_{\rm SN}^2 + \sigma_{\rm rec}^2}{3b^2 s_{\mathbf{g}}^2}\right)^{-1/2}\,, \tag{9}$$

where

$$\rho = \frac{\int_0^\infty \widehat{W}_{\mathbf{g}}(k)\widehat{W}_{\mathbf{v}}(k)C(k)\mathcal{R}^{1/2}(k)P(k)dk}{\left[\int_0^\infty \widehat{W}_{\mathbf{g}}^2(k)P(k)dk\right]^{1/2}\left[\int_0^\infty \widehat{W}_{\mathbf{v}}^2(k)\mathcal{R}(k)P(k)dk\right]^{1/2}}\,. \tag{10}$$

Thus, the likelihood depends explicitly on the parameters b and Ω_m, or on b and $\beta \equiv \Omega_m^{0.6}/b$.

3.2 Joint likelihood for β and b

Figure 1 shows isocontours of the joint likelihood for β and b, corresponding to the confidence levels of 68, 90 and 95%. The maximum of the likelihood is denoted by a dot. The corresponding values of β and b are respectively 0.66 and 0.94. The isocontours of the likelihood are much more elongated along the b-axis than along the β-axis, making the resulting constraints on Ω_m

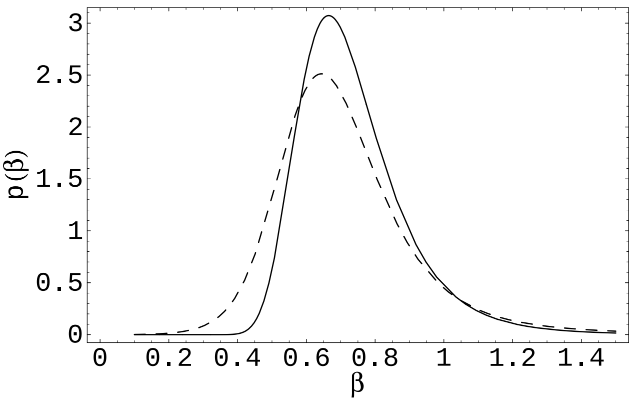

Figure 2: The marginal distribution for β. The result of marginalizing over all possible values of b (from zero to infinity) is shown as a dashed line. The result of marginalizing over the values of b in the range $[0.7, 1.1]$ is shown as a solid line.

much weaker than on β. In practice, therefore, from our analysis we cannot say much about bias, or Ω_m, alone. On the other hand, we can put fairly tight constraints on β.

To do this, we use the fact that the square root of the variance of the PSCz galaxy counts at 8 h^{-1} Mpc is $\sigma_8^{PSCz} \simeq 0.75$ (Sutherland et al. 1999). Combined with WMAP's estimate of σ_8, this yields for the bias of the PSCz galaxies the value about 0.9. Therefore, we adopt here a conservative prior for the bias, namely that it is constrained to lie in the range $[0.7, 1.1]$. These limits are marked in Figure 1 as dashed vertical lines. We marginalize the likelihood over the values of b in this range. The resulting distribution for β is shown in Figure 2 as a solid line. We obtain $\beta = 0.66^{+0.21}_{-0.07}$ (68% confidence limits).

4 Summary and conclusions

The analysis of the LG acceleration performed by S92 and S99 was in a sense more sophisticated than ours. Both teams analysed a differential growth of the gravity dipole in subsequent shells around the LG. Instead, here we used just one measurement of the total (integrated) gravity within a radius of 150 h^{-1} Mpc. Nevertheless, the errors on β we have obtained are significantly smaller than those of S92 and S99. In particular, S99 obtained $\beta = 0.70^{+0.35}_{-0.20}$ at 1σ confidence level. It is striking that while our best value of β is close to theirs, our errors are significantly smaller. The reason is our careful modelling of nonlinear effects. In a previous paper we showed that the coherence function used by S99 greatly overestimates actual decoherence between non-linear gravity and velocity. Tighter correlation between the LG gravity and velocity should result in a smaller random error of β; in the present work we have shown this to be indeed the case.

Acknowledgments

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