Phase-control of directed diffusion in a symmetric optical lattice

M. Schiavoni, L. Sanchez-Palencia, F. Renzoni and G. Grynberg
Laboratoire Kastler-Brossel, Département de Physique de l'Ecole Normale Supérieure, 24, rue Lhomond, 75231, Paris Cedex 05, France.
(Dated: September 29, 2003)

We demonstrate the phenomenon of directed diffusion in a symmetric periodic potential. This has been realized with cold atoms in a one-dimensional dissipative optical lattice. The stochastic process of optical pumping leads to a diffusive dynamics of the atoms through the periodic structure, while a zero-mean force which breaks the temporal symmetry of the system is applied by phase-modulating one of the lattice beams. The atoms are set into directed motion as a result of the breaking of the temporal symmetry of the system.

PACS numbers: 05.40.-a, 05.45.-a

It is now about two centuries that scientists observe and model the motion of microscopic particles in a fluctuating environment. The year 1828 can probably be indicated as the birth date of this field of research, with the observation by Brown [1] of the random motion of particles in a fluid. And it took about a century before that this phenomenon, now known as Brownian motion, was modeled by Einstein [2]. More recently, the problem of modeling molecular motors [3], i.e. microscopic objects moving unidirectionally along periodic structures, has renewed the interest in the field and stimulated much theoretical work devoted to the study of the directed motion in a fluctuating environment in the absence of bias forces. Molecular motors have been modeled by an asymmetric potential (ratchet) and non-gaussian noise [4]. Unidirectional motion in a ratchet potential is also obtained with gaussian noise and an applied periodic force of zero average [4, 5, 6, 7].

In this work we demonstrate the phenomenon of directed diffusion (DD), i.e. directed motion in a fluctuating environment, in a *symmetric* optical lattice. Consider the diffusive dynamics in a periodic potential $U(x)$ of period λ, $U(x+\lambda) = U(x)$, in the presence of a driving force $F(t)$ of period T, $F(t+T) = F(t)$. If the system is symmetric in the sense that $U(-x) = U(x)$ and $F(t+T/2) = -F(t)$, there is no net average transport through the periodic structure [5, 8, 9, 10]. Therefore to observe directed motion the spatiotemporal symmetry of the system has to be broken. For a spatially symmetric potential, the symmetry of the system can be broken by applying a non-monochromatic driving force containing both odd and even harmonics. In the present investigation the driving force has two components of frequencies ω and 2ω and phase difference ϕ. We will demonstrate experimentally the phenomenon of DD for such a configuration, with the phase ϕ playing the role of control parameter for the amplitude and sign of the current of atoms through the lattice.

Our symmetric periodic potential corresponds to a one-dimensional lin⊥lin optical lattice [11]. The periodic structure is determined by the interference of two counterpropagating laser beams (L_1 and L_2), with crossed linear polarizations (Fig. 1). This arrangement results in a periodic modulation of the light polarization, which produces a periodic modulation of the light shifts of the different ground states of the atoms. In this way an atom experiences a periodic potential (*optical potential*), whose amplitude and phase depend on the internal state of the atom. This dependence allows Sisyphus cooling [11] to take place. Indeed, the optical pumping between the different atomic ground states combined with the spatial modulation of the optical potential leads to the cooling of the atoms and to their localization at the minima of the optical potentials, thus producing a periodic array of trapped atoms. The transport of atoms through the lattice is determined by the optical pumping between different ground state sublevels. In fact, atoms at the bottom of a potential well strongly interact with the light and therefore undergo fluorescence cycles. The stochastic process of optical pumping may transfer an atom from a potential well to a neighbouring one corresponding to a different optical potential. This results in the transport of atoms through the lattice. More precisely, in a wide range of lattice parameters the atomic dynamics corresponds to normal diffusion [12, 13].

In order to generate a time-dependent homogeneous force, we apply a phase-modulation to one of the lattice beams, so that to obtain the electric field configuration

$$\vec{E} = E_0 \mathrm{Re}\Big\{ \vec{\epsilon}_x \exp\left[i(kz - \omega_L t)\right] + \vec{\epsilon}_y \exp\left[i(-kz - \omega_L t + \alpha(t))\right] \Big\} . \tag{1}$$

Here E_0 is the (real) amplitude of the electric field, k and ω_L the lattice-field wavevector and frequency, respectively. The modulated phase is $\alpha(t)$. In the laboratory reference frame this laser configuration generates a moving optical potential $U(2kz - \alpha(t))$. To be explicit, consider the case of a $J_g = 1/2 \to J_e = 3/2$ transition, which is the simplest atomic transition for which Sisyphus cooling takes place. In this case the moving bi-potential for the $|g, m = \pm 1/2\rangle$ ground states is $U_\pm(2kz - \alpha(t))$ with $U_\pm(\xi) = U_0[-2 \pm \cos\xi]$, U_0 being the depth of the potential wells. Consider now the dynamics in the moving reference frame defined by the coordinate transformation $z' = z - \alpha(t)/2k$. In this accelerated reference frame the optical potential is stationary. In addition to this potential, the atom, of mass M, experiences also an inertial force F in the z-direction proportional to the acceleration a of the moving frame [14, 15]:

$$F = -Ma = -\frac{M}{2k}\ddot{\alpha}(t) \ . \tag{2}$$

By choosing a phase modulation of the form

$$\alpha(t) = \alpha_0[A\cos(\omega t) + \frac{B}{4}\cos(2\omega t - \phi)] \tag{3}$$

with ϕ constant, we obtain the inertial force

$$F = \frac{M\omega^2\alpha_0}{2k}\,[A\cos(\omega t) + B\cos(2\omega t - \phi)] \tag{4}$$

which is the sum of two forces oscillating at the frequencies ω and 2ω, with phase difference ϕ. Hence, in the accelerated frame the atoms cooled and trapped in the optical lattice experience a force containing both even and odd harmonics, so that our system is suitable for the observation of DD. All the results presented in this work are obtained in the regime of non-adiabatic driving, with the frequency ω of the driving force about equal to the frequency Ω_v of oscillation of the atoms at the bottom of the potential wells.

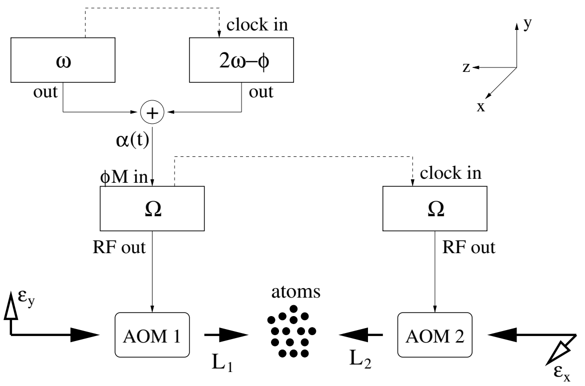

FIG. 1: Sketch of the experimental setup. The phase of the laser field $L1$ is $-kz - \omega_L t + \alpha(t)$, while the phase of the field $L2$ is $kz - \omega_L t$. Here ω_L includes the frequency shift Ω produced by the acousto-optical modulators.

In our experiment ^{85}Rb atoms are cooled and trapped in a magneto optical trap (MOT). This is obtained by applying an inhomogeneous magnetic field, and three orthogonal pairs of counterpropagating $\sigma^{\pm}$ laser fields. We indicate by x, y, z the propagation directions of these fields. At a given instant the MOT magnetic field is turned off and the circularly polarized laser fields along the z-axis are replaced by the two crossed polarized lattice beams. The $\sigma^{\pm}$ laser fields in the x and y directions are left on, so to provide a friction force in the direction orthogonal to the one of the periodic potential. In this way the motion of the atoms in the x and y direction is damped, and the atomic dynamics in the z direction can be studied for longer times. The appropriate (modulated) phase relation between the two lattice fields (Eq. 3) is obtained by using two acousto-optical modulators (AOM), one for each lattice beam (Fig. 1). The AOMs are driven by radio-frequency generators oscillating at $\Omega = 76$ MHz and sharing the same reference clock. One of this radio-frequency generator is phase-modulated by a signal obtained by mixing the output of two oscillators at frequencies ω and 2ω ($\omega \simeq 100$ kHz) and phase difference ϕ. These two oscillators share the same reference clock.

We studied the dynamics of the atoms in the optical lattice by direct imaging with a Charge Coupled Device (CCD) camera. For a given phase ϕ we took images of the atomic cloud at different instants after the atoms have been loaded into the optical lattice. From the images of the atomic cloud we determined the position along the z axis of the center-of-mass (CM) of the atomic cloud as a function of the lattice duration. It should be noted that for the typical time scales of our experiments the measured position of the CM of the atomic cloud in the laboratory and in the accelerated reference frames are approximately equal. In fact the accelerated frame oscillates with an amplitude of about 1 μm, while the typical displacement of the CM associated to the directed diffusion is 100 μm. Furthermore, for a typical frequency $\omega \simeq 100$ kHz the position z in the laboratory frame and the corresponding position in the accelerated frame $z' = z - \alpha(t)/(2k)$, with $\alpha(t)$ given by (3), are equivalent when averaged over a typical exposure time of 1 ms. Therefore for the measurement of the position of the CM of the atomic cloud no coordinate transformation is needed to go from the laboratory frame to the accelerated frame where the description in terms of a static potential and an applied force is valid. We made several measurements for different values of the phase ϕ. We observed that the CM of the atomic cloud moves along the z axis with constant velocity, as shown in the inset of Fig. 2. We determined the CM-velocity as a function of the phase ϕ, with results as in Fig. 2. The experimental results of figures 2 clearly demonstrate the phenomenon of directed diffusion in a symmetric periodic potential: the atoms can be set into a directed motion through a symmetric potential by breaking the temporal symmetry of the system.

The dependence of the CM-velocity on the phase ϕ, shown in Fig. 2, can be explained by examining the temporal symmetries of the system [9, 10]. In fact although the symmetry $F(t + T/2) = -F(t)$ is broken for any value of the phase ϕ, there is an additional temporal symmetry $F(t) = F(-t)$, which implies zero net current through the potential for particular values of ϕ [9, 10]. This symmetry is realized for $\phi = n\pi$, with n integer, and maximally broken for $\phi = (n + 1/2)\pi$. This explains the observed dependence of the CM-velocity on the phase ϕ, and shows that in our system ϕ is the control parameter of the directed diffusion.

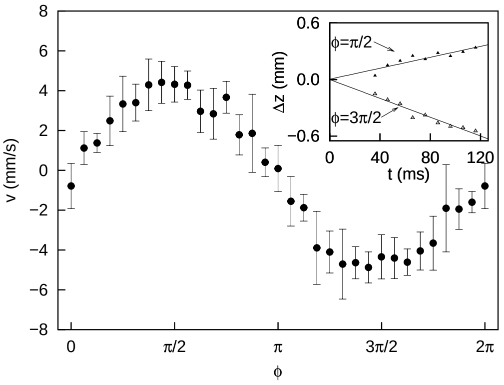

FIG. 2: Velocity of the center-of-mass of the atomic cloud as a function of the phase ϕ. Inset: Displacement along the z axis of the CM of the atomic cloud as a function of the lattice duration for the two values of the phase ϕ corresponding to the maximum velocity in the two opposite directions ($\pm z$), together with the linear fits. The detuning of the lattice fields from atomic resonance is $\Delta = 36$ MHz, the intensity per lattice beam is $I_L = 7$ mW/cm^2. For these parameters the oscillation frequency of the atoms at the bottom of the potential well is $\Omega_v \simeq 105$ kHz. The parameters for the phase-modulation signal $\alpha(t)$ (see Eq. 3) are $\omega = 113$ KHz, $A = 3/4$, $B = 1$ with $\alpha_0 = 10$ rad.

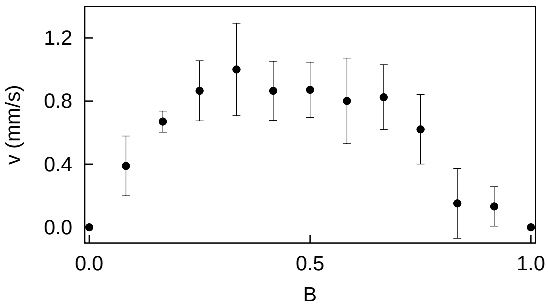

FIG. 3: Velocity of the center-of-mass of the atomic cloud as a function of the amplitude B of the component at 2ω of the driving force, for constant sum of the amplitudes of the two harmonics at ω and 2ω. The parameters for the optical lattice are the same as for Fig. 2. The phase-modulation signal is given by Eq. (3) with $A = 1 - B$, $\omega = 100$ KHz, $\alpha_0 = 12$ rad and $\phi = \pi/2$.

To demonstrate experimentally that directed diffusion is determined by the breaking of the symmetry $F(t + T/2) = -F(t)$, we fix the phase ϕ equal to $\pi/2$, so to maximally break the $F(t) = F(-t)$ symmetry, and study the CM-velocity as a function of the amplitudes of the harmonics of frequencies ω and 2ω of the driving force. We choose a phase-modulation of the form of Eq. (3) with $A = 1 - B$: $\alpha(t) = \alpha_0[(1 - B)\cos(\omega t) + B/4\cos(2\omega t - \phi)]$, so to obtain a force $F = M\omega^2\alpha_0/2k[(1 - B)\cos(\omega t) + B\cos(2\omega t - \phi)]$. Thus, by varying the parameter B we vary the ratio of the amplitudes of the two components of the force at frequencies ω and 2ω, while keeping constant their sum. The experimental results are shown in Fig. 3. We observe that for $B = 0$ and $B = 1$, which correspond to a monochromatic driving force, there is no net transport of atoms. By increasing B from the zero value the atoms are set into directed motion, and a maximum for the CM-velocity is reached for $B \simeq 0.5$, i.e. for about equal amplitudes of the even and odd harmonics. This demonstrates that DD is determined by the breaking of the symmetry $F(t + T/2) = -F(t)$.

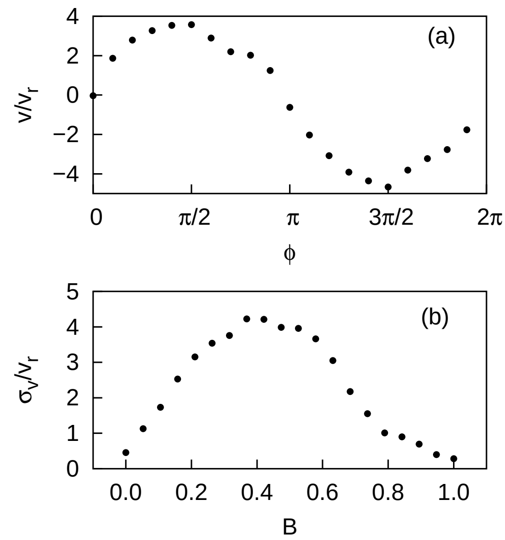

FIG. 4: Results of semiclassical Monte Carlo simulations for the atomic dynamics in the 1D-lin⊥lin optical lattice. The phase-modulation $\alpha(t)$ has the form of Eq. (3), with $A = 1 - B$, $\omega = 0.87\Omega_v$. The lattice parameters are: light shift per beam $\Delta_0' = -150\omega_r$ and lattice detuning $\Delta = -5\Gamma$. Here Γ and ω_r are the width of the excited state and the atomic recoil frequency, respectively. In (a) the CM-velocity in units of the atomic recoil velocity (v_r), is plotted as a function of the phase ϕ, for $\alpha_0 = 8$ and $B = 1/2$. In (b) the amplitude σ_v of the velocity-curve is plotted as a function of the amplitude B of the component at 2ω of the driving force, for constant sum of the amplitudes of the two harmonics at ω and 2ω. Here $\alpha_0 = 3$.

The microscopic mechanism producing a nonzero current of atoms through the optical lattice can be related to the general mechanism of current rectification following harmonic mixing firstly evoked to explain the electronic transport properties of crystals [16], and recently reexamined (Ref. [10] and references therein). For the specific system considered in the present work, the harmonic mixing results in a displacement Δz of the center of oscillation $\langle z(t)\rangle$ of the atoms in a potential well from the well center. Such a displacement originates from the anharmonicity of the potential, and it is quadratic in the amplitude of the field at frequency ω and linear in the amplitude at 2ω: $\Delta z \propto A^2 B$. Therefore a nonzero Δz is obtained only when both components of the force are applied. As the optical pumping rate Γ' (escape rate) toward neighbouring wells increases with the distance from the well center ($\Gamma' \propto \sin^2 k\Delta z$, see Ref. [11]), such a displacement results in an asymmetry between the escape rates toward the left and right wells, and a nonzero current of atoms is produced.

Our experimental observations are supported by semiclassical Monte Carlo simulations for a $J_g = 1/2 \rightarrow J_e = 3/2$ atomic transition. We examined the atomic dynamics in the 1D-lin⊥lin optical lattice for a phase modulation of one of the lattice beams of the form (3). For given amplitudes of the even and odd harmonics, we calculate the CM-velocity as a function of the phase ϕ, with results as in Fig. 4(a). The data are in complete agreement with the experimental findings, and confirm that the cloud of atoms is set into directed motion whenever the temporal symmetry $F(t) = F(-t)$ is broken.

The amplitude of the velocity-curves as the one of Fig. 4(a) has been characterized by the quantity $\sigma_v = [(\sum_{i=1,N} v_{\phi_i}^2 - \langle v \rangle)/N]^{1/2}$ where v_{ϕ_i}, with $i = 1, ..., N$ are the numerical results for the CM-velocity at the phase $\phi = \phi_i$. By plotting (see Fig. 4(b)) the quantity σ_v as a function of the amplitude B of the component at 2ω of the driving force, for constant sum of the amplitudes of the two harmonics at ω and 2ω we recover the behaviour observed in the experiment: a non-zero value of the amplitude B corresponds to the breaking of the $F(t+T/2) = -F(t)$ symmetry, and leads to the directed motion of the atoms.

In conclusion, in this work we demonstrated experimentally the phenomenon of directed diffusion in a symmetric periodic potential. This has been demonstrated with cold atoms in a periodic optical lattice. The same sort of behaviour was previously obtained in an asymmetric periodic potential (ratchet) [17]. The symmetric periodic potential corresponds to a 1D-lin⊥lin optical lattice. Two counterpropagating laser fields produce both the periodic potential and a friction force for the atoms. Furthermore the stochastic process of optical pumping leads to a diffusive dynamics of the atoms through the periodic structure. A force of zero average is applied by phase-modulating one of the lattice fields. Indeed, in an accelerated frame the atoms see a static symmetric periodic potential and an inertial force which breaks the temporal symmetry of the system. The degree of temporal symmetry-breaking of the system can be carefully controlled by varying the parameters of the phase modulation determining the force in the noninertial reference frame. We demonstrated that the atoms can be set into directed motion by breaking the temporal symmetry of the system.

The present realization of directed diffusion has been obtained in the regime of *non-adiabatic* driving, i.e. for a driving force of about the same frequency of the oscillations of the atoms at the bottom of the potential wells. This qualifies our system as testing ground for the recent theory of resonant activation based on logarithmic susceptibilities [8, 18].

Laboratoire Kastler Brossel is an "unité mixte de recherche de l'Ecole Normale Supérieure et de l'Université Pierre et Marie Curie associée au Centre National de la Recherche Scientifique (CNRS)".

[1] R. Brown, Phil. Mag. **4**, 161 (1828).
[2] A. Einstein, Ann. Phys. **17**, 549 (1905).
[3] F. Jülicher, A. Ajdari, and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997).
[4] M.O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993).
[5] A. Ajdari, D. Mukamel, L. Peliti and J. Prost, J. Phys. I (France) **4**, 1551 (1994).
[6] R. Bartussek, P. Hänggi and J.G. Kissner, Europhys. Lett. **28**, 459 (1994).
[7] For a recent review see P. Reimann, Phys. Rep. **361**, 57 (2002).
[8] M.I. Dykman, H. Rabitz, V.N. Smelyanskiy and B.E. Vugmeister, Phys. Rev. Lett. **79**, 1178 (1997).
[9] S. Flach, O. Yevtushenko and Y. Zolotaryuk, Phys. Rev. Lett. **84**, 2358 (2000).
[10] S. Denisov, S. Flach, A.A. Ovchinnikov, O. Yevtushenko and Y. Zolotaryuk, Phys. Rev. E **66**, 41104 (2002).
[11] G. Grynberg and C. Mennerat-Robilliard, Phys. Rep. **355**, 335 (2001).
[12] F.-R. Carminati, M. Schiavoni, L. Sanchez-Palencia, F. Renzoni and G. Grynberg, Eur. Phys. J. D **17**, 249 (2001).
[13] L. Sanchez-Palencia, P. Horak and G. Grynberg, Eur. Phys. J. D **18**, 353 (2002).
[14] L.D. Landau and E.M. Lifshitz, *Mechanics*, (Pergamon Press, Oxford, 1976).
[15] Accelerated optical potentials have been previously used to observe Bloch oscillations and Wannier-Stark ladders in optical lattices. See M. Ben Dahan, E. Peik, J. Reichel, Y. Castin and C. Salomon, Phys. Rev. Lett. **76**, 4508 (1996); S.R. Wilkinson, C.F. Bharucha, K.W. Madison, Q. Niu and M.G. Raizen, Phys. Rev. Lett. **76**, 4512 (1996); O. Morsch, J.H. Müller, M. Cristiani, D. Ciampini and E. Arimondo, Phys. Rev. Lett. **87**, 140402 (2001).
[16] K. Seeger and W. Maurer, Solid State Commun. **27**, 603 (1978).
[17] C. Mennerat-Robilliard, D. Lucas, S. Guibal, J. Tabosa, C. Jurczak, J.-Y. Courtois and G. Grynberg, Phys. Rev. Lett. **82**, 851 (1999).
[18] V.N. Smelyanskiy, M.I. Dykman, H. Rabitz and B.E. Vugmeister, Phys. Rev. Lett. **79**, 3113 (1997).