INHOMOGENEOUS CHAPLYGIN GAS COSMOLOGY

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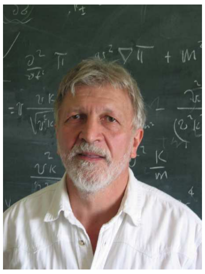

The hypothesis that dark matter and dark energy are unified through the Chaplygin gas is reexamined. Using a generalization of the spherical model which incorporates effects of the acoustic horizon we show that an initially perturbative Chaplygin gas evolves into a mixed system containing cold dark matter in the form of gravitational condensate. Furthermore, by including both condensate and residual gas, we demonstrate that the observed CMB angular and baryonic power spectra are reproduced

An appealing scenario in which dark matter and dark energy are different manifestations of a common structure , may be realized through the Chaplygin gas, an exotic fluid obeying

$$p = -A/\rho \, , \tag{1}$$

which has been extensively studied for its mathematical properties[1]. The cosmological potential of Eq. (1) was first noted by Kamenshchik *et al*[2] who observed that the Chaplygin gas interpolates between matter with $\rho \sim a^{-3}$ at high redshift and a cosmological constant like $\rho \sim \sqrt{A}$ as a tends to infinity. Of particular interest is that Eq. (1) may be obtained from[1,3,4]

$$\mathcal{L}_{\rm BI} = -\sqrt{A}\,\sqrt{1 - g^{\mu\nu}\,\theta_{,\mu}\,\theta_{,\nu}} \, , \tag{2}$$

by evaluating the stress-energy tensor $T_{\mu\nu}$, introducing $u_\mu = \theta_{,\mu}/\sqrt{g^{\alpha\beta}\,\theta_{,\alpha}\,\theta_{,\beta}}$ for the four-velocity, and $\rho = \sqrt{A}/\sqrt{1 - g^{\mu\nu}\,\theta_{,\mu}\,\theta_{,\nu}}$ for the energy density. One recognizes $\mathcal{L}_{\rm BI}$ a Born-Infeld type Lagrangian familiar in the D-brane constructions of string/M theory [1]. The Lagrangian (2) is a special case of the string-theory inspired tachyon Lagrangian [5] in which the constant $\sqrt{A}$ is replaced by a potential $V(\theta)$.

To be able to claim that the Chaplygin gas (or any other candidate) actually achieves unification, one must be assured that initial perturbations can evolve into a deeply nonlinear regime to form a gravitational condensate of superparticles that can play the role of cold dark matter (CDM). In comoving coordinates, the solution for inhomogeneous Chaplygin gas cosmology is[3,4]

$$\rho = \sqrt{A + B/\gamma}. \tag{3}$$

Here γ is the determinant of the induced metric $\gamma_{ij} = g_{i0}\, g_{j0}/g_{00} - g_{ij}$, and B can be taken as constant on the scales of interest. Eq. (3) allows us to implement the Zel'dovich approximation [6]: the transformation from Euler to Lagrange (comoving) coordinates induces $\gamma_{ij} = \delta_{kl} D_i{}^k D_j{}^l$, where $D_i{}^j = a(\delta_i{}^j - b\varphi_{,i}{}^j)$ is the deformation tensor, φ is the velocity potential, and the quantity $b = b(t)$ describes the evolution of the perturbation. The Zel'dovich approximation offers a means of extrapolation into the nonperturbative regime with the help of Eq. (3) and

$$\gamma = a^6 (1 - \lambda_1 b)^2 (1 - \lambda_2 b)^2 (1 - \lambda_3 b)^2, \tag{4}$$

where the λ_i are the eigenvalues of $\varphi_{,i}{}^j$. When one (or more) of the λ's is (are) positive, a caustic forms on which $\gamma \to 0$ and $p/\rho \to 0$, i.e., at the locations where structure forms the Chaplygin gas behaves as dark matter. Conversely, when all of the λ's are negative, a void forms, ρ is driven to its limiting value $\sqrt{A}$, and the Chaplygin gas behaves as dark energy, driving accelerated expansion. For the issue at hand, the Zel'dovich approximation has the shortcoming that the effects of finite sound speed are neglected. Indeed, in the Newtonian limit $p \ll \rho$, an explicit solution for the perturbative density contrast of the pure Chaplygin gas

$$\delta_{\rm pert}(k, a) \;\propto\; a^{-1/4} J_{5/14}(d_{\rm s} k)\,, \tag{5}$$

has been obtained [7]. Here $J_\nu(z)$ is the Bessel function, k the comoving wave number, and $d_{\rm s}$ the comoving sonic horizon given by

$$d_{\rm s} = \frac{2}{7} \frac{\left(1 - \Omega^2\right)^{1/2}}{\Omega^{3/2}} \frac{a^{7/2}}{H_0}\,, \tag{6}$$

with the equivalent matter fraction $\Omega = \sqrt{B/(A+B)} = \sqrt{B}/\rho_{\rm cr}$. Thus, for $d_{\rm s} k \ll 1$, $\delta_{\rm pert} \sim a$, but for $d_{\rm s} k \gg 1$, $\delta_{\rm pert}$ undergoes damped oscillations.

Since the structure formation occurs in the decelerating phase, we can address the question within Newtonian theory by generalizing the spherical model. In the case of vanishing shear and rotation, the continuity and Euler-Poisson equations become

$$\dot{\rho} + 3\mathcal{H}\,\rho = 0\,; \qquad 3\dot{\mathcal{H}} + 3\mathcal{H}^2 + 4\pi G\rho + \vec{\nabla}\cdot\left(\frac{v^2}{\rho}\,\vec{\nabla}\rho\right) = 0, \tag{7}$$

where $\mathcal{H}$ is the local Hubble parameter. It is reasonable to approximate $\delta(t,\vec{x}) \equiv \rho(t,\vec{x})/\bar{\rho}(t) - 1$ by the spherical lump $\delta(t,\vec{x}) = \delta_k(t)\sin(kx)/(kx)$. We then find that $\delta_k(a)$ satisfies [8]

$$a^2\delta_k'' + \frac{3}{2}\,a\,\delta_k' - \frac{3}{2}\,\delta_k(1+\delta_k) - \frac{4}{3}\frac{(a\,\delta_k')^2}{1+\delta_k} + \frac{49}{4}\left(\frac{a}{a_k}\right)^7 \frac{\delta_k}{(1+\delta_k)^2} = 0. \tag{8}$$

where $a_k = (d_{\rm s} k)^{-2/7} a$. Eq. (8) reproduces (5) at linear order and extends the spherical dust model by incorporating the Jeans length through the last term.

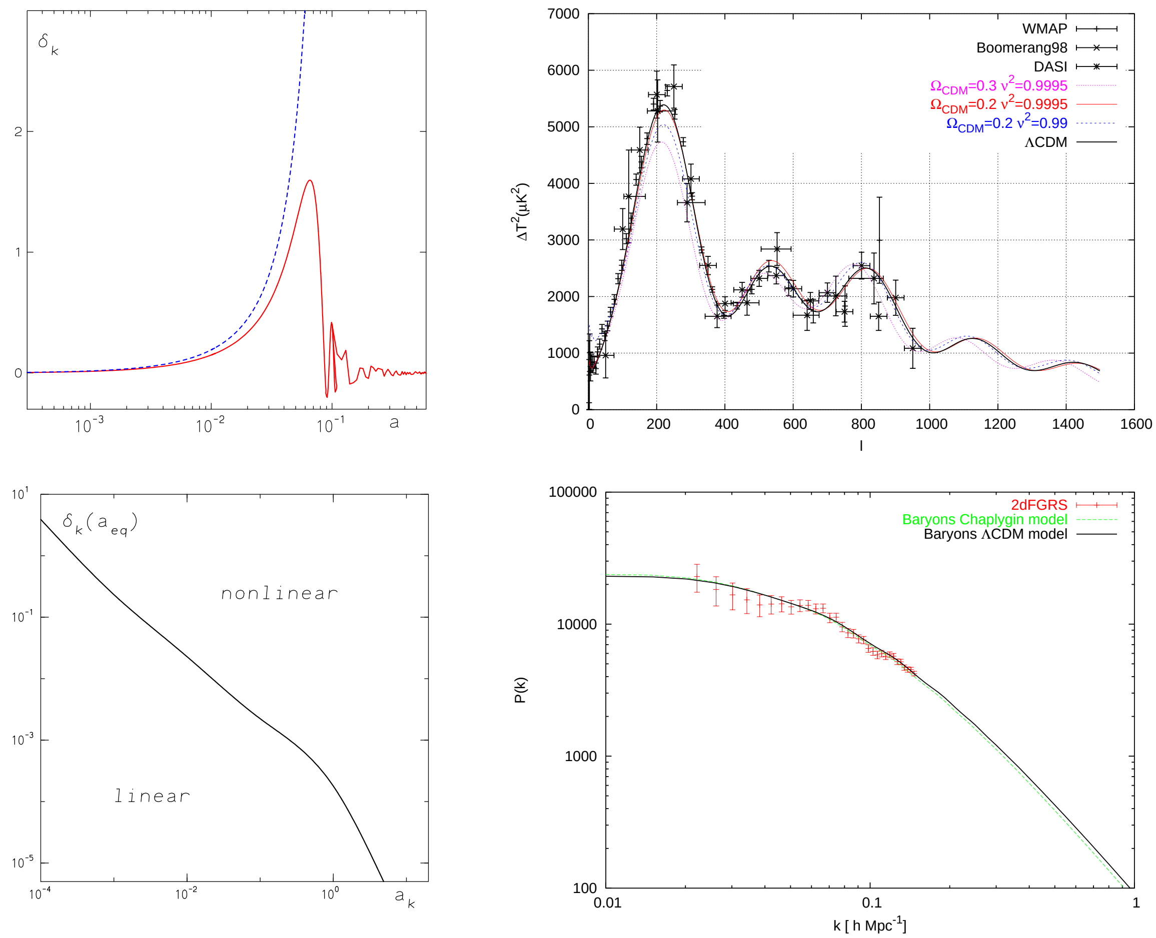

Figure 1: (a) Top left: Evolution of $\delta_k(a)$ in the spherical model, Eq. (8), from $a_{\rm eq} = 3 \times 10^{-4}$ for $a_k = 0.0476$, $\delta_k(a_{\rm eq})$ =0.004 (solid) and $\delta_k(a_{\rm eq})$ =0.005 (dashed). (b) Bottom left: The critical $\delta_k(a_{\rm eq})$ versus a_k. (c) Top right: CMB angular power spectrum for a mixture of Chaplygin gas and condensate with $\Omega_{\rm gas} = 0.7$ and $h = 0.7$, varying $\Omega_{\rm CDM}$ and v^2. (c) Bottom right: Baryon power spectra for the mixed Chaplygin gas with $\Omega_{\rm CDM} = 0.2$ and $v^2 = 0.9995$ and for the ΛCDM model. A normalization factor of 1.35 is used.

In Fig. 1a we show the evolution of two initial perturbations from radiation-matter equality for $a_k = a_{\rm reion} = 1/21$. One sees that for sufficiently small $\delta_k(a_{\rm eq})$, the acoustic horizon can stop $\delta_k(a)$ from growing even in a mildly nonlinear regime. Conversely, for $\delta_k(a_{\rm eq})$ above the threshold, $\delta_k(a) \to \infty$ at finite a just as in the dust model. The critical $\delta_k(a_{\rm eq})$ dividing the two regimes depends strongly on a_k (Fig. 1b). Qualitatively similar conclusions have been reached in a different way by Avelino *et al*[9]. Since the critical δ_k is commensurate with the peak in the conditional probability distribution for spheroidal collapse[10], and a_k is only weakly dependent on the comoving wave number, we can thus be confident that the Chaplygin gas will evolve at high redshift into a mixed system consisting of smoothly distributed gas and gravitational condensate. The latter will participate in hierarchical clustering as CDM.

Homogeneous world models containing a mixture of cold dark matter and Chaplygin gas have been successfully confronted with lensing statistics[11] as well as with supernova and other tests[12]. Clearly, it is only a matter of interpretation to replace 'cold dark matter' by 'Chaplygin droplet matter' (condensate). On the other hand, "unification" has often been misconstrued to mean a mixture of baryonic and perturbative Chaplygin gas only[12–15]. Owing to the damped oscillation of perturbations and the driving decay of the gravitational potential, it is hardly surprising that the baryon plus the purely perturbative Chaplygin gas model is in gross conflict with the

CMB[13,14] and mass power spectrum[15] data. Hence we have undertaken a calculation of the CMB anisotropies and the mass power spectrum in our unification scenario based on the Chaplygin gas and including nonlinear condensate. As the sonic horizon is negligible at recombination and the droplets affect the CMB and the power spectrum only through the gravitational potential, it is adequate to treat them throughout as ordinary cold dark matter, parametrized by $\Omega_{\rm CDM}$, in the perturbation equations. The residual uncondensed gas is parametrized as

$$\bar{\rho}_{\rm gas}(a) = \rho_{\rm cr}\Omega_{\rm gas} \left[v^2 + (1-v^2)/a^6\right]^{1/2} \tag{9}$$

in the background and

$$\bar{v}^2(a) = v^2 \left[v^2 + (1-v^2)/a^6\right]^{-1} \tag{10}$$

in the perturbation equations. The parameters $\Omega_{\rm gas}$ and v are not unrelated. From the statistical distribution of the eigenvalues of the deformation tensor it follows that only in 8% of the cases there is expansion along all three principal axes[6], hence only about 8% of the initial Chaplygin gas should fail to condense. Therefore, we expect the initial fraction of uncondensed gas to be

$$\Omega_{\rm gas}\sqrt{1-v^2}\,(\Omega_{\rm CDM} + \Omega_{\rm gas}\sqrt{1-v^2})^{-1} \simeq 0.08. \tag{11}$$

This equation gives an estimate of the parameter v^2 in terms of $\Omega_{\rm gas}$.

In Fig. 1c we compare the CMB angular power spectrum obtained by implementing (9) and (10) in a modification of the CMBfast[16] code with the WMAP data[17]. Albeit the result is preliminary, and by no means a best fit, it is apparent that the CMB data can be described by an evolved mixture of Chaplygin gas and condensate with parameters satisfying Eq. (11).

In Fig. 1d we exhibit the baryon power spectrum calculated in the two models: the mixed Chaplygin gas and the ΛCDM with the optimal CMB parameters of Fig. 1c. The model spectra have been convolved with the 2dFGRS window function and their amplitude fitted to the power spectrum data[18]. Again, the data is well described in the range of k in which the window function is available. Hence, it may safely be said that unification stands up as a viable scenario.

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