

# GENERALITY OF INFLATION IN CLOSED FRW MODELS

S.A. PAVLUCHENKO

*Sternberg Astronomical Institute, Moscow State University,  
Moscow 119992, Russia*



We investigate the generality of inflation in closed FRW models for a wide class of quintessence potentials. It is shown that inflation is not suppressed for most of them for a wide class of their parameters. This allows us to decide if inflation is common even in case of a closed universe.

## 1 Introduction

Recent observations of supernova type Ia (SNIa)<sup>1</sup> combined with cosmic microwave background (CMB) data<sup>2</sup> and data on large scale structure<sup>3</sup> provide us with evidence that our universe is accelerating now. One can explain this via a presence of a small positive  $\Lambda$  term (cosmological constant). Here we consider one kind of dynamical  $\Lambda$  term, namely, quintessence (see<sup>4,5</sup> for review). It can explain the stage of inflation expansion<sup>6</sup> and accelerating nowadays; this is the reason for the recent increasing interest in it. But theories with a scalar field as the source of expansion have a free parameter – the potential of this scalar field. The aim of this paper is to test some of these potentials that have attracted attention recently.

To speak about generality of inflation – or, in other words, about the probability of inflation for the model with particular potential one need to introduce the measure on initial conditions space and, so, parametrize the space of initial conditions. By the term "probability of inflation" we mean the ratio of the number of solutions experiencing inflation to the number of all possible solutions. By *the number of solutions* we mean the number one can obtain by using an evenly distributed net on the space of initial conditions.

## 2 Main equations

The equations describing the evolution of the universe in a closed FRW model are

Table 1: The dependence of the degree of inflationarity on  $\lambda$  for different powers of the power-law potential.

	$n = 4$			$n = 6$			$n = 8$		
$\lambda$	$10^{-4}$	$10^{-5}$	$10^{-6}$	$10^{-8}$	$10^{-9}$	$10^{-10}$	$10^{-13}$	$10^{-14}$	$10^{-15}$
$+\dot{\varphi}$	32.23	29.90	30.85	24.01	23.34	23.00	17.32	16.93	18.27
$-\dot{\varphi}$	28.41	27.95	29.60	20.42	20.99	21.47	14.73	15.07	16.83
ang.	63.34	63.39	63.37	63.37	63.37	63.37	63.32	63.37	63.37

$$\frac{m_P^2}{16\pi} \left( \ddot{a} + \frac{\dot{a}^2}{2a} + \frac{1}{2a} \right) + \frac{a}{4} \left( \frac{\dot{\varphi}^2}{2} - V(\varphi) \right) = 0,$$

$$\ddot{\varphi} + \frac{3\dot{\varphi}\dot{a}}{a} + \frac{dV(\varphi)}{d\varphi} = 0,$$

and the first integral of the system is

$$\frac{3m_P^2}{8\pi} \left( \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \right) = \left( V(\varphi) + \frac{\dot{\varphi}^2}{2} \right).$$

Also we need to introduce the parametrization. We will use *trigonometrical (angular)*  $(\phi, H)$  and *field*  $(\varphi, H)$  parametrizations (see<sup>7</sup> for details).

Our method is as follows. Starting from the Planck boundary for a given pair of initial conditions  $[(\phi, H)$  or  $(\varphi, H)]$  we numerically calculate the further evolution of the universe to determine whether universe will experience inflation or not.

### 3 Power-law potentials

Power-law potentials are potentials like

$$V(\varphi) = \frac{\lambda\varphi^n}{n}, \quad n \geq 2.$$

These potentials are well studied and they lead to chaotic inflation<sup>8</sup>. They have also attracted attention for their scaling properties<sup>9</sup>. Regarding the degree of inflationarity for this potential, for  $n = 2$  for the angular parametrization we have about 63% inflation and for the field parametrization about 47%. Increasing  $n$  will decrease the inflationarity in case of the field parametrization and will not change it for the angular parametrization. In the angular measure 63% of all possible solutions experience inflation for  $n = 2, 4, 6, 8$ , and in the field measure we have about 30% for  $n = 4$ , 22% for  $n = 6$ , and 17% for  $n = 8$  (see Table 1 for details; three last rows correspond to the cases of initial positive  $\dot{\varphi}$  and negative  $\dot{\varphi}$  for the field measure and last row corresponds to the angular measure). The Damour-Mukhanov potential<sup>10</sup> behaves like a power-law potential in this sense and the probability of inflation is about 63% in case of the angular measure and not less than 47% in case of the field measure (see Table 2 for details).

Table 2: The dependence of the degree of inflationarity on  $q$  (in columns) and  $\varphi_0$  (in rows) for the Damour-Mukhanov potential.

$\varphi_0 \backslash q$	0.5	0.8	1.0	1.5	1.8
0.1	84.94	74.18	67.46	55.46	50.98
0.3	84.93	73.66	67.46	55.46	50.98
0.5	84.63	73.66	67.46	55.46	50.98
0.9	84.63	74.18	67.46	55.46	50.98

#### 4 Inverse power-law potential

Pioneering studies of inverse power-law potentials have been done by Ratra and Peebles<sup>11</sup> and these potentials are like

$$V(\phi) = M^{(4+q)}\phi^{-q}.$$

The dependence of the degree of inflationarity on  $M$  is plotted in Fig. 1(a). The degree of inflationarity is on the  $y$  axis and the power  $q$  is on the  $x$  axis. There are three curves: I corresponds to the case  $M \sim m_P$ , II to  $M \sim 0.7m_P$ , and III to  $M \sim 0.4m_P$ . Note that for this case we can use only the angular measure.

#### 5 Exponential potential

Another interesting potential is exponential one<sup>11</sup>:

$$V(\phi) = V_0 \exp(-\lambda\phi).$$

Our results are the same for a wide range of  $V_0 \sim m_P^4 \div 10^{-10}m_P^4$ . So we have only one free parameter,  $\lambda$ , and the results are plotted in Fig. 1(b). And like in the previous case we can use only the angular measure.

#### 6 Conclusions

In this brief talk we have presented the main results obtained in<sup>7</sup>. We have investigated a wide class of quintessence potentials from the point of view of the generality of inflation. And we have made a weak enough test of them – are they able to provide our universe with inflation? And we obtained answer yes, closed FRW models with a scalar field with these potentials experience inflation for a wide range of their parameters. So inflation is general for a wide class of cosmological models.

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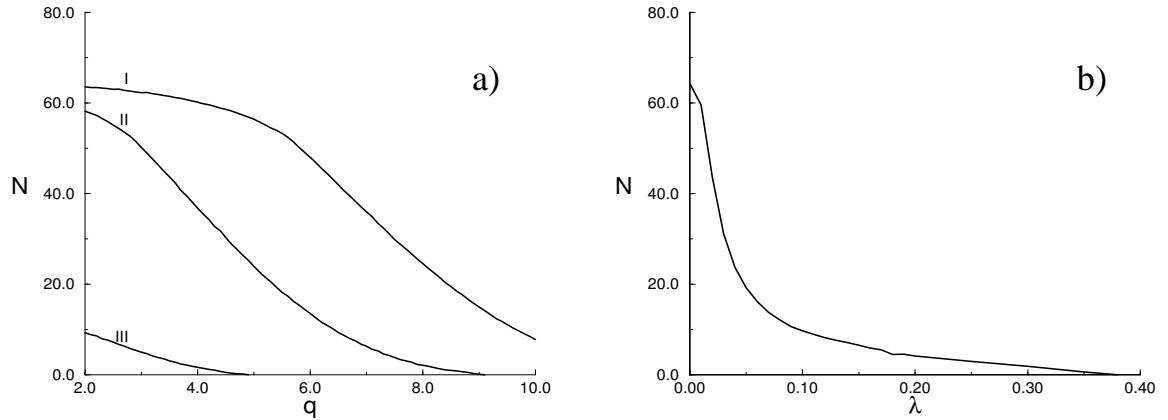


Figure 1: The dependence of the degree of inflationarity on the power  $q$  for the case of inverse power-law potential (a) and the same dependence but on  $\lambda$  for the case of exponential potential (b) (see text for details).

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